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Collapse of an ellipsoidal cavity attached to a flat wall

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A numerical model of the collapse of a gas cavity in the form of a semi-ellipsoid of rotation adjacent to a solid flat wall in a liquid is constructed. Calculations were compared with experiments in which nontrivial dynamics of such a cavity was observed, leading to the formation of a cumulative jet directed from the wall. The hydrodynamic mechanism of the cumulative jet formation has been established, and the main factors leading to the collapse dynamics observed in experiments have been identified. According to the model, these are the shape of the cavity, the proximity of the cavity to the surface and the pressure drops..

Keywords: collapse of a gas cavity in a liquid, cumulative jet.

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The study of bubble dynamics in liquids in the vicinity of solid surfaces has remained an important research topic for many years. The work of Rayleigh [1], who was the first to study the spontaneous formation and collapse of bubbles in liquid in an effort to explain the damage to driving propellers, may be regarded as a pioneering study into cavitating cavities in liquid. With the advent of numerical methods, the influence of non-spherical deviations on the evolution of a bubble in the vicinity of a rigid wall was analyzed by the boundary element method, which is a convenient tool in ideal liquid models [2–4]. This method was used in [5] to study the influence of deviation of the initial shape of a bubble from the spherical one and the thickness of the liquid layer between it and the wall on the dynamics of collapse. The authors of [6] have also attempted to factor in the non-sphericity of a bubble in a viscous liquid. In the case of an initially spherical bubble, models were refined through the introduction of additional parameters, such as surface tension [7], wall curvature [8], etc., into the physical model of the phenomenon. All these diverse examples are tied by a common pattern of formation of a cumulative jet as a result of collapse of a bubble: this jet is oriented toward the solid wall in the vicinity of which the bubble was located. However, a different scenario of bubble collapse was discovered in experiments [9] in the process of combustion of a stoichiometric propane-oxygen mixture in a bubble in water on a rigid wall in the form of a flat disk. The burnt gas bubble first expanded to a certain limit, assuming a shape close to a semi-ellipsoid of rotation, and then collapsed. A cumulative jet formed in the process of collapse near the axis of symmetry and then transformed into a vortex ring. The novel feature here is the orientation of this cumulative jet, which was directed away from the wall to which the bubble was adjacent. The phenomenon resembles the formation of a vortex ring when thermals rise, but gravity in these experiments is co-directional with propagation of the forming jet, which excludes a baroclinic mechanism. Thus, a qualitatively new scenario of collapse of a gas cavity near a wall leading to the formation of a jet and a vortex ring was discovered.

The aim of the present study is to construct a relevant numerical model and clarify the mechanism of formation of a cumulative liquid jet directed away from the wall during the collapse of a gas cavity attached to this wall. The collapse stage, which is crucial to understanding the mechanism of formation of a cumulative jet, is analyzed in the model. The model is tested by comparing the results of calculations and experiments on combustion of $3\,\mathrm{cm}^3$ of a stoichiometric propane-oxygen mixture performed under the conditions probed in [9].

Half-space $z < 0$ filled with liquid and bounded by a rigid flat wall at $z = 0$ is considered. A gas cavity in the form of a semi-ellipsoid of rotation about the z axis is positioned on the wall. The liquid is assumed to be ideal and incompressible, the motion is potential, and the force of gravity and surface tension are neglected. Numerical calculations and the comparison with experimental data start from the moment of maximum expansion of the cavity, which is taken as the initial time. Under the assumptions made above, the wall may be regarded as a symmetry plane, and the original problem is reduced to the problem of collapse of an ellipsoidal bubble in an infinite liquid. This problem is axisymmetric and is solved in cylindrical variables $(r,\ \varphi,\ z)$. Equations are written in a dimensionless form. Length, time, potential, pressure, and density are made nondimensional with a, $[\rho a^2/(P_\infty - P_i)]^{1/2}$, $[a^2(P_\infty - P_i)/\rho]^{1/2}$, $(P_\infty - P_i)$, and ρ, respectively. Here, a is the semi-axis perpendicular to axis z, P_i is the pressure in the bubble at the initial time, P_∞ is the pressure at infinity, and ρ is the liquid density. The liquid at infinity is assumed to be at rest, and the gas pressure inside the bubble is uniform and equal to the pressure in liquid at the bubble boundary. A system of equations, which includes the Laplace equation for the

