04

Steady states of relativistic electron-positron plasma diode

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Steady states of a diode with relativistic electron and positron flows being supplied by opposite electrodes are considered. The particles move in the plasma without collisions. For a fixed potential difference between the electrodes, the solutions are characterized by three dimensionless parameters: the electric field strength at the left electrode ε_0, the interelectrode distance δ, and the relativistic factor of emitted electrons γ_0. Branches of solutions are plotted in the $\{\varepsilon_0, \delta\}$-plane. They qualitatively coincide with those in the nonrelativistic case. As the relativistic factor increases, the branches shrink along the ε_0 axis and stretch along the δ axis.

Keywords: plasma diode, relativistic electron and positron beams, steady states.

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Electron-positron plasma with relativistic electron and positron flows is found in various high-energy astrophysical objects [1] (specifically, pulsars) emitting radio-frequency radiation the nature of which has not been clarified yet [2]. In addition, electron-positron plasma is being studied extensively in laboratory conditions [3].

It was hypothesized in [4] that radio-frequency pulsar emission is caused by fluctuations of the electric field in plasma induced by the instability of steady states. A model of a pulsar diode with electrons and positrons supplied from opposite boundaries is proposed as a means to probe these processes. Steady states of a nonrelativistic diode were investigated in [5]. In the present study, the steady states of a relativistic diode are examined. All solutions are found, and their dependence on the relativistic factor is studied. It is demonstrated that the solutions agree qualitatively with similar solutions for a nonrelativistic diode.

A planar diode with distance d and potential difference U between the electrodes is considered. it is assumed that relativistic monoenergetic electron and positron flows with velocities $v_{e,0}$ and $v_{p,0} = -v_{e,0}$ and densities $n_{e,0} = n_{p,0} = n_0$ are supplied by opposite electrodes. Owing to the equality of electron and positron rest masses ($m_e = m_p = m_0$), the energies of incoming particles are equal. Particles move in the interelectrode gap without collisions. A particle reaching any electrode is absorbed at it.

Let us introduce electron P_e and positron P_p momenta

$$P_e = \gamma_e m_e v_e, \quad P_p = \gamma_p m_p v_p, \quad \gamma_{e,p} = \left[1 - v_{e,p}^2/c^2\right]^{-1/2}. \tag{1}$$

Here, c is the speed of light in vacuum and $\gamma_{e,p}$ are the relativistic factors of electrons and positrons. Just as in a nonrelativistic diode [5], the steady-state potential distributions (PDs) have a wave-like shape. If potential barrier height $e|\varphi_m|$ is equal to $W_0 = (\gamma_{e,0} - 1)m_e c^2$, a certain fraction of electrons is reflected from the potential barrier (electron virtual emitter, e-VE), while other electrons pass through it. This electron beam „splitting" is attributable to the fact that actual electrons have a certain momentum spread. Following the work of Bursian [6], we may introduce reflection coefficient r_e equal to the ratio of the density of reflected electrons to the density of emitted electrons. The current density of electrons overcoming the barrier is $j_e = (1 - r_e)\, j_{e,0}$, where $j_{e,0} = e n_{e,0} v_{e,0}$ is the electron current density at the left boundary (e is the electron charge). Coefficient of reflection r_p of positrons from a positron virtual emitter (p-VE) is introduced in a similar fashion.

The electron concentration in the steady case is determined using the law of conservation of energy and the continuity equation. The law of conservation of energy

$$(\gamma_e(z) - 1)m_e c^2 - e\varphi(z) = (\gamma_{e,0} - 1)m_e c^2 \tag{2}$$

allows one to express velocity $v_e(z)$ in terms of potential $\varphi(z)$. If electron reflection is neglected, we obtain the following from the continuity equation:

$$j_e = e n_e(z) v_e(z) = e n_{e,0} v_{e,0} = j_{e,0}. \tag{3}$$

Concentrations of electrons and positrons n_e and n_p in the gap with reflection taken into account are

$$n_{e,p}(z; r_{e,p}) = \frac{j_{e,p,0}}{e v_{e,p}(z)} \alpha_{e,p}(z; r_{e,p}),$$

$$\alpha_e(z; r_e) = (1 + r_e)\Theta(z_m - z) + (1 - r_e)\Theta(z - z_m),$$

$$\alpha_p(z; r_p) = (1 + r_p)\Theta(z - z_M) + (1 - r_p)\Theta(z_M - z). \tag{4}$$

