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Quasilinear modification of the ion distribution function in the presence of ion Bernstein waves in tokamak plasma

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Received May 14, 2024
Revised July 12, 2024
Accepted July 12, 2024

In this paper, a quasilinear equation is derived that describes the evolution of the ion distribution function due to the ion Bernstein waves of intermediate frequency range. It is shown that the quasilinear equation can be reduced approximately to a one-dimensional equation in the space of transverse ion velocities, and the diffusion coefficient is proportional to the wave's power absorbed.

Keywords: ion Bersnstein wave, quasilinear diffusion.

DOI: 10.61011/TPL.2024.11.59661.19991

A number of anomalous phenomena accompanying the propagation of high-power microwave beams were discovered in experiments on electron cyclotron resonance heating in toroidal magnetic plasma confinement setups. Notable among them is the emergence of groups of accelerated ions [1,2] with no linear mechanisms of interaction with microwaves. However, this phenomenon may be treated as a consequence of low-threshold decay of a microwave into two upper hybrid waves [3]. The primary instability is saturated via a cascade of decays of primary daughter waves (with a high-frequency upper hybrid wave and a low-frequency ion Bernstein (IB) wave excited at each step). The excitation of exactly this type of daughter waves was verified by modeling the two-plasmon parametric decay of a microwave with extraordinary polarization by the particle-in-cell (PIC) method [4]. The interaction of daughter IB waves with ions may explain the effect of generation of groups of accelerated ions during electron cyclotron resonance heating. The quasilinear evolution of the ion distribution function needs to be analyzed in order to obtain a quantitative estimate of the result of the wave−particle interaction. In the present study, we derive a quasilinear equation characterizing the evolution of the ion distribution function. The effect of toroidal drift of ions in a non-uniform magnetic field with a finite curvature of the lines of force is taken into account. The magnetic shear effect, which induces a longitudinal component of the wave vector of a wave as a result of reprojection, is neglected. However, this effect is weaker than the one considered in the present study if condition $\Delta x_d/(\rho_i q) < 1$ is satisfied, where q is the safety factor, $\rho_i = v_{ti}/\omega_{ci}$ is the ion Larmor radius, v_{ti} is the ion thermal velocity, ω_{ci} is the ion cyclotron frequency, and Δx_d is the spatial size of the region where an IB wave is excited as a result of a cascade of decays.

Let us consider the local kinetic equation for collisionless plasma in non-uniform magnetic field $\mathbf{B} = B(x)\mathbf{e}_z$ that characterizes distribution function

$$\left(\frac{\partial}{\partial t} + v_j\frac{\partial}{\partial x_j} + v_{di,j}\frac{\partial}{\partial x_j} + \frac{Ze}{m_i}\left(E_j + e_{jkl}v_k B_l\right)\frac{\partial}{\partial v_j} + \omega_{ci}e_{jkz}v_k\frac{\partial}{\partial v_j}\right)f_i = 0, \tag{1}$$

where summation over repeating indices is implied, e_{jkz} is a fully antisymmetric unit tensor, Ze is the ion charge, $\mathbf{v}_{di} = \mathbf{e}_y(v_\perp^2 + 2v_z^2)/(R\omega_{ci})$ is the ion drift velocity, and R is the radius of curvature of a magnetic field line (the major radius of the setup). Equation (1) is written in an arbitrary coordinate system. While the Cartesian coordinate system $(x,\ y,\ z)$ is more convenient for descriptions of waves, the cylindrical coordinate system $(v_\perp,\ \theta,\ v_z)$, where θ is the azimuthal angle, is better suited for characterizing the motion of a particle in a magnetic field. The relation between the transverse components of velocity in both coordinate systems is set by equations $v_x = v_\perp\cos\theta$ and $v_y = v_\perp\sin\theta$. The solution to Eq. (1) is sought here in the form of a Taylor series expansion in wave amplitude $f_i = \bar{n}f_0 + f^{(1)}$, where $\bar{n}$ is the background density, $f_0(v_\perp, v_z)$ is the equlibrium ion distribution function independent of gyro angle θ, and $f^{(1)}$ is the linear correction to the distribution function with its frequency $\omega \gg \omega_{ci}$ and transverse wave number $q_\perp \gg 1/\rho_i$ „imposed“ by the electric field of an IB wave with amplitude A_0 propagating strictly transverse to the magnetic field

$$\mathbf{E} = \frac{\mathbf{A}}{2}\exp(i\mathbf{q}\cdot\mathbf{r} - i\omega t) + \text{c.c.} \tag{2}$$

The wave vector, which has components $\mathbf{q} = (q_x, q_y, 0)$, is the solution of the dispersion equation for longitudinal (IB) waves. The amplitude of the electric field of an IB wave has components $\mathbf{A} = -i(q_x, q_y, 0)A_0$. Inserting this expansion for the distribution function of non-equilibrium plasma into

