

Dynamics of microcavities induced by extremely short pulses in a three-level medium

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Based on the numerical solution of the system of equations for the density matrix together with the wave equation for the electric field strength, the dynamics of atomic populations and polarization of the medium during collisions of unipolar extremely short pulses of half-cycle pulses in a three-level medium is studied. The possibility of ultrafast switching of the state of the medium and the possibility of guiding optical microresonators at each resonant transition of the medium are shown. The considered multilevel system is an example of a non-stationary spatially inhomogeneous medium, the optical characteristics of which quickly change both in space and in time under the action of extremely short pulses.

Keywords: non-stationary media, extremely short pulses, attosecond pulses.

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Introduction

Electromagnetic pulses of an attosecond duration are obtained experimentally by method of generation and summation of the high-order harmonics [1–3]. Attosecond pulses are of high interest because of their use in the study and control of electrons motion in a matter [4–7], which has been a subject for the Nobel award in physics [8] recently and emphasizes the relevancy of studies in this area. But the pulses obtained in practice have several half-waves with the electric field strength of the opposite polarity. [1–4]. Obviously, for the study and faster control of quantum systems and for other applications of ultrafast optics, pulses of extremely short duration are required — unipolar or half-cycle [9–15] pulses. They can be obtained if the initial multi-cycle pulse is cleared from all half-waves except one. An important characteristic of the pulses of this kind is their electrical area, which is defined at a given point in space as an integral of the field strength \mathbf{E} over time t [9,14]

$$S_E(r) = \int \mathbf{E}(\mathbf{r}, t) dt. \quad (1)$$

Obviously, for ordinary multi-cycle pulses, this value is always close to zero and therefore may be neglected. The electrical area is a critical physical quantity, thus, the effect of half-cycle pulses on microobjects is determined precisely by their electric area, related to its atomic scale, when the pulse duration is shorter than the period of electron rotation in the atom. [15–20]. Therefore, it is of practical interest to obtain half-cycle pulses with a large electrical area. The results of recent studies in generation and application of

unipolar pulses for analysis of superfast processes dynamics in a matter are provided in papers [21–25] and referenced literature, as well as in the chapter of the monograph [26].

In the last decades non-steady media have been extensively studied [27–31] (which, for instance, may include metamaterials, conducting zinc oxides [32–34]), the optical properties of which (dielectric permittivity) may rapidly vary with time, e.g., under the action of incident radiation pulses. With a fast change in the refractive index in such media, the nature of refraction and reflectance of the sample radiation propagating in them changes significantly [35,36]. At that, optical characteristics may change not only with time, but also in space [36]. Such non-steady and spatially inhomogeneous media may have a slew of interesting applications, e.g., ultra-fast computations [27–31], creation of a non-threshold laser source [37] and etc. The results of the studies in this field are summarized in papers [27–31,36].

Another simple example of an unsteady spatially inhomogeneous medium is an ordinary resonant medium (for example, consisting of atoms, molecules, and quantum dots) through which a sequence of extremely short pulses propagates, coherently interacting with the medium. [38–40]. Coherent interaction occurs when the pulse duration and time intervals between them are shorter than the relaxation time of the medium polarization T_2 [41]. The propagating pulse leaves behind fluctuations in the polarization of the medium that exist over time. T_2 [42]. At the same time, each subsequent pulse will interact with these fluctuations, either swinging or extinguishing them. This will lead to the formation of spatial gratings of atomic populations that can

be induced and ultrafastly controlled using a sequence of extremely short pulses that do not overlap in the medium, see papers [38–40] and [43,44].

If extremely short pulses collide in a medium, then the population difference in the overlap region may have a near constant value. While outside of the overlap region this difference changes stepwise — it has either constant value, or the population difference occurs in the medium due to the coherent mechanism described above. This means that a dynamic microcavity [45–47] appears in the medium; the properties of such microcavity can be controlled with an increase in the number of collisions between pulses. The analytical theory of how such a microcavity can be formed, discussed in [48–50], is based on a number of assumptions, for example, a low-field approximation and a rare-field medium. Numerical calculations were carried out mainly when the medium was simulated in a two-level approximation [45–47,50]. The results of the studies are summarized in paper [51].