Here, $\Theta(x)$ is the Heaviside step function and z_m (z_M) is the e-VE (p-VE) position, where the potential is φ_m (φ_M). In order to determine the PD, we insert electron and positron concentrations (4) into the Poisson equation. It is convenient to write this equation in dimensionless form. To do this, we use the initial energy of electrons and the Debye length at the left boundary as the units of energy and length [7]:

$$W_0 = (\gamma_0 - 1)m_0 c^2,$$

$$\lambda_D = \left[(2\tilde{\varepsilon}_0 W_0)/(e^2 n_0)\right]^{1/2} = \left[(2\tilde{\varepsilon}_0 m_0 c^3)/(e j_0)\right]^{1/2} F(V_0),$$

$$F(V_0) = \left(\frac{eV_0}{m_0c^2}\right)^{3/4}\left(\frac{eV_0}{m_0c^2}+2\right)^{1/4}\left(\frac{eV_0}{m_0c^2}+1\right)^{-1/2}. \quad (5)$$

Here, $\tilde{\varepsilon}_0 \approx 8.854 \cdot 10^{-12}\,\mathrm{C^2/(N{\cdot}m^2)}$ is the permittivity of vacuum, $j_0 = j_{e,0} = j_{p,0}$ is the current density of particle beams, $\gamma_0 = \gamma_{e,0} = \gamma_{p,0}$ is the relativistic factor of particles at the boundaries, and $V_0 = W_0/e$ is the accelerating voltage. The dimensionless coordinate, potential, and electric field strength are $\xi = z/\lambda_D$, $\eta = e\varphi/(2W_0)$, and $\varepsilon = eE\lambda_D/(2W_0)$. The Poisson equation in dimensionless variables is written as

$$\eta'' = \frac{(\gamma_0^2-1)^{1/2}}{\gamma_0}\left(\frac{\alpha_e(\xi;r_e)[\gamma_0+2(\gamma_0-1)\eta]}{\{[\gamma_0+2(\gamma_0-1)\eta]^2-1\}^{1/2}} - \frac{\alpha_p(\xi;r_p)[\gamma_0+2(\gamma_0-1)(V-\eta)]}{\{[\gamma_0+2(\gamma_0-1)(V-\eta)]^2-1\}^{1/2}}\right). \quad (6)$$

In the $\gamma_0 \to 1$ limit, this equation transforms into the corresponding equation for a nonrelativistic diode. The potential at the boundaries satisfies conditions $\eta(0) = 0$, $\eta(\delta) = V$, where $\delta = d/\lambda_D$, $V = eU/2W_0$. Thus, the steady-state solutions in a relativistic diode are specified by parameters δ, V, and γ_0 (in a nonrelativistic diode, only δ and V remain).

The potential distributions are wave-like functions. Let us denote the minimum and maximum PD values as η_m and η_M. Inequalities $-1/2 < \eta_m < \min\{0, V\}$, $\max\{0, V\} < \eta_M < V + 1/2$ hold true for these values. Just as in the nonrelativistic case [5], there are four types of solutions (see the table).

In the present study, only the solutions with $V = 0$ are examined. Owing to the symmetry of the problem, the PDs must be odd-symmetric about the center of the gap, and only two of the four types of solutions listed in the table are feasible: solutions 1 and 4.

The solutions for a nonrelativistic diode were obtained in [5] and are shown in the upper panel of the figure.

Branch n_{2s} corresponds to homogeneous ($s = 0$) and inhomogeneous ($s > 0$) solutions type 1, while branches d_0, d_{00}, and d_{11} correspond to solutions type 4. All these solutions are also relevant to a relativistic diode.

