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Optical microcavities created by unipolar light pulses in a medium (review)

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The results of the authors' recent work are discussed, which for the first time demonstrates the possibility of forming and ultrafast controlling optical microcavities formed by a single-moment overlap of unipolar pulses in a resonant medium. The possibility of implementing attosecond switching of medium states using half-cycle pulses is discussed, i.e. the possibility of rapidly changing the optical properties of the medium in space and time.

Keywords: extremely short pulses, attosecond pulses, optical microcavities, spatiotemporal photonic crystals, attosecond optical switching.

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1. Introduction

Obtaining ultrashort electromagnetic pulses of up to an attosecond duration in recent decades has become an essential topic of modern optics [1–5]. Attosecond pulses are widely used for the study and control of the electrons dynamics in a matter [6–9]. The relevancy of the studies in this area was proved by a awarding the Nobel prize in physics [10]. Traditional pulse duration reduction methods used in lasers with passive mode-locking and high-order harmonic generation systems make it possible to obtain bipolar pulses [1–9]. They contain several field half-waves of the opposite polarity. Extremely short pulses in a given spectral range are obtained if all half-waves are removed from a multi-cycle pulse, except one. Such pulses are called unipolar or half-cycle, see [11–19], papers [20-27] and chapter in the multi-author monograph [28].

For the unipolar pulses critical physical value is their electric area $\mathbf{S}_E$, defined as an integral of electric field strength over time $\mathbf{E}(\mathbf{r}, t)$ at a given point in space [29–31],

$$\mathbf{S}_E = \int \mathbf{E}(\mathbf{r}, t)dt. \tag{1}$$

For conventional multicycle pulses, the electric area is always close to zero, and, hence, may be ignored. To date, the issue of existence, acceptance and effect of unipolar pulses on microobjects has been well studied, these issues are outlined in papers [20–27] and in the monograph [28]. Half-cycle pulses are capable of rapid and unidirectional excitation of electrons in atoms, molecules, and nanoscale structures, which makes them promising for ultrafast control of the quantum systems behavior [32–39], ultra-high time resolution holography [40], and other interesting applications [22–28,41].

In parallel with the interest in unipolar pulses in recent decades, the so-called non-steady media with optical characteristics (for example, dielectric constant) varying over time, have attracted much interest in optics [42–45]. The use of half-cycle pulses to control the properties of such media, as outlined below, has only recently been discussed. If changes in the optical properties of a medium occur rapidly and periodically over time, then such media are called temporary photonic crystals (TPC) [46]. If changes in the optical parameters of a medium occur in space and with time, these media are called spatiotemporal photonic crystals (STPC) [47]. One the one hand, the interest to such media is basically of academic nature. Thus, a rapid change in the optical properties of the medium can lead to a change in the reflectance and refraction propagating in such test radiation media [48–50]. On the other hand, such media can have many applications, for example, intensification of light [51], creating a threshold-free laser source [52] and etc.

The urgent issue is to find suitable media where such rapid changes in the refractive index can be implemented. Today, conducting zinc-tin oxides are used as such media [49,53–56]. Such media near the point with zero permittivity are featuring high nonlinearity. However, they require the use of complex manufacturing technologies, so there is a continuous search for more accessible media for the implementation of TPC and STPC.

For various tasks of ultrafast optics, it is pivotal to find faster ways to change the properties of the medium. In recent years, ultrashort laser pulses (multi-cycle pulses with a duration of tens of femtoseconds) have been used to rapidly modulate the parameters of such non-steady media [49,54,55]. In this case, the switching of the medium properties occurs at a time of several femtoseconds. For a faster switching of the media properties the attosecond pulses may be used [57]. To implement ultrafast attosecond switching („attosecond optical switching"), as shown below, the half-cycle unipolar pulses can be used, since they have the shortest duration in a given spectral range. For example, when passing through a medium, a sequence of long multi-cycle [58–61] and extremely short pulses [62–74] can provide a distribution of quantum level populations of a medium in space according to a harmonic or other law (in this case, we usually talk about formation of a grating of atomic populations in the medium); this distribution can change with the passage of each subsequent pulse [63–65].

In the earlier studies [58-61] and cited literature the gratings were described as created by long multicycle resonance pulses that were not overlapped in the medium instantaneously. The gratings created this way were further applied in echo-holography [60,61]. Obviously, this approach has no prospects in ultrafast optics, since it does not allow fast changes in the state of the medium due to the long pulse duration and is not considered below. The gratings created in this case always have a harmonic shape and actually appeared at a given resonance transition of the medium, since the central frequency of the multi-cycle pulse coincided with the frequency of the given resonance transition.

Below, the new type of *inharmonious shaped structures* — dynamic microcavities (DM), which occur when half-cycle pulses overlap in the medium, will be discussed; this significantly distinguishes the tasks discussed below from those previously discussed in [58–61]. It is the extremely short single-cycle and subcycle pulses rather than long multi-cycle pulses that contribute to the emergence of such structures. Moreover, in our case, when unipolar pulses without a carrier frequency are used, the interaction with the medium is non-resonant, and the DM, as will be seen below, *emerges at each resonance transition of the multilevel medium*. Therefore, the case of multi-cycle pulses is not of interest to us and is not considered below.

The problems of obtaining half-cycle pulses have also led to the problem of controlling their temporal shape. The possibility of obtaining unipolar pulses of unusual shapes, such as square and triangular shape, was demonstrated [16,17,75–78]. The pulses of unusual shape may be used for the frequency-selective spectroscopy of quantum systems [74] and superfast control of quantum qubits [79,80]. Meanwhile, the issue of using such pulses for superfast control of the medium properties in space and time, according to our data, has not been raised or studied before. These issues have been investigated and reviewed in papers published in recent years. The results of recent articles that address these issues will be discussed later in this review.

Previously, the possibility of creating population difference gratings under the action of extremely short pulses in the medium was studied [62–68]. The results of these studies are summarized in the mini-review [73]. The induced gratings were, primarily, of a harmonic shape. To our knowledge, the possibility of obtaining other types of structures, including non-harmonic shaped unipolar pulses, has not been discussed or studied. And in the performed studies the medium was simulated in a two-level approximation. Other levels were taken into account for the case of non-overlapping pulses, in the approximation of low excitation, etc. [63–65]. A detailed study of the dynamics of such structures under strong excitation and taking into account the ionization of a multilevel medium was not considered.

When further studying the interaction of half-cycle pulses with a matter, a new phenomenon (that cannot be provided using conventional multi-cycle pulses) was predicted and studied — creation and control of dynamic microcavities (DM) [81–91], which occur when a pair of the half-cycle pulses simultaneously overlaps („collides") at some point inside the environment. In this case, the population difference at any resonance transition of the medium has an almost constant value within the pulse overlap region (with a size practically equivalent to the spatial size of the pulse). Outside of the overlap region, it has a different meaning or changes in space according to some law (a population grating appears). The populations difference between the media in the pulse overlap region and outside of it indicates the formation of a „microcavity". Parameters of this microcavity can be changed if the pulses re-enter the medium and collide in it. In this context, below we'll use the term „dynamic microcavity" (DM), taking into account possible fast change of its parameters under the action of half-cycle pulses.

DM occurs when a sequence of half-cycle pulses coherently interacts with the medium, i.e. the pulse duration and delays between them are shorter than the relaxation time of the polarization of the medium. T_2 [92-98]. An important issue is the applicability of the conventional two-level approximation of the medium and consideration of ionization in creating and controlling DM in coherent interaction of the half-cycle pulses with the medium discussed below. These issues have been studied recently in papers [86,89,95–97] and etc. in the studies described in details below.

This paper summarizes the latest results of the studies by authors [81–91], obtained by theoretical analysis and numerical modeling, where for the first time a new phenomenon was predicted and studied in detail - the possibility of creating and superfast control of dynamic microcavities. They emerge when the sequence of half-cycle attosecond pulses is instantaneously overlapped („collides") at some point of the resonance medium. The paper outlines the theory of formation of such microcavities and the recently proposed analytical approach jcite87,88,90, which predicts the possibility of their creation and superfast control of their properties with an increase in the number of collisions between pulses.

The results of numerical computations of DM for a three-level medium, the parameters of which have values similar to those for a hydrogen atom, are presented. Cases of strong and weak excitations are considered. A criterion has been established showing at which parameters of the excitation pulses the shapes of the observed structures will be distinct. Parameters of such microcavities are evaluated — Q factors and reflectivity of mirrors.

The considered phenomena of DM creation demonstrate one of the possibilities of the attosecond switching of medium properties using half-cycle pulses and unipolar pulses of unconventional shape (square and triangular), which has not been discussed in the literature so far. Also, when creating and controlling microcavities, there is a rapid change in the optical properties of the medium both, in space and with time. That is, such medium is a new type of the spatiotemporal photon crystal (STPC). Finding the right materials to create such media, as noted above, is an urgent problem. A pivotal issue of the applicability of a two-level approximation and the possibility of forming such structures when taking into account the ionization of the medium in such problems is discussed.