To date, rapid changes in the refractive index of unstable media (such as metamaterials and conductive media based on zinc oxide, etc.) are realized using long (multi-cycle) femtosecond pulses of laser radiation. [28–34]. The use of half-cycle pulses evidently makes it possible to control the media properties much faster, which can be used to implement the mechanism of the attosecond optical switching. [52]. Given the growing interest to the optics of non-steady and spatially inhomogeneous media, as well as half-cycle unipolar pulses, this study is focused on the dynamics of polarization and population differences in the collision of half-cycle attosecond pulses in a three-level medium based on the numerical solution of a system of equations for a density matrix, together with the wave equation for electric field strength. The possibility of forming microcavities with controlled parameters at each resonant transition of the medium is shown, which expands the applicability of the previously obtained results in a two-level approximation [45–47,50].

Theoretical model and results of numerical modelling

To carry out numerical calculations, the following system of equations was used for the density matrix of a three-level medium, which was solved numerically in conjunction with the wave equation for the electric field strength [53]:

$$\begin{aligned} \frac{\partial}{\partial t} \rho_{21} &= -\rho_{21}/T_{21} - i\omega_{12}\rho_{21} \\ &- i \frac{d_{12}}{\hbar} E(\rho_{22} - \rho_{11}) - i \frac{d_{13}}{\hbar} E\rho_{23} + i \frac{d_{23}}{\hbar} E\rho_{31}, \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial}{\partial t} \rho_{32} &= -\rho_{32}/T_{32} - i\omega_{32}\rho_{32} \\ &- i \frac{d_{23}}{\hbar} E(\rho_{33} - \rho_{22}) - i \frac{d_{12}}{\hbar} E\rho_{31} + i \frac{d_{13}}{\hbar} E\rho_{21}, \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{\partial}{\partial t} \rho_{31} &= -\rho_{31}/T_{31} - i\omega_{31}\rho_{31} \\ &- i \frac{d_{13}}{\hbar} E(\rho_{33} - \rho_{11}) - i \frac{d_{12}}{\hbar} E\rho_{32} + i \frac{d_{23}}{\hbar} E\rho_{21}, \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{\partial}{\partial t} \rho_{11} &= \frac{\rho_{22}}{T_{22}} + \frac{\rho_{33}}{T_{33}} + i \frac{d_{12}}{\hbar} E(\rho_{21} - \rho_{21}^*) \\ &- i \frac{d_{13}}{\hbar} E(\rho_{13} - \rho_{13}^*), \end{aligned} \quad (5)$$

$$\begin{aligned} \frac{\partial}{\partial t} \rho_{22} &= -\rho_{22}/T_{22} - i \frac{d_{12}}{\hbar} E(\rho_{21} - \rho_{21}^*) \\ &- i \frac{d_{23}}{\hbar} E(\rho_{23} - \rho_{23}^*), \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{\partial}{\partial t} \rho_{33} &= -\frac{\rho_{33}}{T_{33}} + i \frac{d_{13}}{\hbar} E(\rho_{13} - \rho_{13}^*) + i \frac{d_{23}}{\hbar} E(\rho_{23} - \rho_{23}^*), \end{aligned} \quad (7)$$

$$P(z, t) = 2N_0 d_{12} \text{Re} \rho_{12}(z, t) + 2N_0 d_{13} \text{Re} \rho_{13}(z, t), \quad (8)$$

$$\frac{\partial^2 E(z, t)}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E(z, t)}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial^2 P(z, t)}{\partial t^2}. \quad (9)$$

In this system of equations the variables have the following meaning: $\rho_{11}, \rho_{22}, \rho_{33}$ — variables for the 1-st, 2-d and 3-d states of the medium, respectively, $\rho_{21}, \rho_{32}, \rho_{31}$ — off-diagonal variables of the density matrix elements influencing the media polarization dynamics, $\omega_{12}, \omega_{32}, \omega_{31}$ — resonance transition frequencies, \hbar — reduced Planck constant, and d_{12}, d_{13}, d_{23} — dipole moments of transition. The equations also contain relaxation time T_{ik} , which is insignificant in the analysis of the processes under consideration, since pulse durations and intervals between them were chosen much shorter than this time.

The medium was excited by a sequence of the counter-moving unipolar pulses of attosecond duration, which had a Gaussian shape:

$$E_1(z = 0, t) = E_{01} e^{-\frac{(t-\Delta_1)^2}{\tau^2}}, \quad (10)$$

$$E_2(z = L, t) = E_{02} e^{-\frac{(t-\Delta_2)^2}{\tau^2}}. \quad (11)$$

Here $\Delta_1 = \Delta_2 = 2.5\tau$ — delays chosen so that the pulses are overlapped in the center of the medium. Zero boundary conditions were selected at the boundaries of the integration region. Propagation of unipolar pulses delineated by a one-dimensional wave equation can be realized in coaxial waveguides [54].