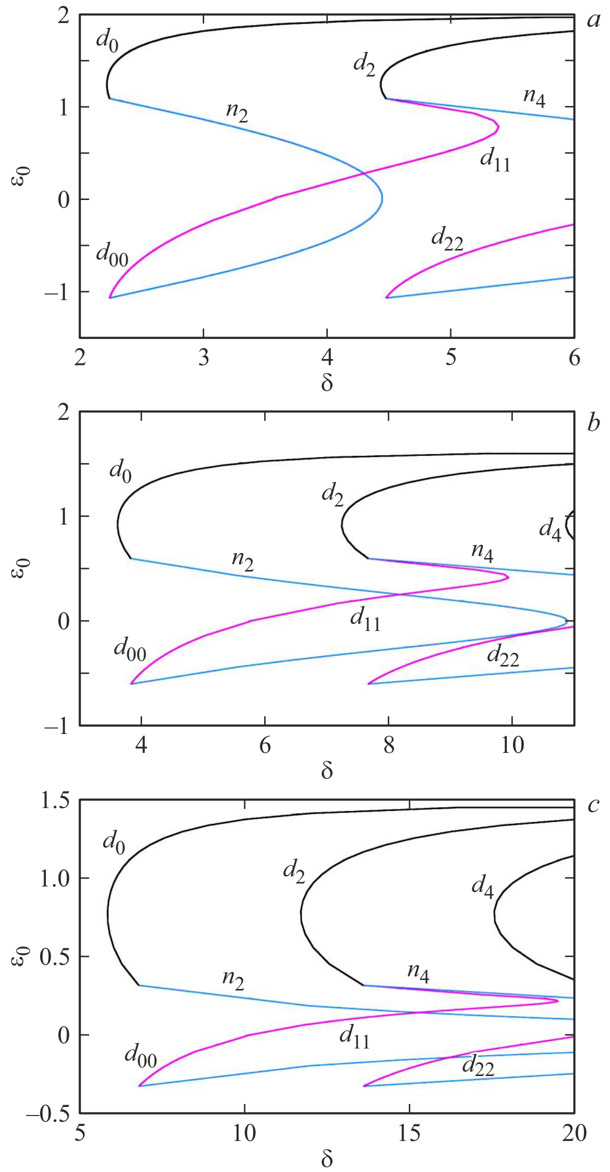
To examine the steady-state potential distributions, we multiply Eq. (6) by $2d\eta/d\xi$ and integrate once in ξ from ξ_1 to ξ:

$$[\eta'(\xi)]^2 - [\eta'(\xi_1)]^2 = G(\eta(\xi)) - G(\eta(\xi_1)), \quad (7)$$

where

$$G(x) = \sqrt{\gamma_0+1}\Big[\alpha_e(\xi;r_e)\sqrt{[\gamma_0+2(\gamma_0-1)x]^2-1} + \alpha_p(\xi;r_p)\sqrt{[\gamma_0+2(\gamma_0-1)(V-x)]^2-1}\Big] \times \left(\gamma_0\sqrt{\gamma_0-1}\right)^{-1}.$$

Equation (7) is the basic relation that allows one to find and investigate the PDs for all branches.



Steady-state solution branches for $\gamma_0 = 1$ (*a*), 3 (*b*), and 10 (*c*). $V = 0$.

Let us first consider the mode without particle reflection. Here, $\alpha_e = \alpha_p = 1$, and all PD minima (maxima) are equal to each other. These solutions lie on branches n_{2s}. The η_m minimum value is bound to field strength ε_0 at the left boundary by relation (7) at $\eta(\xi) = 0$, $\eta(\xi_1) = \eta_m$. At $\varepsilon_0 > 0$ ($\varepsilon_0 < 0$), the minimum (maximum) is located at the left electrode. The maximum $|\varepsilon_0|$ value is attained at $\eta_m = -1/2$:

$$|\varepsilon_{0,\max}| = \frac{(\gamma_0+1)^{1/4}}{\gamma_0^{1/2}} \times \left(\sqrt{\gamma_0+1} + \sqrt{\gamma_0+1+4\gamma_0V+4(\gamma_0-1)V^2} - \sqrt{\gamma_0+1+4\gamma_0(V+1/2)+4(\gamma_0-1)(V+1/2)^2}\right)^{1/2}. \quad (8)$$