2. Formation of optical microcavities in the resonance medium under the action of half-cycle pulses

Let's consider the physical principle of DM formation influenced by the extremely short pulses that are instantaneously overlapped („collided") in the medium [82–92]. Passing through the resonant medium, the pulses leave behind polarization oscillations at a resonance transition frequency at each point of the medium's thin layer. These polarization oscillations propagate following the pulses. The polarization waves are emerged [61–63], running towards each other following the incident pulses. Thus, the pulses, propagating towards each other, collide in the overlap region and, passing on, interact with a polarization wave induced by another pulse. At the same time, in some areas of the medium, these polarization oscillations will be dampened by the incident pulse, while in others, on the contrary, they will be enhanced. When such a passing pulse interacts with the polarization wave already existing in the medium outside the pulse overlap region, this results in formation of „Bragg" population difference gratings. Schematic representation of this process is given in Fig. 1.

At the same time, in collision area of the medium, the population difference has almost constant value; thus, we may say that a microcavity arises in the medium, the parameters of which can change with the passage of each subsequent pulse. In this context, as mentioned earlier, it can be said that there exists a DM induced and controlled by a sequence of the half-cycle pulses. The studies in this area begin with paper [81]. It illustrates unusual behavior

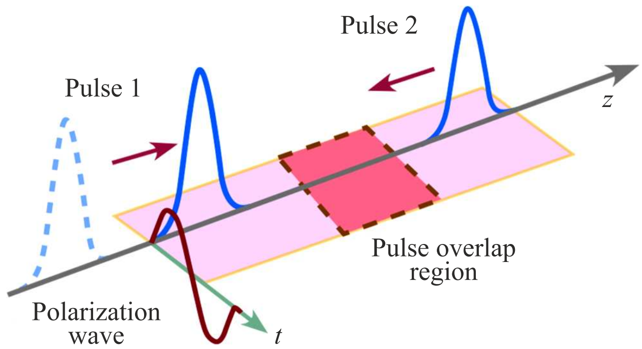

Figure 1. Interaction of the extended media with a pair of extremely short pulses propagating towards each other.

of polarization of a two-level medium during passage of a sequence of extremely short pulses overlapped in the medium.

In the present study, it is shown that narrow, consecutive sections with a size of less than the transition wavelength may occur in the medium, with harmonic polarization waves formed in these sections. These waves can run in opposite directions in different parts of the medium and are localized within these areas. At that, local „quasi-resonators" with a size of less than the medium transition wavelength for the polarization waves emerge in the medium. These results are of interest, not only because of the unconventional nature of the phenomenon itself, they also clearly illustrate the possibility of both, switching the properties of the medium and controlling its radiation, which can be achieved by changing the direction of movement of the polarization wave. [81].

However, in our opinion, DM based on changes in the atomic populations of the medium in space are of greater practical interest. In this case, this change may have an non-harmonic shape if the incident unipolar pulses have an unconventional shape, e.g., square shape. As to out knowledge, the possibility of creating microcavities during collision of unipolar pulses in a medium was first theoretically shown in paper [82]. In this study, the dynamics of polarization and population differences in the collision of unipolar pulses were studied.

The pulses had a non-harmonic temporal shape (square) and overlapped in a two-level medium. The first pulse acted as a $\pi/2$ pulse, i.e. it transferred the medium to a state with zero population difference. Computation demonstrated unconventional dynamics of the system: in the pulse overlap region, the population difference was almost constant, and on the edges it either changed periodically in space (in this case, a periodic grating of population differences emerged), or had a value different from its value in the pulse overlap region. In the first case, the dynamic microcavity with Bragg-like mirrors emerged. In the second case, a quasi-resonator of a different type also appeared due to an abrupt change of refractive index between the pulse overlap region and the regions of the medium located outside this region.

In further study [83] the dynamics of DM during collision of square pulses acting like 2π-pulses of self-induced transparency (SIT) [98] in a two-level medium was investigated. At that, during the pulse action, its first half transfers the medium from the ground state to the excited one, and the second half returns the medium to the ground state. Numerical computations demonstrated that population difference has an almost constant value in the pulse overlap region (less than the wavelength of the medium transition) and changes abruptly at the boundary of the overlap region. Obviously, the formation of a DM also occurs here. At that, the DM parameters can be controlled by increasing the number of collisions between the pulses — it can be activated and deactivated, its spatial characteristics may be changed. The estimates have shown that Q-factor of such microcavities can reach about 1000 in a dense medium.

An analytical approach allowing to calculate the dynamics of such a microcavity in a two-level medium under the action of square pulses was proposed in the study jcite84. This approach is considered somewhat less trustworthy because it is impossible to predict the behavior of polarization and population differences on the pulse edges based on the pulses duration and steepness. Numerical computations were carried out to study the role of the pulse edges in the study.

In the first mentioned studies [82–84] the microcavities' dynamics was investigated using square pulses and the medium was simulated in a two-level approximation. The dynamics of microcavities in the collision of half-cycle weak Gaussian pulses in a two-level medium was studied in more detail in paper [85]. The possibility of forming microcavities with Bragg-like mirrors in the form of population difference gratings has been shown. This study also demonstrates that microcavities may be formed and controlled during collision of Gaussian SIT pulses in a three-level medium.

The possibility of inducing of dynamic resonators in the collision of single-cycle SIT pulses in a two-level medium was theoretically studied in paper [86]. In this study, in contrast to those mentioned above, microcavity and „mirrors" appeared only in the pulse overlap region — the population difference in this region had a constant value, and at the edges the population difference grating contained a small number of periods, much shorter than the wavelength of the medium transition. With the growing number of collisions between the pulses the amount of periods of this structure increased.

The study [87] described the dynamics of DM in collision of pulses that have a non-conventional shape — square and triangular — by analysis for a multi-level medium and based on numerical computations for a three-level medium. As the number of collisions between pulses increased, the shape of these microcavities was greatly distorted. The formation of microcavities in the collision of Gaussian pulses in a three-level medium was studied in article [89]. Whereas no SIT pulses were used in the numerical computations. In this study, it is shown that the investigated three-level medium can be a STPC, since when pulses collide, the parameters of the medium change rapidly both, in space and in time.

Thus, the results of numerical computations performed for the two-level and three-level media clearly demonstrated that its is possible to create DMs, the parameters of which can be controlled with an increase in the number of collisions between pulses. These studies have set the task of an analytical description of such structures. This approach was offered for the first time in [87,88] and discussed in detail in [91]. The fundamentals of this approach and conclusions made are discussed in the next section.

3. Analytical theory of microcavities formation

Let's briefly discuss the analytical approach suggested earlier [87,88,91]. Considering the medium to be sparse, which makes it possible to neglect the change in pulses shape during propagation, to calculate the population of bound states, a well-known expression for an approximate solution of Schrodinger equation in the first order of perturbation theory may be used [99]:

$$w_{1k} = \frac{d_{1k}^2}{\hbar^2}\left|\int E(t)e^{i\omega_{1k}t}dt\right|^2. \tag{2}$$

Here ω_{1k} — frequency of the medium transition from its ground state to level k, d_{1k} — dipole moment of this transition. It should be emphasized that w_{1k} means the k- level of population when only the ground state is initially populated.

Then, the effect of a sequence of pulses on an extended medium is reduced to the effect of a pair of pulses with a variable delay Δ per single atom [62–65,73]. In case of an extended medium the delay $\Delta \sim 2z/c$ (for a pair of pulses running towards each other) defines the difference between the moments when pulses come into a given point of the medium with a coordinate z. Consequently, the expressions obtained describe the distribution of populations at each point in the medium.

Let the system be effected by a pair of half-cycle pulses having a Gaussian shape, the expressions for which are written as

$$E(t) = E_1 \exp\lfloor -t^2/\tau_1^2 \rfloor + E_2 \exp\lfloor -(t-\Delta)^2/\tau_2^2 \rfloor$$

and electric areas of which are given as $S_{E,1,2} = E_{1,2}\tau_{1,2}\sqrt{\pi}$ (from this point onward τ_i — duration of each pulse, and E_i — amplitude, $i = 1, 2$). The delay between pulses is denoted as Δ. Then, using (2) we may obtain an expression for the states population [73,74,78]:

$$w_{1k} = \frac{d_{1k}^2}{\hbar^2} S_{E1}^2 \exp\left[-\frac{\omega_{1k}^2\tau_1^2}{2}\right] + \frac{d_{1k}^2}{\hbar^2} S_{E2}^2 \exp\left[-\frac{\omega_{1k}^2\tau_2^2}{2}\right]$$
$$+ 2\frac{d_{1k}^2}{\hbar^2} S_{E1}S_{E2} \exp[-\omega_{1k}^2(\tau_1^2+\tau_2^2)/4]\cos(\omega_{1k}\Delta). \tag{3}$$

This expression is similar to the corresponding formula for the total emission intensity during the interference of two monochromatic light waves [100]. In this context, with low fields, we can say that the effect of half-cycle pulses on a quantum system occurs due to interference of pulses' electric areas or interference of amplitudes of bound states [74,101,102] (the concept of area interference was first introduced in these studies).

To calculate the media bound states populations in the pulse overlap region we need to put a delay $\Delta = 0$ into expression (3) which results in the next simple expression for a population:

$$w_{1k} = \frac{d_{1k}^2}{\hbar^2}\left(S_{E1}\exp\left[-\frac{\omega_{1k}^2\tau_1^2}{4}\right] + S_{E2}\exp\left[-\frac{\omega_{1k}^2\tau_2^2}{4}\right]\right)^2. \tag{4}$$

Outside the pulse overlap region, the population is calculated using the expression (3). It shows formation of a harmonic grating of population differences on both sides of the pulse overlap region. Thus, a Bragg-mirror microcavity appears in the medium in the form of atomic populations gratings with every resonance transition of the multilevel medium.