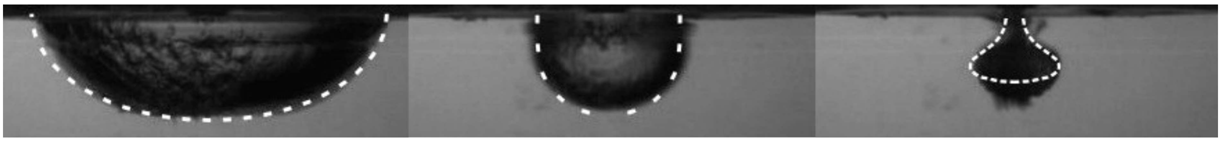

Figure 1. Calculated bubble contours in the axial plane (dashed lines) superimposed onto the shadow images obtained 0 (a), 3 (b), and 3.8 ms (c) after the onset of collapse.

velocity potential in liquid and the equation of state of gas in the bubble and is supplemented by kinematic and dynamic (Cauchy–Lagrange integral) conditions at the boundary, is solved to determine the dynamics of the cavity boundary:

$$\nabla^2\phi = 0, \qquad p_c v_c^{\gamma} = p_i, \qquad \frac{d\mathbf{r}}{dt} = \nabla\phi,$$

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}|\nabla\phi|^2 + p - 1 = 0,$$

r is the dimensionless radius vector of boundary points; $\nabla\phi$ values are taken at the bubble boundary; $p = p_c - p_i$; p_i and p_c are the dimensionless initial and current gas pressures in the bubble; $v_c = V_c/V_i$; V_i, V_c are the initial and current volumes of the ellipsoid, which are equal to twice the volume of the bubble; and γ is the adiabatic index.

The cavity surface deformation and the characteristics of liquid at the surface are calculated using the boundary element method detailed in [2,3]. This approach is applicable only up to the moment of collision of the cavity surface elements with each other, but provides the means to obtain detailed data characterizing the dynamics of cavity boundaries in the process of collapse. The potential of motion satisfies the Laplace equation within a certain region Ω (occupied by liquid) with piecewise-smooth boundary S (the bubble surface) and is considered to be a sufficiently smooth function. Since the symmetry plane is inside the bubble (and not between two bubbles as in [5]), the boundary integral equation takes the form

$$c(q)\phi(q) + \int\limits_S \phi(q')\frac{\partial}{\partial n}\left(\frac{1}{|q-q'|}\right)dS = \int\limits_S \frac{\partial}{\partial n}\big(\phi(q')\big)\frac{1}{|q-q'|}dS,$$

where point $q \in \Omega$, $q' \in S$, $\frac{\partial}{\partial n}$ is a derivative along the normal to S, $c(q) = 4\pi$ if $q \in \Omega$, and $c(q) = 2\pi$ if $q \in S$. Choosing point q on bubble surface S, one may obtain equations for determining the potential if its normal derivative (the velocity of the boundary) is known, and vice versa. The computational problem is simplified in axisymmetric formulations, since the potential and its derivative do not depend on angle φ and integration over this variable may be performed analytically. The two-dimensional boundary in variables (r, z) is replaced by a set of segments; quadratic approximation was used in calculations both for these segments and for the functions of potential and its normal derivative.

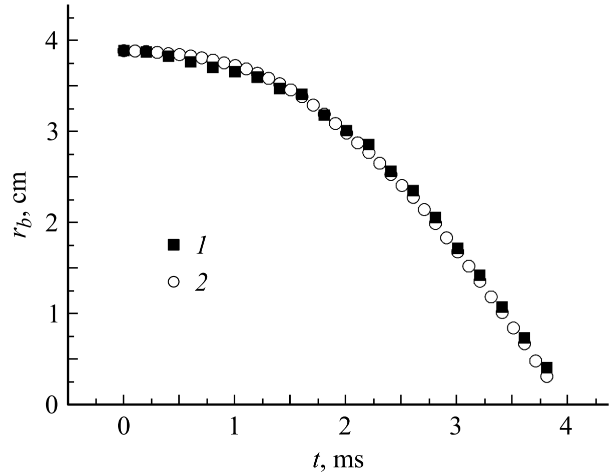

Figure 2. Temporal variation of experimental (*1*) and calculated (*2*) values of bubble boundary radius r_b near the wall.