Types of steady-state solutions

Solution type	Reflection	r_e	r_p	η_m	η_M
1	No	$r_e = 0$	$r_p = 0$	$\eta_m > -1/2$	$\eta_M < V + 1/2$
2	e	$0 < r_e < 1$	$r_p = 0$	$\eta_m = -1/2$	$\eta_M < V + 1/2$
3	p	$r_e = 0$	$0 < r_p < 1$	$\eta_m > -1/2$	$\eta_M = V + 1/2$
4	e, p	$0 < r_e < 1$	$0 < r_p < 1$	$\eta_m = -1/2$	$\eta_M = V + 1/2$

Using (7) with $\eta(\xi) = \eta_M$, $\eta(\xi_1) = \eta_m$ and taking into account that $\eta'(\xi)$ vanishes at these points, we obtain the following relation: $\eta_M = V - \eta_m$. In the case of $\gamma_0 \gg 1$ and $V = 0$, the domain of existence of solutions without particle reflection shrinks significantly along ε_0, since $|\varepsilon_{0,\max}| \sim \gamma_0^{-3/4}$, and the PDs take the form of functions of an oscillatory nature with a small amplitude.

To construct the PDs, one needs to solve Eq. (7) for the potential derivative and integrate in η. This yields an implicit $\eta(\xi)$ dependence. At $|\eta_m| \ll 1$, the solution may be obtained analytically. The relation between ε_0 and η_m is written as $\varepsilon_0^2 = 4/[\gamma_0(\gamma_0 + 1)]\,\eta_m^2$ in this case. The solution then takes the following form within the $(\xi \le \xi_m)$ interval:

$$\xi = \int_\eta^0 \frac{dx}{\sqrt{\varepsilon_0^2 + [G(x) - G(0)]}} = \frac{[\gamma_0(\gamma_0 + 1)]^{1/2}}{2} \arcsin\left(\frac{\eta}{\eta_m}\right). \quad (9)$$

It follows that $\delta(\gamma_0) = 4\xi_m = [\gamma_0(\gamma_0 + 1)]^{1/2}\pi$ at the right boundary of branch n_2. This formula reveals that interelectrode distance $\delta(\gamma_0)$ is equal to $\sqrt{2}\pi$ in the nonrelativistic limit $(\gamma_0 \to 1)$ and tends to infinity in approximate proportion to γ_0 at $\gamma_0 \gg 1$. Thus, the solution branches on plane $\{\varepsilon_0, \delta\}$ stretch along δ as γ_0 increases. The lower two panels of the figure, where this plane is shown for $\gamma_0 = 3$ and 10, illustrate this.

Let us now consider the mode with reflection of particles of both types. Both electrons and positrons are reflected at the corresponding virtual emitters. Minimum η_m and maximum η_M PD values are $-1/2$ and $V + 1/2$, respectively. Two types of solutions are feasible. The first one (d_{2s} branches) has virtual emitters located near the corresponding emitters; the solutions of the second type ($d_{i,i}$ branches) have the e-VE located to the right of the p-VE. In both cases, $r_e = r_p$.

In the case of PDs belonging to branch d_{2s}, a wave with maximum $\eta_{\max} < V + 1/2$ cannot exist in the region to the left of the e-VE. A wave with minimum $\eta_{\min} > -1/2$ also cannot exist in the region to the right of the p-VE. The simplest solution type is a wave with a minimum located near the left electrode and a maximum located near the right electrode. Solutions with a higher number of waves are constructed by adding an integer number of half-waves between the e-VE and the p-VE. These solutions have all $\eta_{\min} = \eta_m = -1/2$ and all $\eta_{\max} = \eta_M = V + 1/2$.