An expression similar to (3) and (4) can also be obtained when a pair of unipolar pulses of non-conventional shape, e.g., square shape, impacts the multilevel medium. The expression for the field strength of pulses can be written as

$$E_1(t) = \begin{cases} E_{01}, & 0 < t < \tau_1, \\ 0, & \text{else}, \end{cases} \tag{5}$$

$$E_2(t) = \begin{cases} E_{02}, & \tau_1 + \Delta < t < \Delta + \tau_1 + \tau_2, \\ 0, & \text{else}. \end{cases} \tag{6}$$

By introducing an electric area of pulses $S_i = E_{0i}\tau_i$ and taking the case of identical duration pulses for convenience, $\tau_1 = \tau_2 = \tau$, we'll obtain the following expression for the states population [65,87]:

$$w_{ik} = 2\,\frac{d_{ik}^2}{\tau^2\omega_{ik}^2\hbar^2}\,(1 - \cos\omega_{ik}\tau) \times \left(S_1^2 + S_2^2 + 2S_1S_2\cos\omega_{ik}(\tau + \Delta)\right). \tag{7}$$

In this expression, the population of states is also determined by the sum of the squares of the electric areas and periodically depends on the delay between them, which is completely identical to the expression for the total intensity at the interference of two monochromatic light waves.

An analytical theory of microcavities formation, showing the possibility of their creation and ultrafast control when impacted by half-cycle pulses of Gaussian shape, was considered in details in the article [91]. The results of analysis of the quantum level populations performed in the low-field approximation in the first order of perturbation theory are in good consistence with numerical solution of the system of equations for the density array. In high fields, when the perturbation theory is not applicable, numerical computations of microcavities dynamics during collision of Gaussian SIT pulses in a two-level medium were carried out. The result significantly depends on the polarity of pulses: when the pulses of the same polarity collide, the medium is practically not excited and the gratings are not formed.

The situation changes dramatically when pulses of opposite polarity collide — a microcavity with Bragg-like mirrors appears in the medium only in the pulse overlap region. In the rest of the region, the medium remains unexcited, as it should be after the passage of the SIT pulses. With growth of collisions between the pulses the spatial frequency of these gratings increased.

Thus, the proposed analytical approach predicts an important result — formation of an optical microcavity with Bragg-like mirrors in the form of harmonic population gratings with every resonance transition of a multilevel medium. At that, the physical mechanism of DM formation in a low-field is based on the interference of electric areas of pulses. This result is universal, since no specific type of medium was described in this approach. It has a heuristic value. The results of numerical computations for a three-level medium, the parameters of which have values similar to those for a hydrogen atom, are presented below. However, the analytical approach is valid in low fields, and it neglects the polarization dynamics of the medium and ionization of the system. A more thorough investigation of DM requires numerical computations. This also raises the issue of validity of a two-level approximation in such problems, which is discussed below.

4. Validity of using the two-level approximation and allowing for ionization of the medium

It is worth noting that all of the above-mentioned concerning the possibility of forming a DM in a two-level medium is also true for the multi-level media. Let's discuss in detail the results of the latest research in this area. The study [95] was focused on dynamics of coherence (polarization) of the five-level resonance media under the action of a pair of half-cycle pulses with a delay between them. Additionally, we found that the response of the multilevel medium has good quantitative matching with the results for a two-level model when the amplitude of the exciting pulses is below a certain threshold.

A more precise quantitative criterion is that the electric area of the pulse should be less than the atomic measure of the system introduced in [36,37]. It is valid when the duration of the excitation pulses is shorter than the inverse frequencies of atomic transitions, which is true for the parameters used in numerical computations with results discussed below. This criterion is discussed in details below. This conclusion is proved by numerical solution of Schrödinger equation obtained allowing for the quantum well [103].

In particular, creation of atomic populations gratings in a three-level media by the non-overlapped pulses was described in papers [90,93,96]. The conclusion from the study jcite90 turned out to be interesting: it studied numerically the dynamics of population gratings under the action of half-cycle pulses that do not overlap simultaneously in a three-level medium. The parameters of the medium (transition frequencies, dipole moments) were the same as for a hydrogen atom. The system dynamics was demonstrated to be different at various transitions of the three-level media. The formation of quasi-harmonic population gratings was observed at one transition, and the formation of a DM similar to that described above was observed during other transition. Thus, DMs may emerge also in the multi-level media.

The effect of ionization was taken into account based on the numerical solution of temporal Schrödinger equation for a rectangular quantum well [97]. In this study the peculiarities of the non-linear interference of pulses' electric areas were examined. Numerical computations have demonstrated that the curve of the particle's bound state population in a quantum well versus change in the pulses delay has a characteristic form of „beatings" in case when the electric area of the pulse is comparable and larger than the atomic measure of the system. This result differs from simple harmonic dependence of populations on delay, see (3) and (7), as predicted by perturbation theory for a small field amplitude of exciting pulses. These results show the possibility of controlling the probability of particle ionization in a quantum well until its complete suppression at a certain delay between pulses. It is shown that DM and gratings of populations difference may also occur in a multi-level media, even taking into account ionization. [97].

Gratings are saved in a multilevel medium due to the fact that a short pulse passing through the medium always leaves behind the coherence oscillations of the medium with each resonance transition [61,90,91,93,96]. These oscillations exist for a period when the medium polarization is relaxed T_2. Each subsequent pulse coherently interacts with polarization oscillations, which leads to emergence of a microcavity populations grating at each medium transition. Thus, the use of a two-level model seems justified here.

It is also interesting to note the results of other recent studies confirming the presence of a number of coherent effects, such as Rabi oscillations (predicted earlier in two-level media) under the influence of attosecond pulses in multilevel media. Papers [104,105] outline the spectrum of high-order harmonics of helium atom excited by XUV-attosecond pulses. In this case the changes in the atomic populations of Rabi frequency were observed on of the resonance transitions. Experimentally, Rabi oscillations have been observed in semiconductors [106] and in a helium atom excited by femtosecond pulses generated by free electron lasers [107].

5. Numerical simulation results

In this section, we will briefly discuss the issue of formation and control of optical microcavities and „Bragg" population difference gratings. Let's consider two cases when the pulses may be considered „weak" and „strong" in relation to their effect on the studied three-level medium.

To demonstrate the dynamics of the population difference gratings and DMs emerging during collision of a sequence of pulses, the Maxwell-Bloch system of equations was numerically solved for the density matrix of a three-level medium, its polarization P, as well as the wave equation for the electric field E [108]:

$$\frac{\partial}{\partial t}\rho_{21} = -\rho_{21}/T_{21} - i\omega_{12}\rho_{21} - i\,\frac{d_{12}}{\hbar}\,E(\rho_{22}-\rho_{11}) - i\,\frac{d_{13}}{\hbar}\,E\rho_{23} + i\,\frac{d_{23}}{\hbar}\,E\rho_{31}, \tag{8}$$

$$\frac{\partial}{\partial t}\rho_{32} = -\rho_{32}/T_{32} - i\omega_{23}\rho_{32} - i\,\frac{d_{23}}{\hbar}\,E(\rho_{33}-\rho_{22}) - i\,\frac{d_{12}}{\hbar}\,E\rho_{31} + i\,\frac{d_{13}}{\hbar}\,E\rho_{21}, \tag{9}$$

$$\frac{\partial}{\partial t}\rho_{31} = -\rho_{31}/T_{31} - i\omega_{13}\rho_{31} - i\,\frac{d_{13}}{\hbar}\,E(\rho_{33}-\rho_{11}) - i\,\frac{d_{12}}{\hbar}\,E\rho_{32} + i\,\frac{d_{23}}{\hbar}\,E\rho_{21}, \tag{10}$$

$$\frac{\partial}{\partial t}\rho_{11} = \frac{\rho_{22}}{T_{22}} + \frac{\rho_{33}}{T_{33}} + i\,\frac{d_{12}}{\hbar}\,E(\rho_{21}-\rho_{21}^{*}) - i\,\frac{d_{13}}{\hbar}\,E(\rho_{13}-\rho_{13}^{*}), \tag{11}$$

$$\frac{\partial}{\partial t}\rho_{22} = -\rho_{22}/T_{22} - i\,\frac{d_{12}}{\hbar}\,E(\rho_{21}-\rho_{21}^{*}) - i\,\frac{d_{23}}{\hbar}\,E(\rho_{23}-\rho_{23}^{*}), \tag{12}$$

$$\frac{\partial}{\partial t}\rho_{33} = -\frac{\rho_{33}}{T_{33}} + i\,\frac{d_{13}}{\hbar}\,E(\rho_{13}-\rho_{13}^{*}) + i\,\frac{d_{23}}{\hbar}\,E(\rho_{23}-\rho_{23}^{*}), \tag{13}$$

$$P(z,t) = 2N_0 d_{12}\mathrm{Re}\rho_{12}(z,t) + 2N_0 d_{13}\mathrm{Re}\rho_{13}(z,t) + 2N_0 d_{23}\mathrm{Re}\rho_{32}(z,t), \tag{14}$$

$$\frac{\partial^2 E(z,t)}{\partial z^2} - \frac{1}{c^2}\,\frac{\partial^2 E(z,t)}{\partial t^2} = \frac{4\pi}{c^2}\,\frac{\partial^2 P(z,t)}{\partial t^2}. \tag{15}$$

In this system of equations, the parameter values have the following meaning: $\rho_{21}, \rho_{32}, \rho_{31}$ — off-diagonal elements of the density matrix associated with the polarization of the medium; $\rho_{11}, \rho_{22}, \rho_{33}$ — populations of the 1-st, 2-d and 3-d states of the medium, respectively, $\omega_{12}, \omega_{32}, \omega_{31}$ — frequencies of resonance transitions of the three-level medium, d_{12}, d_{13}, d_{23} — dipole moments. Equations (8)–(15) also contain relaxation members T_{ik}. System of equations (8)–(15) was solved numerically; equations of density matrix (8)–(13) were solved by the 4th order Runge-Kutta method, the wave equation (15) was solved by method of finite differences.