Eq. (1) and isolating the terms of the first and second orders in electric field amplitude, we obtain the equations for linear corrections to the distribution function

$$\left(-i\alpha + i\lambda\cos(\theta-\psi) - \frac{\partial}{\partial\theta}\right) f^{(1)} = -\frac{\bar{n}Ze}{2m_i\omega_{ci}}\mathbf{A}\frac{\partial f_0}{\partial \mathbf{v}}, \quad (3)$$

where $\alpha = (\omega - \omega_{di})/\omega_{ci}$, $\omega_{di} = q_y v_{di}$, $\lambda = q_\perp v_\perp/\omega_{ci}$, $\psi = \arctan(q_y/q_x)$. Integrating Eq. (3), we find

$$f^{(1)} = i\frac{\bar{n}Ze}{2m_i\omega_{ci}} A_0 \exp\big(i\lambda\sin(\theta-\psi) - i\alpha\theta\big) \times \int_\infty^\theta d\theta' \exp\big(i\alpha\theta' - i\lambda\sin(\theta'-\psi)\big) \times \left[q_x\cos\theta'\frac{\partial}{\partial v_\perp} + q_y\sin\theta'\frac{\partial}{\partial v_\perp}\right] f_0(v_\perp, v_z). \quad (4)$$

Let us use representation

$$\exp\big(-i\lambda\sin(\theta'-\psi)\big) = \sum_{p=-\infty}^{\infty} J_p(\lambda)\exp\big(-ip(\theta'-\psi)\big),$$

where J_p — Bessel function. Since $q_x = q_\perp\cos\psi$, $q_y = q_\perp\sin\psi$, and $\cos\psi\cos\theta' + \sin\psi\sin\theta' = \cos(\theta'-\psi)$, integration over the azimuthal angle in expression (4) yields the following result:

$$f^{(1)} = c\frac{q_\perp A_0}{2B}\sum_{p=-\infty}^{\infty}\frac{\exp\big(i\lambda\sin(\theta-\psi) - ip(\theta-\psi)\big)}{\alpha - p} \times \frac{pJ_p(\lambda)}{\lambda}\frac{\partial}{\partial v_\perp} f_0. \quad (5)$$

Using (1), (2), and (5), we obtain the following for the stationary part of the distribution function of the equation of the second order in wave amplitude:

$$\frac{\partial}{\partial t} f_0 = -\frac{Ze}{2m_i}\left[A_x\left(\cos\theta\frac{\partial}{\partial v_\perp} - \sin\theta\frac{\partial}{v_\perp\partial\theta}\right) + A_y\left(\sin\theta\frac{\partial}{\partial v_\perp} + \cos\theta\frac{\partial}{v_\perp\partial\theta}\right)\right] f^{(1)^*} + \ldots. \quad (6)$$

Integrating the terms containing the θ-derivative over the azimuthal angle (inclusive of integrating by parts), we obtain

$$\frac{\partial}{\partial t} f_0 - \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp D(v_\perp, v_z)\frac{\partial}{\partial v_\perp} f_0 = 0, \quad (7)$$

where $D(v_\perp, v_z) = -c^2\frac{q_\perp^2|A_0|^2}{2B^2}\sum_{p=-\infty}^{\infty}\mathrm{Im}\left(\frac{1}{\alpha(v_\perp, v_z) - p}\right)\frac{p^2 J_p(\lambda)^2}{\lambda^2}$ is the quasilinear diffusion coefficient. The approach proposed in [5] makes it easy to demonstrate that an infinite sum of products of Bessel functions is equal to

$$\mathrm{Im}\left(\sum_{m=-\infty}^{\infty}\frac{m^2 J_m^2(\lambda)}{\alpha - m}\right) = \pi\alpha^2 J_\alpha(\lambda)^2\,\mathrm{Im}\big(\cot(\pi\alpha)\big) = -\alpha^2 J_\alpha(\lambda)^2\sum_{l=-\infty}^{\infty}\delta(\alpha - l). \quad (8)$$

Note that the parameters of an IB wave in the intermediate frequency range satisfy limit relations $\lambda \gg 1$ and $\omega/\omega_{ci} \gg 1$. Using the asymptotic expression for the Bessel function

$$J_\alpha(\lambda) \approx \sqrt{\frac{2}{\pi}}\frac{1}{\sqrt[4]{\lambda^2 - \alpha^2}}\cos\Psi,$$

where $\Psi = \sqrt{\lambda^2 - \alpha^2} - \alpha\arccos\left(\frac{\alpha}{\lambda}\right) - \frac{\pi}{4} \gg 1$ [6], and taking the rapidly oscillating nature of these functions into account, we retain the non-oscillating part in expression (8):

$$J_\alpha(\lambda)^2 \approx \frac{1}{\pi\sqrt{\lambda^2 - \alpha^2}} H(\lambda - \alpha),$$