The influence of the trailing edge of the pulse of the opposite polarity can be ignored if its duration is longer and the amplitude is weaker than that of the main half-wave. [55]. Therefore, the trailing edge of a pulse in this study was neglected. The results of numerical solving the Schrödinger time equation also showed that medium ionization taken into account does not lead to disappearance of the population gratings [56]. All this makes the approximations used in the paper (neglecting the pulse trailing edge of the opposite polarity and medium ionization) justified to some extent.

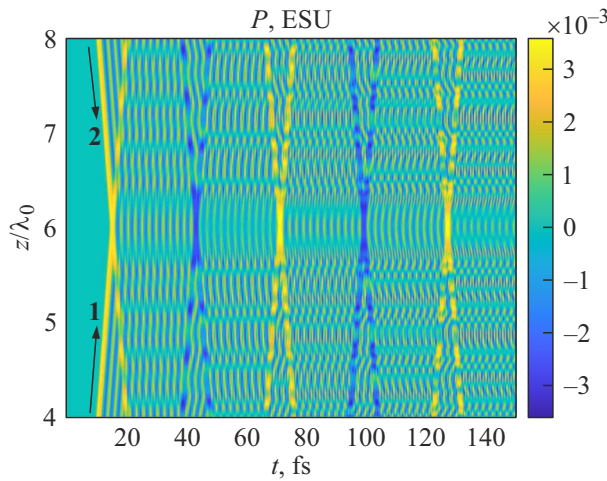


Figure 1. Spatiotemporal dynamics of polarization of a three-level medium $P(z, t)$.

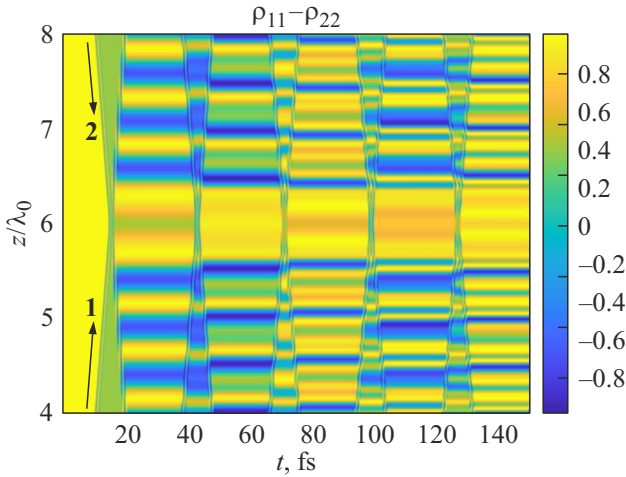


Figure 2. Spatiotemporal dynamics of population difference $\rho_{11} - \rho_{22}$ of a three-level medium.

Parameters of numerical computations were as follows. Pulse amplitudes $E_{01} = E_{02} = 2 \cdot 10^5$ ESU, pulse duration $\tau = 580$ as. Parameters of a three-level medium $\omega_{12} = 2.69 \cdot 10^{15}$ rad/s ($\lambda_{12} = \lambda_0 = 700$ nm), $d_{12} = 20D$, $\omega_{13} = 2.5\omega_{12}$, $d_{23} = 0$, $\omega_{23} = \omega_{13} - \omega_{12}$, $d_{13} = 1.5d_{12}$, $N_0 = 10^{14}$ cm $^{-3}$, relaxation time $T_{1k} = 1$ ns. Excitation pulses (10), (11) propagated towards each other and collided in the center of the three-level medium at a point $z_c = 6\lambda_0$.

The results of numerical computation for the selected parameters are shown in Fig. 1–4, where the spatiotemporal dynamics of the medium polarization and population differences for each medium transition are demonstrated. Direction of propagation of the first two pulses 1 and 2 is shown by arrows. The first collision occurs at a moment of time $t = 15.2$ fs, the second collision — at a moment 42.3 fs, third collision — at 71.7 fs and etc. The systems dynamics after 5 pulse collisions is illustrated in these Figures in

whole. From Fig. 2–4 we may see that after every collision near the pulse overlapping region ($z_c \sim 6\lambda_0$) the population difference has practically constant value, different for each resonance transition. Population difference gratings appear at the edges of this region, i.e., at each resonant transition of the medium, a microcavity with Bragg-like mirrors appears. The shape of these gratings is harmonic only after the first collision between the pulses.