At the initial moment, a pair of unipolar pulses with a Gaussian shape were sent from the edges of the integration region into the medium:

$$E(z = 0, t) = E_{01} e^{-t^2/\tau^2}, \qquad (16)$$

$$E(z = L, t) = E_{02} e^{-t^2/\tau^2}. \qquad (17)$$

Total length of the calculated region was $L = 12\lambda_0$. The medium was located between the points$z_1 = 2\lambda_0$ and $-z_2 = 10\lambda_0$. The pulses reached the boundary of the integration region, reflected from it, and returned to the medium, each time overlapping in the center at point $z_c = 6\lambda_0$. A three-level medium modeled similar to the first three levels of hydrogen atom was considered. Parameters used in the numerical computation are given in the table. The model parameters were taken from [109].

In computations the unipolar pulses are considered which have only one half-wave of the field strength. The effect of the trailing edge of opposite polarity on the system as described in paper [78], in some cases may be neglected, and this issue is not discussed further.

The degree of excitation of the system bound states, depending on the parameters of the impacting pulses, can be described by comparing the electric area of the pulse S_E with the characteristic atomic scale of the system S_a (atomic measure of area) [36–39]. S_E is a quantitative measure of the incident pulse effect on the system; the values for the pulses parameters given in the table are $S_{E1} = 2.38 \cdot 10^{-8}$ V·s/cm and $S_{E2} = 1.19 \cdot 10^{-7}$ V·s/cm, respectively.

Let's estimate for the examined system (hydrogen atom) the characteristic sizes, having taken the radius of the first Bohr orbit as$a_B = 0.053$ nm. Using the atomic measure for electric area determined in [36] we'll obtain $S_a = 2\hbar/ea_B = 2.48 \cdot 10^{-7}$ V·s/cm. Thus, for the selected systems S_a relates to S_E as $S_a/S_{E1} \approx 10$ and $S_a/S_{E2} \approx 2$. Hence, the first case may be considered as „weak", and the second — as „strong" effect of pulses on the system.

The results of numerical modeling of system (8)–(15) are shown in Fig. 2 and 3 for pulses with amplitudes E_{01} and E_{02} respectively; other parameters are given in the table.

As can be seen in Figs. 2 and 3, in both cases, after the first pulse collision, the dependence of the populations difference for all transitions takes the form of Bragg gratings with a constant plateau in the overlap region (microcavity localized in the region $(5.7-6.3)\lambda_0$). Instantaneous values of the populations difference distribution after the first pulse collision ($t = 6$ fs) are shown in Fig. 4, *a* for the „weak" excitation case; in Fig. 2 and in Fig. 5, *b* for the „intense" excitation case (case in Fig. 3), respectively. As expected, the modulation depth of the gratings is inversely proportional to the ratio S_a/S_{E1} only with weak excitation.

It should be stressed, that only for„weak" excitation in Fig. 2 a distinct harmonic grating with a period of $\lambda_0/2$ may be observed (however, the beatings with a structure peculiar to the multi-level systems are observed); for strong excitation as seen from Fig. 4, *b*, the structures of the populations difference have a more complicated nature: the beatings are prevailing.

Parameters used in numerical computation

Frequency (wavelength λ_0) transition $1 \to 2$	$\omega_{12} = 1.55 \cdot 10^{16}$ rad/s ($\lambda_{12} = \lambda_0 = 121.6$ nm)
Dipole moment of transition $1 \to 2$	$d_{12} = 3.27$ D
Frequency (wavelength) transition $1 \to 3$	$\omega_{13} = 1.84 \cdot 10^{16}$ rad/s ($\lambda_{13} = 102.6$ nm)
Dipole moment of transition $1 \to 3$	$d_{13} = 1.31$ D
Frequency (wavelength) transition $2 \to 3$	$\omega_{23} = 2.87 \cdot 10^{15}$ rad/s ($\lambda_{23} = 656.6$ nm)
Dipole moment of transition $2 \to 3$	$d_{23} = 12.6$ D
Concentration of atoms	$N_0 = 1 \cdot 10^{14}$ cm^{-3}
Filed amplitude 1,2	$E_{01} = 3.2 \cdot 10^5$ V/cm $E_{02} = 1.6 \cdot 10^6$ V/cm
Parameter τ	$\tau = 140$ as

As seen in Fig. 2, *a–c*, with every following pulse the shape of the grating is varied, yet, its base period remains the same. In Fig. 5, *a* and *b* the instantaneous values of the populations difference after the fourth collision of pulses ($t = 20.5$ fs) are presented for „weak" and „strong" excitation, respectively. By comparing Fig. 4, *a* and 5, *a*, it is clearly seen that the microcavity in the center is deepened, and, simultaneously, the structure of the gratings on both of its sides becomes more complicated: thus, on the sides in the regions $(3.4-4)\lambda_0$ and $(8-8.6)\lambda_0$ the „channels", occur the depth of which is comparable with the depth of the microcavity.

At the same time, as can be seen in Fig. 3, *a–c*, although for the case of „strong" pulses we may almost immediately create the regions with an almost complete inversion of populations with every transition of the medium, the shape of the gratings on the sides degrade with the growing number of injected pulses. So, after the fourth collision, as can be seen in Fig. 5, *b*, we may talk neither about periodic structure, nor about preservation of the microcavity shape.

Therefore, in this section we've briefly summarized the two opposite scenarios of microcavities formation — with involvement of „weak" and „strong" pulses for a three-level medium with parameters similar to the hydrogen atom [109]. As can be seen, although for strong pulses (Fig. 3) we manage to create a microcavity and gratings with significant population inversion after the first collision, their structure and period are not preserved during further pulses injecting. At the same time, in case of „weak" pulses (Fig. 2), the inversion for each of the system transitions remains small even after repeated passage of pulses through

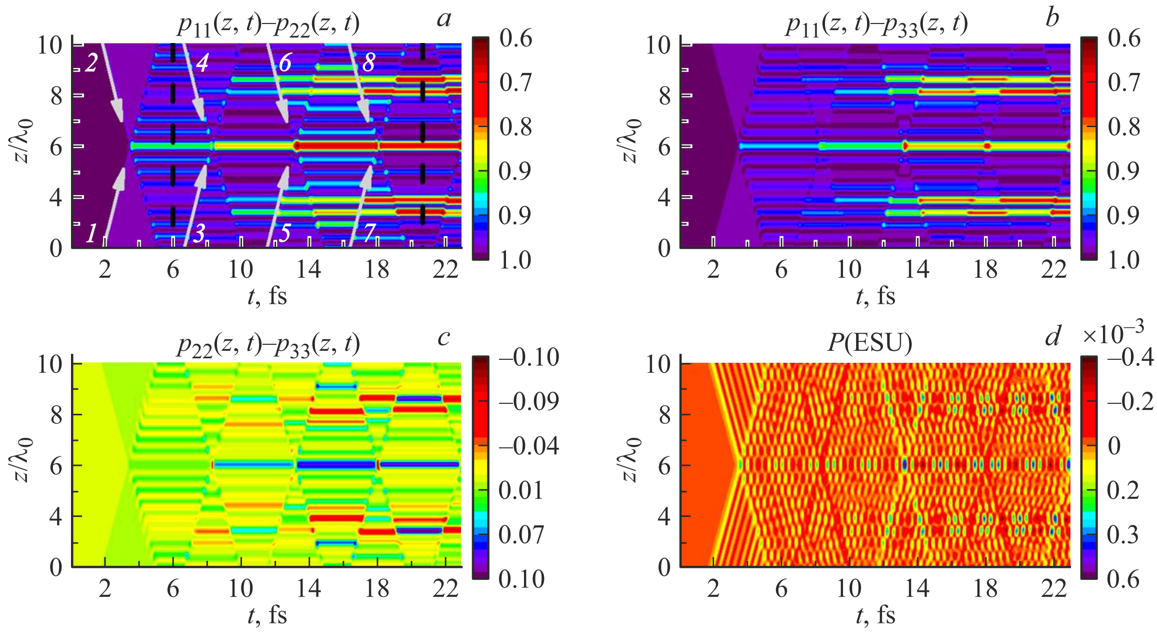

Figure 2. Dynamics of the populations difference: $\rho_{11}-\rho_{22}$ (a), $\rho_{11}-\rho_{33}$ (b), $\rho_{22}-\rho_{33}$ (c); d — polarization dynamics $P(z,t)$ when subcycle pulses of Gaussian shape pass through the medium, their numbers and propagation directions are shown by arrows with a text. The amplitude of pulses was $E_{01} = 3.2 \cdot 10^5$ V/cm, ratio $S_a/S_{E1} \approx 10$; other computation parameters are provided in the table.