$H(\lambda - \alpha)$ is the Heaviside step function. In view of the above, the quasilinear diffusion coefficient takes the form

$$D(v_\perp, v_z) = c^2\frac{|A_0|^2}{2B^2}\frac{\omega^2}{v_\perp^2}\frac{\omega_{ci}^2}{q_\perp}\frac{H(v_\perp - \omega/q_\perp)}{\sqrt{v_\perp^2 - \omega^2/q_\perp^2}} \times \sum_{l=-\infty}^{\infty}\delta\left(\omega - l\omega_{ci} - q_y\frac{v_\perp^2 + 2v_z^2}{R\omega_{ci}}\right), \quad (9)$$

where $\delta(\ldots)$ is the delta function, which is the real part of the inverse ion propagator of Eq. (3). Following the constant-energy resonance model [7], we obtain an approximate relation

$$D(v_\perp) \approx c^2\frac{|A_0|^2}{4B^2}\frac{\omega^2}{v_\perp^2}\frac{\omega_{ci}}{q_\perp} \times \sum_{l=-\infty}^{\infty}\frac{H(v_\perp - \omega/q_\perp)}{\sqrt{v_\perp^2 - \omega^2/q_\perp^2}}\frac{R\omega_{ci}^2/q_y}{\sqrt{\omega/\omega_{ci} - l}}\Bigg|_{v_\perp = \sqrt{\frac{\omega - l\omega_{ci}}{q_y/(R\omega_{ci})}}}. \quad (10)$$

Let us find the fraction of wave energy lost as a result of attenuation by ions:

$$Q = \frac{iA_0^*}{8\pi}\mathbf{q}\cdot\mathbf{j} = i\frac{\omega_{pi}^2}{\omega_{ci}}\frac{q_\perp^2|A_0|^2}{(8\pi)^2}\int v_\perp\cos(\theta-\psi) \times \sum_{p=-\infty}^{\infty}\frac{\exp\big(i\lambda\sin(\theta-\psi) - ip(\theta-\psi)\big)}{\alpha - p}\frac{pJ_p(\lambda)}{\lambda}\frac{\partial f_0}{\partial v_\perp}d\mathbf{v}.$$

Integration over the azimuthal angle yields the following expression:

$$Q = \frac{\omega_{pi}^2|A_0|^2}{(8\pi)^2}\sum_{l=-\infty}^{\infty}\frac{\omega}{q_\perp}\frac{H(v_\perp - \omega/q_\perp)}{\sqrt{v_\perp^2 - \omega^2/q_\perp^2}} \times \frac{R\omega_{ci}^2/q_y}{\sqrt{\omega/\omega_{ci} - l}}\frac{\partial}{\partial v_\perp} f_0\Bigg|_{v_\perp = \sqrt{\frac{\omega - l\omega_{ci}}{q_y/(R\omega_{ci})}}}. \quad (11)$$

Expression (11), which is proportional to the imaginary part of the linear susceptibility of plasma [8], specifies the energy density of a longitudinal wave in a non-uniform magnetic field. Comparing expressions (10) and (11), we find that the diffusion coefficient in the velocity space in Eq. (7) is proportional to the magnitude of specific losses of an wave:

$$D = Q \Big/ \bar{n} m_i \frac{\omega_{ci}}{\omega} \frac{v_\perp^2}{4\pi} \frac{\partial f_0}{\partial v_\perp}\bigg|_{v_\perp = \sqrt{\frac{\omega - l\omega_{ci}}{q_y/(R\omega_{ci})}}}. \tag{12}$$

Thus, the diffusion coefficient in the velocity space may be determined by analyzing the energy release of an IB wave, which does not require the calculation of the spatial distribution of electric fields and allows one to limit oneself to the examination of behavior of ray trajectories corresponding to a beam of waves for calculating their ion cyclotron absorption.

Note that it follows from (10) that the range of transverse velocities of ions capable of interacting with an IB wave corresponds to inequality

$$v_\perp^{tail} \approx \sqrt{\frac{R\omega_{ci}^2}{q_y}} > \frac{\omega}{q_\perp} \geqslant v_{ti}. \tag{13}$$

Inequality (13) is in reasonable agreement with the range of transverse velocities at which „tails" emerge in the function of ion distribution over these velocities [1,2].

Quasilinear equation (7) with diffusion coefficient (10), which characterizes the evolution of the ion distribution function as a result of absorption of power of ion Bernstein waves of the intermediate frequency range, may be used to analyze the efficiency of ion cyclotron heating and interpret the data from various toroidal plasma confinement experiments and applied in problems related to the characterization of ion acceleration in ionospheric and space plasma.

Funding

This study was supported by the Russian Science Foundation (grant No. 23-72-00024).

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by D.Safin