Branches d_{2s} lie in the upper part of the (ε_0, δ) plane. The relation between ε_0 and reflection coefficient r for them is determined using Eq. (7) applied within the $[0, \xi_m]$ interval. It can be shown that the minimum of ε_0 at $\gamma_0 \gg 1$ is attained at $r = 0$ and decreases with increasing γ_0 as $\gamma_0^{-3/4}$; the maximum, in turn, is achieved at $r = 1$ and tends to $\sqrt{2}$ irrespective of V (this is $\sqrt{2}$ times lower than at $\gamma_0 = 1$). Adding an integer number of wavelengths to the PDs, we obtain new branches of solutions d_{2s}.

At the same time, solutions type 2 (branches $d_{i,i}$) allow for the existence of a wave with $\eta_{\max} = V + 1/2$ and minimum potentials $\eta_{\min} > -1/2$ within the $\xi < \xi_M$ region. Equation (7) applied within the $[\xi_m, \xi_M]$ interval provides an opportunity to establish the relation between η_m and reflection coefficient r. Condition $\eta_m \le 0$ yields a constraint on r:

$$r_{\lim} = \Big[\sqrt{\gamma_0 + 1} + \sqrt{\gamma_0 + 1 + 4\gamma_0 V + 4(\gamma_0 - 1)V^2} - \sqrt{\gamma_0 + 1 + 2\gamma_0(2V + 1) + (\gamma_0 - 1)(2V + 1)^2}\Big] \times \Big[\sqrt{\gamma_0 + 1 + 2\gamma_0(2V + 1) + (\gamma_0 - 1)(2V + 1)^2} + \sqrt{\gamma_0 + 1 + 4\gamma_0 V + 4(\gamma_0 - 1)V^2} - \sqrt{\gamma_0 + 1}\Big]^{-1}. \quad (10)$$

At $\gamma_0 \gg 1$ and $V = 0$, we find $r_{\lim} \to 1/(2\gamma_0)$. This implies that $\eta_{\min} \to -1/2$, and the solutions from branches $d_{i,i}$ transform into the solutions from branch d_{2s}; i.e., branches $d_{i,i}$ vanish.

Limit r value (10) corresponds to $\varepsilon_0 = 0$. The minimum potential is as the left electrode in this case. At $0 < r < r_{\lim}$, the expression for ε_0 is derived from relation (7) applied within the $[0, \xi_M]$ interval.

The possibility of existence of a wave with $\eta_{\min} = \eta_m = -1/2$ and $\eta_{\max} < \eta_M = V + 1/2$ in the region to the right of point ξ_m is proven in a similar fashion. To determine the relation between $\eta_{\max}$ and r, one needs to analyze (7) within the $[\xi_m, \xi_{\max}]$ interval. The value of r is also bounded from above here, and this value corresponds to zero field strength at the right electrode. The limit r value is $r_{\lim}$ (10). Thus, boundary value $r = r_{\lim}$ corresponds to zero electric field strengths at both electrodes.

Solution branches $d_{i,i}$ are constructed as follows. A total of i minima located to the left of the p-VE and i maxima located to the right of the e-VE are fixed, and parameter r is varied from $r_{\lim}$ (10) to zero. In this case, $|\varepsilon_0|$ varies from zero at $r = r_{\lim}$ to the maximum value corresponding to the boundary of the mode without particle reflection (formula (8) with $r = 0$). Thus, solution branches $d_{i,i}$ lie either in the $(0, \varepsilon_{0,\max})$ band or in the $(-|\varepsilon_{0,\max}|,0)$ band and originate at the points where branches n_s end.

It is evident from the figure that several solutions emerge when δ exceeds a certain value, and their number increases with δ. This is observed at all γ_0 values.

Thus, steady states of a diode with counter-propagating flows of relativistic electrons and positrons and zero potential difference between the electrodes were examined. It was demonstrated that, just as in the nonrelativistic case [5], the PDs are wave-like functions. A complete classification of all solutions in the modes both with and without particle reflection was presented.

Conflict of interest

The authors declare that they have no conflict of interest.

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