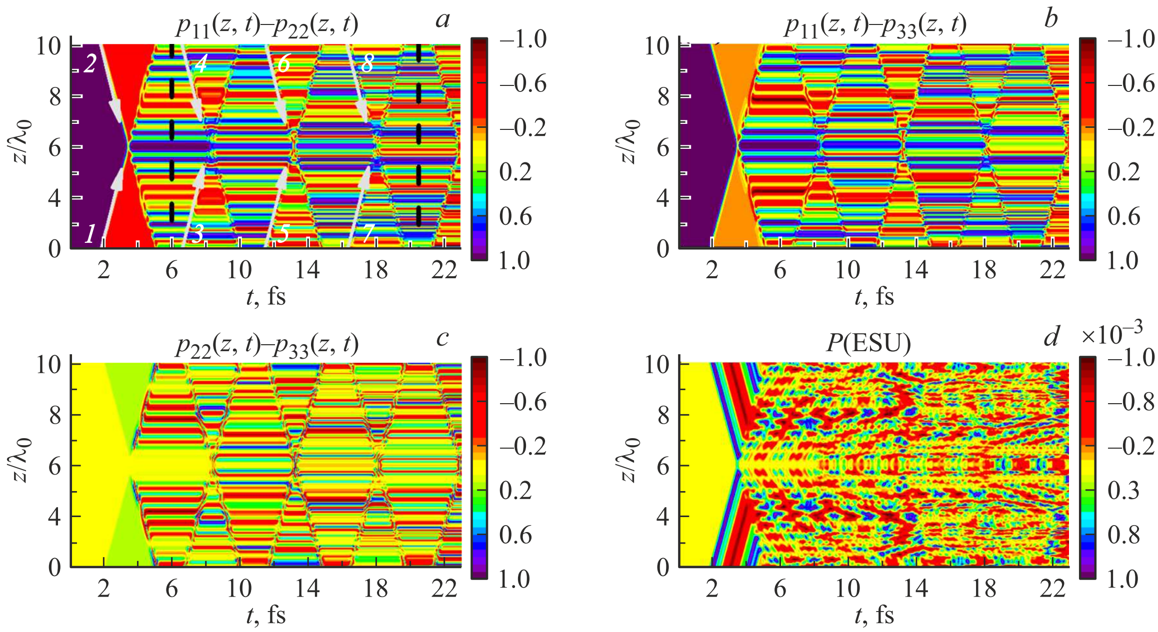

Figure 3. Dynamics of the populations difference: a — $\rho_{11}-\rho_{22}$, b — $\rho_{11}-\rho_{33}$, c — $\rho_{22}-\rho_{33}$; d — polarization dynamics $P(z,t)$ when subcycle pulses of Gaussian shape pass through the medium, their numbers and propagation directions are shown by arrows with a text. The amplitude of pulses was $E_{02} = 1.6 \cdot 10^6$ V/cm, ratio $S_a/S_{E2} \approx 2$; other computation parameters are provided in the table.

the medium, but the overall structure of the resonators and gratings does not collapse. To monitor the high-quality gratings and microcavities, it is necessary that the medium is not strongly excited by incident half-cycle pulses, i.e., the electric area of the pulses should be less than its atomic measure, $S_E \ll S_a$. Also, the results of numerical modeling clearly show the possibility of switching the state of the medium in attosecond time scale using the half-cycle pulses [57].

It should be noted that this mechanism differs from the traditional method of grating formation based on the interference of monochromatic light beams overlapping in

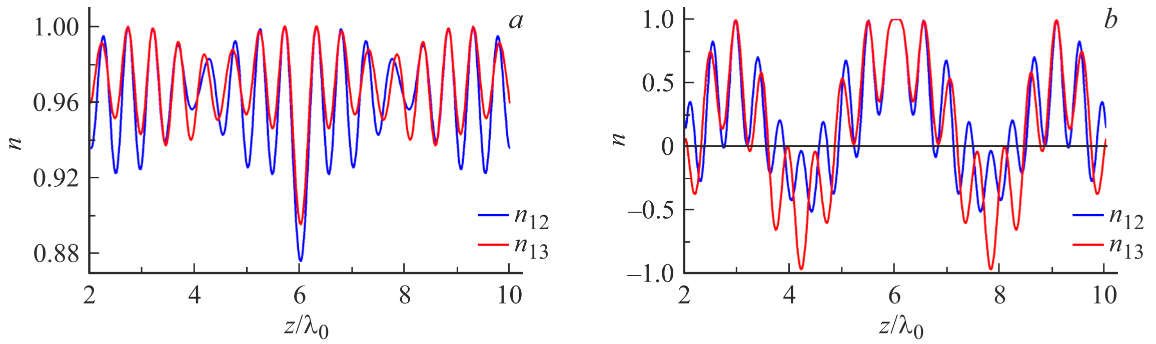

Figure 4. Instantaneous values of the populations difference distribution after the first pulses collision of amplitude: a — E_{01}, b — E_{02}, the cross-sections are denoted by black streaks in Fig. 2,a and Fig. 3,a correspondingly; blue — $\rho_{11} - \rho_{22}$, red — $\rho_{11} - \rho_{33}$. The computation parameters are given in the table

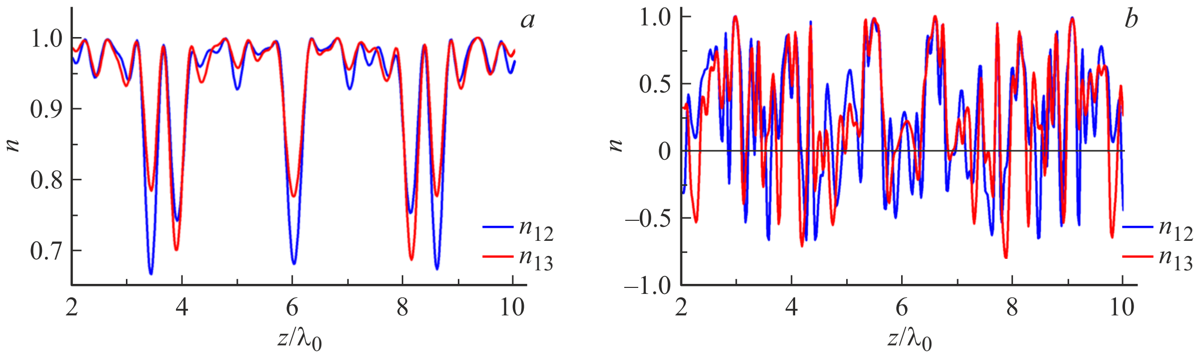

Figure 5. Instantaneous values of the populations difference distribution after the fourth pulses collision of amplitude: a — E_{01}, b — E_{02}, the cross-sections are denoted by black streaks in Fig. 2,a and Fig. 3,a correspondingly; blue — $\rho_{11} - \rho_{22}$, red — $\rho_{11} - \rho_{33}$. The computation parameters are given in the table.

the medium [110]. In our case, the half-cycle pulses collide in the medium and their direct interference, in the ordinary sense, is impossible. Then creation of gratings, as shown above, occurs due to interference of the electric areas of pulses [74,101,102] (this is true in case of weak pulses). In case of stronger fields, the creation of gratings occurs due to the interference of polarization waves induced by the transmitted pulse with the incident pulse [61–64,90,91].

Once again, we emphasize that creation of these structures in the form of DM is possible only with the help of extremely short pulses that overlap in the medium, and is not possible with the use of long multi-cycle pulses. This distinguishes the problems considered here from the previously considered cases of harmonic population gratings, which were created using long quasi-monochromatic multi-cycle pulses that do not overlap in the medium [58–61], although physical mechanism of Bragg gratings arising here is based on the coherent interaction of pulses with the medium, similar to the case of long pulses.

It should also be noted, that the results of numerical computations in [90] for a three-level medium based on a hydrogen atom have shown that microcavities can be induced only at one of the medium transitions when the half-cycle pulses do not overlap in the medium.

6. Estimation of Bragg gratings reflectivity

Such gratings of population difference and, consequently, refractive index can serve as Bragg mirrors in optical fibers [111,112], various sensors jcite113, etc. To estimate the reflectivity of the grating, let's assume that the grating was created in a waveguide, i.e. there is a modulation of the refractive index in the fiber (for simplicity, we assume a harmonic dependence) according to the expression

$$n = n_0 + \Delta n \cos \frac{2\pi}{\Lambda} z,$$

n_0 — some average refractive index, Δn — modulation amplitude, Λ — structure period. The reflectivity R of grating can be found using expression [112]

$$R(L, \lambda) = \frac{\Omega^2 (\sinh sL)^2}{\Delta k^2 (\sinh sL)^2 + s^2 (\cosh sL)^2},$$

where L — length of the structure, λ — wavelength of test pulse, $s = \sqrt{\Omega^2 - \Delta k^2}$, $\Omega \cong \pi \Delta n/\lambda$, $\Delta k = k - \pi/\Lambda$ — frequency tuning, $k = 2\pi n_0/\lambda$ — wavenumber.