With every further collision their shape becomes a bit more complicated. A more complex dynamics of gratings was also observed in the case of collision of unusually shaped pulses (rectangular and triangular) in the three-level medium. [49]. Physical mechanism of these structures formation was outlined earlier. It is related to the interaction of medium polarization fluctuations induced by each previous pulse with each following pulse [38–47]. Obviously, this mechanism differs from the traditional method of grating formation based on the interference of

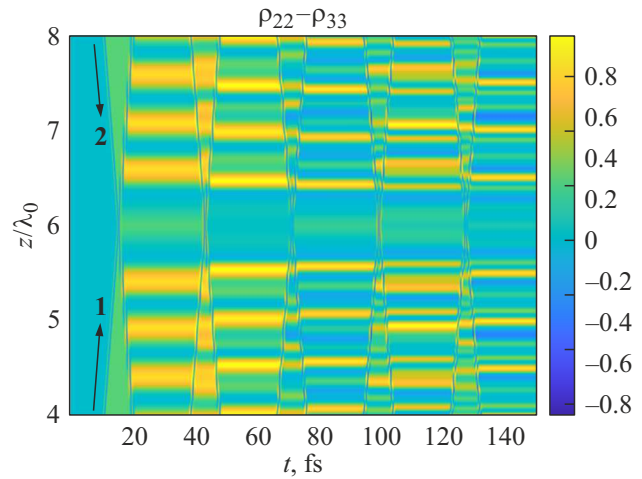


Figure 3. Spatiotemporal dynamics of population difference $\rho_{22} - \rho_{33}$ of a three-level medium.

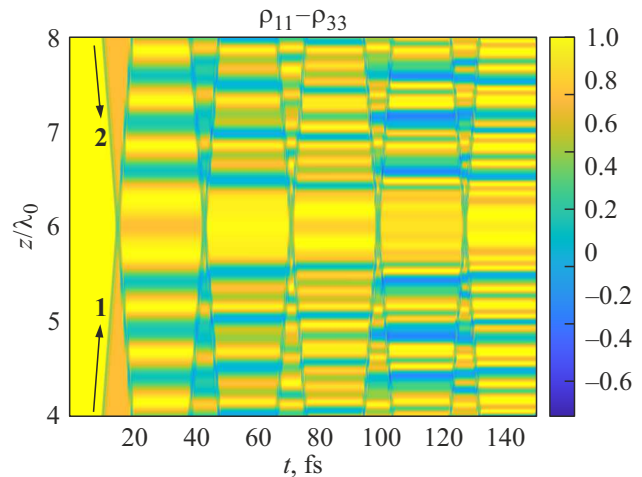


Figure 4. Spatiotemporal dynamics of population difference $\rho_{11} - \rho_{33}$ of a three-level medium.

long multi-cycle monochromatic light beams overlapping in the medium. [57–59].

Thus, formation of microcavities, previously analyzed numerically [45–50] in a two-level medium and predicted theoretically in the approximation of a low-field and a rare-field medium, persists in the three-level medium. From Fig. 1–4 we also see that the medium properties can be controlled both, in space and in time when the half-cycle pulses collide. The shape of the light-induced can be controlled by changing the number of collisions between pulses.

Conclusion

In the study numerical computation is used to demonstrate the possibility of inducing and ultrafast control of the properties of dynamic microcavities with Bragg-like mirrors at each resonant transition of a three-level medium during collision of the half-cycle pulses. This expands the field of applicability of early studies performed in approximations of a two-level model, low-field and etc. [45–50]. The considered three-level medium is an example of a spatially inhomogeneous and unstable medium, the properties of which may change rapidly both in space and in time under the action of the half-cycle pulses. At the same time, half-cycle pulses can change the properties of the medium much faster than conventional multi-cycle femtosecond laser pulses. The obtained results clearly demonstrate the possibility of an attosecond optical switching by means of the unipolar pulses. The results obtained open up new research horizons in optics of the non-steady media and attosecond physics. The studied dynamic microcavities may also be of interest for creating optical memory systems based on atomic coherence fabricated using half-cycle pulses, in contrast to the earlier studied schemes that use multi-cycle pulses [60–63].

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Conflict of interest

The authors declare that they have no conflict of interest.

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