The following strategy was used to determine the evolution of the boundary shape: since the initial shape of the bubble surface and the potential at the surface are known, we may resolve the discretized form of the boundary integral to find the normal velocity at the bubble surface. Knowing the potential, we may also calculate the tangential velocity at the surface and thus obtain the total gradient of potential at the surface or, in other words, the magnitude and direction of the boundary element velocity. This allows us to calculate the shape of the bubble boundary at the next time step and subsequently calculate a new potential distribution using the dynamic condition.

The value of a is determined directly from a shadow image, and V_i is derived in MatLab from a shadow image under the assumption of its axial symmetry. Uncontrolled heat losses in the process of expansion of a burnt gas charge [10] make it impossible to determine P_i. Therefore, P_i is varied, and its value is chosen so as to match the calculations with experimental data. The dimensional parameters of the problem are as follows: $a = 3.88$ cm, $\rho = 1$ g/cm^3, $P_\infty = 10^5$ Pa, $P_i = 0.11P_\infty$, $V_i = 146.3$ cm^3, and $\gamma = 1.24$ [10].

Figure 1 presents the calculated bubble contours in the axial plane (dashed lines) superimposed onto the shadow images obtained 0, 3, and 3.8 ms after the onset of collapse.

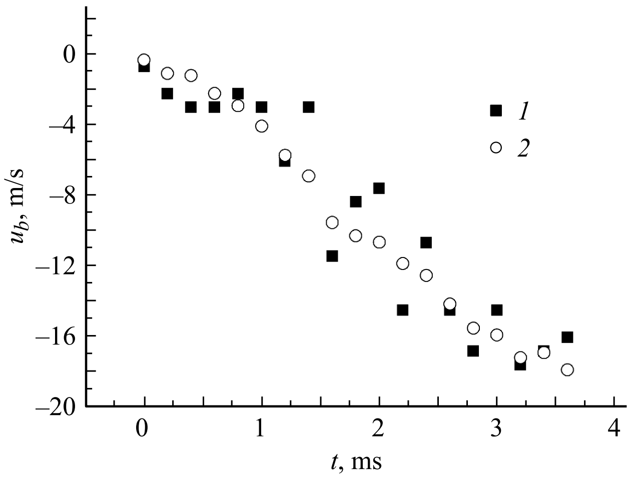

Figure 3. Experimental (*1*) and calculated (*2*) velocities $u_b = \Delta r_b/\Delta t$ of the bubble boundary near the wall at different moments in time with intervals $\Delta t = 0.2$ ms between them.

The temporal variation of experimental and calculated values of bubble boundary radius r_b near the wall is illustrated in Fig. 2. Figure 3 shows experimental and calculated velocities u_b of the bubble boundary near the wall at different moments in time with intervals $\Delta t = 0.2$ ms between them: $u_b = \Delta r_b/\Delta t$. It follows from Fig. 1 that the experimental and calculated boundaries agree qualitatively (specifically, in terms of the formation of a mushroom-shaped structure at the end of collapse). Figures 2 and 3 reveal a quantitative agreement between the experimental and calculated parameters characterizing the collapse of the bubble boundary near the wall. This qualitative and quantitative agreement between the results of calculations and experiments indicates that the constructed model is adequate to the observed process of cavity collapse.

The constructed model provides an opportunity to establish the mechanism of formation of a cumulative jet directed away from the wall. The formation of a mushroom-shaped structure implies that the boundaries of a bubble near the wall converge at an accelerated rate; i.e., the emergence of an annular jet several millimeters in thickness converging toward the axis is observed. According to the data in Fig. 3, the rate of jet convergence increases with time, eventually reaching ~ 20 m/s; thus, a significant kinetic energy is stored in it. When the annular jet converges, it slows down, inducing an increase in pressure on the wall surface in the vicinity of the symmetry axis. The pressure force imparts momentum to the liquid perpendicular to the surface, thus forming a cumulative jet with its kinetic energy borrowed from the energy of the converging annular jet. Thus, the cumulative jet directed away from the wall is formed from the annular jet converging toward the symmetry axis near the wall. In addition to clarifying the mechanism of formation of the cumulative jet, the model suggests that the key factors leading to its emergence are the geometric shape of a gas cavity, its proximity to the wall, and the pressure drop.

Conflict of interest

The authors declare that they have no conflict of interest.

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