Obviously, an increase in the depth of modulation, which is varied using parameters of incident pulses, leads to an

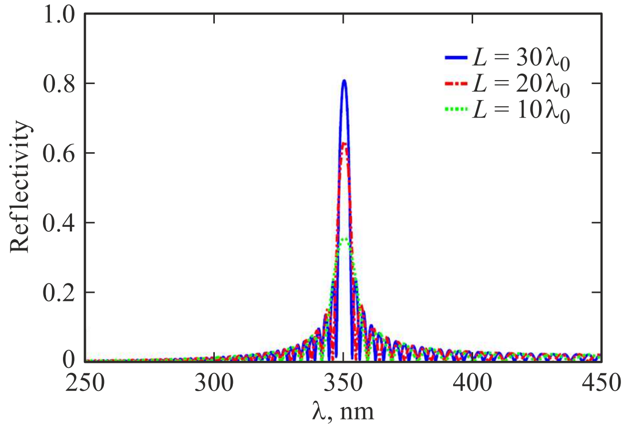

Figure 6. Reflectivity of Bragg grating induced in a two-level medium $R(L, \lambda)$ versus length of the grating L and wavelength of the test pulse λ for various lengths of the grating L; $d_{12} = 20\,\mathrm{D}$, $T_2 = 0.5\,\mathrm{ps}$, $\Lambda_0 = 700\,\mathrm{nm}$, $N_0 = 10^{19}\,\mathrm{cm}^{-3}$, grating period $\Lambda = \lambda_0/2$, population difference of the medium's levels varies from -1 to $+1$.

increase in reflectivity. Therefore, the induced gratings can serve as Bragg mirrors with rapidly changing parameters or may be used as highly reflective mirrors in dynamic microcavities.

Let's review the case of a primitive two-level resonance medium as a more specific example. Then, frequency dependence of the modulation of refractive index Δn can be expressed through the medium parameters as follows [114]:

$$\Delta n(\omega) = \frac{1}{\hbar} \frac{4\pi\omega_0 d_{12}^2 N_0(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + \frac{4\omega^2}{T_2^2}} \Delta\rho,$$

where d_{12} — dipole moment of transition, N_0 — concentration of atoms of the medium, T_2 — time of the medium phase relaxation, ω_0 — frequency of resonance transition, $\Delta\rho$ — depth of modulation of the levels' populations difference.

Figure 6 shows an example of dependence of reflectivity$R(L, \lambda)$ for a periodic grating induced in a layer of a two-level medium.

As can be seen in Fig. 6, the reflectivity of the Bragg grating can reach quite high values even at relatively small concentrations of atoms corresponding to gaseous media. With this value, the Q factors of microcavities formed by a pair of such Bragg mirrors for example in Fig. 6 may vary within $\approx 1-10$. A further increase in the reflectivity of Bragg mirrors and, consequently, the Q-factor of microcavities is limited due to strong distortion of exciting pulses in the medium with a higher density of atoms. In this case, the resulting structures have a more complex and non-periodic shape, which also results in lower values of reflectance. Thus, in practice, it becomes necessary to maintain a balance between a sufficiently large amplitude of the refractive index modulation, on the one hand, and the regularity and periodicity of the structure of the gratings themselves, on the other hand. It should be stressed again that the main advantage of such microcavities is primarily their dynamic nature, i.e. the possibility of superfast switching of their parameters at a times of units of femtoseconds or even hundreds of attoseconds. Note that the estimates made above are approximate, as they are performed for stationary structures. In our case the induced structures like resonators are dynamic, they exist for a period of the medium's phase memory timeT_2. Therefore, the obtained estimates of reflectivity and Q factor of DM are true for the time shorter than T_2 until the atomic coherence is not destroyed.

7. Other methods of creation of gratings and quasi-resonators and a discussion of possible applications of DM

It is also interesting to note the possibility of creating a quasi-resonator when a single powerful single-cycle pulse is propagating in a resonant medium [115]. In this case, the nonlinear self-action of such a pulse in the medium leads to its complete stop and emergence of a bound structure of field and matter (oscillon).

Recently, the possibilities of inducing plasma periodic structures using high-power focused femtosecond laser pulses have been examined and possible mechanisms for the occurrence of these structures have been discussed, see [116–120] and literature below. Such structures made, for instance, in a fused quartz are interesting in terms of creation of optical memory [121].

Also the possibility of creation of atomic populations (electromagnetically induced photonic lattice, EIPL) emerging due to electromagnetically induced transparency in the alkali metal pairs and the properties of such atomic populations are actively studied. But at the same time, long multi-cycle powerful light beams with a spatially periodic intensity profile are used to create the gratings, see paper [122]. When creating such structures the atomic coherence also plays an essential role. Such structures may be of interest for topological photonics [123].

Atomic coherence is also important for creating quantum optical memory on atomic assemblies, which has been widely discussed recently [124–126]. The creation of such a memory is possible based on the phenomenon of slow light and photonic echo emerging due to effect of the long multi-cycle light pulses.

Note that, in our opinion, the above-mentioned DMs induced by half-cycle pulses can also be used to create optical memory due to atomic coherence. The results of numerical computations carried out in [115] have shown the possibility of stopping and saving of light in such quasi-resonators for a period of time about the time of the phase memory of the medium T_2.

The advantage of the DM discussed above is that such structures can be induced at each resonance transition of the medium, in contrast to the resonant multi-cycle pulses. Also, the use of half-cycle pulses makes it possible to control these structures superfast at times of about half the field period, which provides extensive capabilities for their use to create optical memory, study the dynamics of ultrafast processes in matter and other interesting applications.

8. Conclusion

Progress in reducing the electromagnetic pulses duration has led to the task of obtaining pulses of maximum duration in a given spectral range — unipolar half-cycle pulses containing only one half-wave of the field. Thus, the new effects in the interaction of such pulses with a matter could be predicted and investigated. These phenomena occur at time intervals of about half the oscillation period of the light wave and are impossible in case of conventional multi-cycle pulses. In the review we've summarized the results of recent studies showing that DMs may be formed during collision of the unipolar half-cycle attosecond pulses.

In the given analysis we've summarized the micro-resonators inducing and controlling options *with each resonance transition* for a multi-level media. Although the dynamics of a system in a multi-level medium may differ slightly from the dynamics of a two-level medium, the studies conducted show the possibility of such structures in a multi-level medium, which expands the applicability of the results of early research obtained in a two-level approximation. Due to the universality of the results obtained above, such structures seem to be real in any medium, the relaxation times of populations T_1 and polarization T_2 of which exceed the duration of subcycle attosecond pulses and the delay between them. We may expect differences in the shapes of these structures and their depth of modulation in different media, depending on the parameters of a particular media.

In case of low fields such structures emerge physically due to the interference of electric areas of the incident pulses. In case of high fields, the pulses induce polarization waves in the medium, which interact with each subsequent pulse resulting in appearance of Bragg-like gratings of population differences on the edges from the pulse overlap region. With the growing number of collisions it is possible to control characteristics of such structures — their activation, deactivation, variation in reflectivity, spatial frequency of gratings for a time of about hundreds of attoseconds. Q-factor of such microcavities can reach about 1000 in a dense medium [83,91].

During the passage of pulses, a rapid change in the characteristics of the medium occurs both, in space and in time due to the appearance of subwavelength gratings of atomic populations. Therefore, such medium represents itself a new type of spatiotemporal photon crystals [84,96] being extensively studied today [47]. The conducted studies show that parameters of a medium may be changed superfast if subcycle pulses are used in addition to the conventional methods of changing the refractive index described in the literature [127]. They also show the possibility of attosecond optical switching at subcycle time scales [57]. The DMS discussed above may be of interest for creating optical memory systems based on atomic coherence, the parameters of which can be controlled in an superfast manner by using half-cycle attosecond pulses. The described phenomena open up new research directions both, in the optics of unipolar pulses and in the optics of unsteady media, physics of spatiotemporal photonic crystals and attosecond physics. This review is an expanded version of an early mini-review [128].

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Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] F. Krausz, M. Ivanov. Rev. Mod. Phys., **81**, 163 (2009).
[2] E.A. Khazanov. Quant. Electron., **52**, 208 (2022).
[3] K. Midorikawa. Nature Photonics, **16**, 267 (2022).
[4] M.Yu. Ryabikin, M.Yu. Emelin, V.V. Strelkov. Phys. Usp., **66**, 360, (2023).
[5] S. Severino, K. Ziems, M. Reduzzi, A. Summers, H.-W. Sun, Y.-H. Chien, S. Gräfe. J. Biegert. Nature Photonics, **18**, 731 (2024).
[6] F. Calegari, G. Sansone, S. Stagira, C. Vozzi, M. Nisoli. J. Phys. B: Atomic, Molecular and Optical Phys., **49**, 062001 (2016).
[7] M.T. Hassan, T.T. Luu, A. Moulet, O. Raskazovskaya, P. Zhokhov, M. Garg, N. Karpowicz, A.M. Zheltikov, V. Pervak, F. Krausz, E. Goulielmakis. Nature, **530**, 66 (2016).
[8] D. Ertel, D. Busto, I. Makos, M. Schmoll, J. Benda, H. Ahmadi, M. Moioli, F. Frassetto, L. Poletto, C.D. Schröter, T. Pfeifer, R. Moshammer, Z. Mašín, S. Patchkovskii, G. Sansone. Science Advances, **9** (35), 7747 (2023).
[9] H.Y. Kim, M. Garg, S. Mandal, S. Mandal, L. Seiffert, T. Fennel, E. Goulielmakis. Nature, **613**, 662 (2023).
[10] NobelPrize.org URL: https://www.nobelprize.org/prizes/physics/2023/press-release
[11] H.-C. Wu, J. Meyer-ter Vehn. Nature Photon, **6**, 304 (2012).
[12] J. Xu, B. Shen, X. Zhang, Y. Shi, L. Ji, L. Zhang, T. Xu, W. Wang, X. Zhao, Z. Xu. Sci. Rep., **8**, 2669 (2018).
[13] S.V. Sazonov. JETP Lett., **114** (3), 132 (2021).
[14] M.M. Glazov, N.N. Rosanov. Phys. Rev. A, **109** (5), 053523 (2024).

[15] A.V. Bogatskaya, E.A. Volkova, A.M. Popov. Phys. Rev. E, **105**, 055203 (2022).
[16] E. Ilyakov, B.V. Shishkin, E.S. Efimenko, S.B. Bodrov, M.I. Bakunov. Optics Express, **30**, 14978 (2022).
[17] A.S. Kuratov, A.V. Brantov, V.F. Kovalev, V.Yu. Bychenkov. Phys. Rev. E, **106**, 035201 (2022).
[18] N.N. Rosanov. Optics Lett., **49** (6), 1493 (2024).
[19] Q. Xin, Y. Wang, X. Yan, B. Eliasson. Phys. Rev. E, **107**, 035201 (2023).
[20] S.V. Sazonov. Opt. Spectr., **130** (12), 1573 (2022).
[21] N.N. Rosanov, R.M. Arkhipov, M.V. Arkhipov. Phys. Usp., **61**, 1227 (2018).
[22] R.M. Arkhipov, M.V. Arkhipov, N.N. Rosanov. Quant. Electron., **50** (9), 801 (2020)].
[23] R.M. Arkhipov, M.V. Arkhipov, A.V. Pakhomov, P.A. Obraztsov, N.N. Rosanov. JETP Letters, **117** (1), 8 (2023).
[24] N.N. Rosanov. Phys. Usp., **66**, 1059 (2023).
[25] D. Michalache. Romanian Reports in Physics, **76**, 402 (2024).
[26] N.N. Rosanov, M.V. Arkhipov, R.M. Arkhipov, A.V. Pakhomov. Contemprorary Physics, **64** (3), 224 (2023).
[27] N.N. Rosanov, M.V. Arkhipov, R.M. Arkhipov. Phys. Usp. DOI: 10.3367/UFNe.2024.07.039718.
[28] N.N. Rosanov, M.V. Arkhipov, R.M. Arkhipov. *Teragertsovaya fotonika*, edited by V.Ya. Panchenko, A.P. Shkurinov (RAS, M., 2023), pp. 360–393 (in Russian).
[29] J.D. Jackson. *Classical Electrodynamics* (John Wiley & Sons, 1962).
[30] E.G. Bessonov. Sov. Phys. JETP, **53**, 433 (1981).
[31] N.N. Rosanov. Opt. Spectr., **107**, 721 (2009).
[32] P.H. Bucksbaum. In: *AIP Conference Proceedings* (American Institute of Physics, 1994), vol. 323, No. 1, p. 416–433.
[33] D. Dimitrovski, E.A. Solov'ev, J.S. Briggs. Phys. Rev. A, **72** (4), 043411 (2005).
[34] A.S. Moskalenko, Z.-G. Zhu, J. Berakdar. Phys. Rep., **672**, 1 (2017).
[35] R.M. Arkhipov, M.V. Arkhipov, I. Babushkin, A. Demircan, U. Morgner, N.N. Rosanov. Opt. Lett., **44**, 1202 (2019).
[36] R.M. Arkhipov, M.V. Arkhipov, A.V. Pakhomov, N.N. Rosanov. JETP Lett., **114**, 129 (2021).
[37] N. Rosanov, D. Tumakov, M. Arkhipov, R. Arkhipov. Phys. Rev. A, **104** (6), 063101 (2021).
[38] A. Pakhomov, M. Arkhipov, N. Rosanov, R. Arkhipov. Phys. Rev. A, **105**, 043103 (2022).
[39] R. Arkhipov, P. Belov, A. Pakhomov, M. Arkhipov, N. Rosanov. JOSA B, **41** (1), 285 (2024).
[40] R.M. Arkhipov, M.V. Arkhipov, N.N. Rosanov. JETP Lett., **111**, 484 (2020).
[41] C. Wesdorp, F. Robicheaux, L.D. Noordam. Phys. Rev. Lett., **87** (8), 083001 (2001).
[42] A.B. Shvartsburg. Physics-Uspekhi, **48** (8), 797 (2005).
[43] E. Galiffi, R. Tirole, S. Yin, H. Li, S. Vezzoli, P.A. Huidobro, M.G. Silveirinha, R. Sapienza, A. Alú, J.B. Pendry. Advanced Photonics, **4** (1), 014002 (2022).
[44] C. Caloz, Z.L. Deck-Leger. IEEE Trans. Antennas and Propagation., **68** (3), 1583 (2019).
[45] V.M. Levkovskaya, A.V. Kharitonov, S.S. Kharintsev. Opticheskii Zhurnal, **91** (5), 5 (2024).
[46] E. Lustig, Y. Sharabi, M. Segev. Optica, **5** (11), 1390 (2018).
[47] Y. Sharabi, A. Dikopoltsev, E. Lustig, Y. Lumer, M. Segev. Optica, **9** (6), 585 (2022).
[48] J. Mendonça, P. Shukla. Phys. Scr., **65**, 160 (2002).
[49] E. Lustig, O. Segal, S. Saha, E. Bordo, S.N. Chowdhury, Y. Sharabi, A. Fleischer, A. Boltasseva, O. Cohen, V.M. Shalaev, M. Segeev. Nanophotonics, **12**, 1 (2023).
[50] Z. Dong, H. Li, T. Wan, Q. Liang, Z. Yang, B. Yan. Nat. Photonics, **18**, 68 (2024).
[51] J.B. Pendry, E. Galiffi, P.A. Huidobro. Optica, **8**, 636 (2021).
[52] K. Xu, M. Fang, J. Fen, C. Wang, G. Xie, Z. Huang. Opt. Lett., **49**, 842 (2024).
[53] C. Liu, M.Z. Alam, K. Pang, K. Manukyan, J.R. Hendrickson, E.M. Smith, Y. Zhou, O. Reshef, H. Song, R. Zhang, H. Song, F. Alishahi, A. Fallahpour, A. Almaiman, R.W. Boyd, M. Tur, A.E. Willner. Opt. Lett., **46** (14), 3444 (2021).
[54] K. Pang, M.Z. Alam, Y. Zhou, C. Liu, O. Reshef, K. Manukyan, M. Voegtle, A. Pennathur, C. Tseng, X. Su, H. Song, Z. Zhao, R. Zhang, H. Song, N. Hu, A. Almaiman, J.M. Dawlaty, R.W. Boyd, M. Tur, A.E. Willner. Nano Lett., **21** (14), 5907 (2021).
[55] S. Saha, O.Segal, C. Fruhling, E. Lustig, M. Segev, A. Boltasseva, V.M. Shalaev. Optics Express, **31** (5), 8267 (2023).
[56] A. Boltasseva, V.M. Shalaev, M. Segev. Optical Materials Express, **14** (3), 592 (2024).
[57] M.T. Hassan. ACS Photonics, **11**, 334 (2024).
[58] I.D. Abella, N.A. Kurnit, S.R. Hartmann. Phys. Rev., **141**, 391 (1966).
[59] E.I. Shtyrkov, V.S. Lobkov, N.G. Yarmukhametov. JETP Lett., **27** (12), 648 (1978).
[60] E.I. Shtyrkov, V.V. Samartsev. Physics Status Solidi A, **45**, 647 (1978).
[61] E.I. Shtyrkov. Opt. Spectr., **114** (1), 96 (2013).
[62] R.M. Arkhipov, M.V. Arkhipov, I. Babushkin, A. Demircan, U. Morgner, N.N. Rosanov. Opt. Lett., **41**, 4983 (2016).
[63] R.M. Arkhipov, M.V. Arkhipov, I. Babushkin, A. Demircan, U. Morgner, N.N. Rosanov. Scientific Reports, **7**, Art. No. 12467 (2017).
[64] R. Arkhipov, A. Pakhomov, M. Arkhipov, I. Babushkin, A. Demircan, U. Morgner, N.N. Rosanov. Scientific Reports, **11**, Art. No. 1961 (2021).
[65] O.O. Diachkova, R.M. Arkhipov, M.V. Arkhipov, A.V. Pakhomov, N.N. Rosanov. Laser Physics, **33** (4), 045301 (2023).
[66] H. Zhang, S. Zhang, S. Li, X. Ma. Opt. Commun., **462**, 125182 (2020).
[67] S. Zhang, S. Li, Y. Bai, K. Huang. J. Nanophoton., **17** (1), 016013 (2023).
[68] N.N. Rosanov, V.E. Semenov, N.V. Vyssotina. Laser Physics, **17**, 1311 (2007).
[69] A. Pusch, J.M. Hamm, O. Hess. Phys. Rev. A, **82** (2), 023805 (2010).
[70] D.V. Novitsky. Phys. Rev. A, **8**4, 013817 (2011).
[71] R.M. Arkhipov, M.V. Arkhipov, A.V. Pakhomov, I. Babushkin, N.N. Rosanov. Opt. Spectr., **123**, 610 (2017).
[72] R.M. Arkhipov, A.V. Pakhomov, M.V. Arkhipov, D.O. Zhiguleva, N.N. Rosanov. Opt. Spectr., **124**, 541 (2018).
[73] R.M. Arkhipov. JETP Lett., **113**, 611 (2021).
[74] R.M. Arkhipov, M.V. Arkhipov, A.V. Pakhomov, O.O. Diachkova, N.N. Rosanov. Radiophysics and Quantum Electronics, **66** (4), 286 (2024).
[75] A. Pakhomov, M. Arkhipov, N. Rosanov, R. Arkhipov. Phys. Rev. A, **106** (5), 053506 (2022).
[76] A. Pakhomov, N. Rosanov, M. Arkhipov, R. Arkhipov. Opt. Lett., **48** (24), 6504 (2023).

[77] R.M. Arkhipov, M.V. Arkhipov, A.V. Pakhomov, O.O. Diachkova, N.N. Rosanov. JETP Letters, **117** (8), 574 (2023).
[78] R. Arkhipov, A. Pakhomov, M. Arkhipov, A. Demircan, U. Morgner, N. Rosanov, I. Babushkin. Optics Express, **28** (11), 17020 (2022).
[79] M.V. Bastrakova, N.V. Klenov, A.M. Satanin. Phys. Solid State, **61**, 1515 (2019).
[80] M.V. Bastrakova, N.V. Klenov, A.M. Satanin. JETP, **131**, 507 (2020).
[81] R.M. Arkhipov, M.V. Arkhipov, N.N. Rosanov. Opt. Spectr., **130** (9), 1121 (2022).
[82] R.M. Arkhipov, M.V. Arkhipov, A.V. Pakhomov, O.O. Dyachkova, N.N. Rosanov. Opt. Spectr., **130** (11), 1443 (2022).
[83] O.O. Diachkova, R.M. Arkhipov, M.V. Arkhipov, A.V. Pakhomov, N.N. Rosanov. Opt. Commun., **538**, 129475 (2023).
[84] R.M. Arkhipov, O.O. Diachkova, M.V. Arkhipov A.V. Pakhomov, N.N. Rosanov. Applied Physics B, **130**, 52 (2024).
[85] O. Diachkova, R. Arkhipov, A. Pakhomov, N. Rosanov. Opt. Commun., **565**, 130666 (2024).
[86] R. Arkhipov, A. Pakhomov, O. Diachkova, M. Arkhipov, N. Rosanov. Opt. Lett., **49** (10), 2549 (2024).
[87] R.M. Arkhipov. Bulletin of the Lebedev Physics Institute, **51** (5), S366 (2024).
[88] R.M. Arkhipov, N.N. Rosanov. Opt. i spektr., **132** (5), 532 (2024) (in Russian).
[89] R. Arkhipov. arXiv preprint arXiv:2402.16122
[90] R. Arkhipov, M. Arkhipov, A. Pakhomov, O. Diachkova, N. Rosanov. Phys. Rev. A, **109**, 063113 (2024).
[91] R. Arkhipov, A. Pakhomov, O. Diachkova, M. Arkhipov, N. Rosanov. JOSA B, **41** (8), 1721 (2024).
[92] P.G. Kryukov, V.S. Letokhov. Sov. Phys. Usp., **12**, 641 (1970).
[93] R. Arkhipov. Laser Physics, **34** (6), 065301 (2024).
[94] N.V. Znamenskii, V. Sazonov. JETP Lett., **85**, 358 (2007).
[95] A. Pakhomov, N. Rosanov, M. Arkhipov, R. Arkhipov. JOSA B, **41** (1), 46–54 (2024).
[96] R.M. Arkhipov, M.V. Arkhipov, N.N. Rosanov. Opt. i spektr., **132** (4), 434 (2024) (in Russian).
[97] R.M. Arkhipov, O.O. D'yachkova, P.A. Belov, M.V. Arkhipov, A.V. Pakhomov, N.N. Rosanov. ZhETF, **166** (8), 162 (2024). (in Russian).
[98] L. Allen, J.H. Eberly. *Optical resonance and two-level atoms* (Wiley, N.Y., 1975).
[99] L.D. Landau, E.M. Lifshitz. *Quantum mechanics* (Pergamon, Oxford, 1974).
[100] M. Born, E. Wolf. *Principles of Optics* (Cambridge University Press, 2019).
[101] R.M. Arkhipov, M.V. Arkhipov, I. Babushkin, A.V. Pakhomov, N.N. Rosanov. JETP Letters, **114** (5), 250–255 (2021).
[102] R.M. Arkhipov, M.V. Arkhipov, A.V. Pakhomov, N. Rosanov. Laser Physics, **32** (6), 066002 (2022).
[103] P.A. Belov, R.M. Arkhipov. Micro and Nanostructures, **180**, 207607 (2023).
[104] A. de las Heras, C. Hernández-García, J. Serrano, T. Popmintchev, L. Plaja. *European Quantum Electronics Conference (EQEC 2023), Technical Digest Series* (Optica Publishing Group, 2023), paper ee_3_5.
[105] A. De las Heras, C. Hernández-García, J. Serrano, A. Prodanov, D. Popmintchev, T. Popmintchev, L. Plaja. arXiv:2404.04053, 2024.
[106] O.D. Mücke, T. Tritschler, M. Wegener, U. Morgner. Phys. Rev. Lett., **87** (5), 057401 (2001).
[107] S. Nandi, E. Olofsson, M. Bertolino, S. Carlström, F. Zapata, D. Busto, C. Callegari, M. Di Fraia, P. Eng-Johnsson, R. Feifel, G. Gallician, M. Gisselbrecht, S. Maclot, L. Neoričić, J. Peschel, O. Plekan, K.C. Prince, R.J. Squibb, S. Zhong, P.V. Demekhin, M. Meyer, C. Miron, L. Badano, M.B. Danailov, L. Giannessi, M. Manfredda, F. Sottocorona, M. Zangrando, J.M. Dahlström. Nature, **608**, 488 (2022).
[108] A. Yariv. *Quantum Electronics* (John Wiley & Sons, N.Y., London, Toronto, 1975).
[109] S.E. Frisch. *Optical spectra of atoms* (State Publishing House of Physical and Mathematical Literature, M.-L., 1963).
[110] H.J. Eichler, P. Günter, D.W. Pohl. *Laser-Induced Dynamic Gratings* (Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1981).
[111] S.A. Vasil'ev, O.I. Medvedkov, I.G. Korolev, A.S. Bozhkov, A.S. Kurkov, E.M. Dianov. Quantum Electron., **35** (12), 1085 (2005).
[112] R. Kashyap. *Fiber Bragg Gratings* (Acad. Press, Amsterdam, Boston, Heidelberg, 2009).
[113] R. Rohan, K. Venkadeshwaran, P. Ranjan. J. Optics, **53** (1), 282 (2024).
[114] R.W. Boyd, B.R. Masters. *Nonlinear Optics*, 3rd edn, (Academic, NY, 2008).
[115] M. Arkhipov, R. Arkhipov, I. Babushkin, N. Rosanov. Phys. Rev. Lett., **128** (20), 203901 (2022).
[116] A.V. Bogatskaya, A.M. Popov. Opt. i spektr., **132** (1), 47 (2024) (in Russian).
[117] A.E. Rupasov, I.V. Gritsenko, N.I. Busleev, G.K. Krasin, Yu.S. Gulina, A.V. Bogatskaya, S.I. Kudryashov. Opt. i spektr., **132** (1), 83 (2024) (in Russian).
[118] Yu.S. Gulina, A.E. Rupasov, G.K. Krasin, N.I. Busleev, I.V. gritsenko, A.V. Bogatskaya, S.I. Kudryashov. Pis'ma v ZhETF, **119** (9), 638 (2024) (in Russian).
[119] S. Kudryashov, A. Rupasov, M. Kosobokov, A. Akhmatkhanov, G. Krasin, P. Danilov, B. Lisjikh, A. Abramov, E. Greshnyakov, E. Kuzmin, M. Kovalev, V. Shur. Nanomaterials, **12** (23), 4303 (2022).
[120] A.V. Bogatskaya, E.A. Volkova, A.M. Popov. Phys. Plasmas, **31**, 073901 (2024).
[121] H. Wang, Y. Lei, L. Wang, M. Sakakura, Y. Yu, G. Shayeganrad, P.G. Kazansky. Laser & Photonics Reviews, **16** (4), 2100563 (2022).
[122] J. Yuan, S. Liang, Q. Yu, C. Li, Y. Zhang, M. Xiao, Z. Zhang. Adv. Phys. Res., 2400082 (2024).
[123] J. Gao, Z.S. Xu, Z. Yang, V. Zwiller, A.W. Elshaari. Nanophotonics, **1** (1), 34 (2024).
[124] S.A. Moiseev, K.I. Gerasimov, M.M. Minnegaliev, E.S. Moiseev, A.D. Deev, Yu.Yu. Balega. arXiv preprint arXiv:2410.01664 (2024).
[125] S.A. Moiseev, K.I. Gerasimov, M.M. Minnegaliev, E.S. Moiseev. arXiv preprint arXiv:2408.09991 (2024).
[126] S.A. Moiseev, M.M. Minnegaliev, K.I. Gerasimov, E.S. Moiseev, A.D. Deev, Yu.Yu. Balega. Phys. Usp., **67** (2024) DOI: 10.3367/UFNe.2024.06.039694.
[127] J.B. Khurgin. Laser & Photonics Reviews, **18** (4), 2300836 (2024).
[128] R.M. Arkhipov, M.V. Arkhipov, A.V. Pakhomov, O.O. Diachkova, N.N. Rosanov. Bulletin of the Russian Academy of Sciences: Physics **89** (1), 63 (2025).

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