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Simulation of high power electrical atmospheric discharges using medium energy proton accelerators

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An experiment is proposed to create conditions for the occurrence of electrical atmospheric discharges by irradiating a neutral rain cloud with a flow of high-energy protons. It is shown that such an approach can eliminate problems in existing methods of lightning research, allowing one to observe the natural development of its leader at a given time and in a given area of the earth's surface. Estimates are made of the size and density of the rain cloud, the possibility of transporting protons through the atmosphere and their accumulation on the cloud, the irradiation time, and the amount of charge that must be imparted to the cloud to form an electrical breakdown in the atmosphere. The calculation results show that the proposed model can be implemented in practice.

Keywords: Linear accelerator of medium-energy protons, density of rain clouds, passage of protons through the atmosphere, multiple scattering, electrostatic field, beam of charged particles, ionization of air.

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Introduction

There are two primary reasons why lightning remains a phenomenon that is not fully understood to this day. First, it is hard to study it in the natural environment, since ground areas are affected at random (an average of 2–4 lightning strikes per a square kilometer of the Earth occur every year [1]) and new means and methods of detection are required. The so-called triggered lightning is a widely used method of artificial initiation of atmospheric electrical discharges. It involves launching small rockets either with a grounded wire (classical triggering) towed upward for a few hundred meters or with an ungrounded wire (altitude triggering) a few hundred meters in length, the end of which is positioned approximately 100 m above the ground, into a thundercloud [2]. However, 90% of classical triggered lightning events and discharges striking high-altitude objects are upward-moving, and only 10% are downward stepped leaders, which makes them almost „untraceable“ [1]. For example, the mechanisms of their initiation are unknown, since processes proceeding in the cloud environment cannot be probed [1–4].

Second, the lack of similarity between the phenomena of atmospheric electrical breakdown of different scales does not allow one to simulate natural processes in their entirety in experiments with a long laboratory spark. Specifically, the average channel length ($L \sim 3$ km) and leader speed ($\upsilon \sim 10^6$ m/s) of actual lightning in a cloud-to-ground gap are several orders of magnitude greater than the corresponding parameters of a laboratory spark ($L \sim 10$ m and $\upsilon \sim 10^4$ m/s) [1,4]. The maximum recorded length and duration of an intercloud discharge are 768.8 km and 17.7 s [5] (the average length of an intercloud discharge is on the order of several kilometers [1]). In contrast to a laboratory spark, lightning is an electrodeless discharge [4]. The cloud environment is a very poor conductor; charges in it are scattered on the surface of droplets and are not transported directly to the lightning leader, instead acting only as a source of the electric field that initiates and supports the leader process [1]. In addition, the potential of a downward leader at the ground may differ significantly from the potential at the starting point, while a laboratory spark always carries the potential of the electrode [1].

Thus, fundamentally new study approaches are needed to overcome the mentioned barriers on the way toward understanding the nature of atmospheric electrical discharges and developing reliable means of lightning protection.

The aim of the present study is to assess the possibility of charging a rain cloud to an electric field strength at which an atmospheric electrical discharge is initiated and evolves naturally in a cloud-to-ground or cloud-to-cloud gap. This approach provides an opportunity to establish the conditions for initiation of lightning artificially and observe its evolution in the natural environment.

1. Physical model of the proposed experiment

Figure 1 illustrates the proposed method for investigation of atmospheric electrical discharges. When a natural or artificial (e.g., sprayed from an airplane) rain cloud approaches,

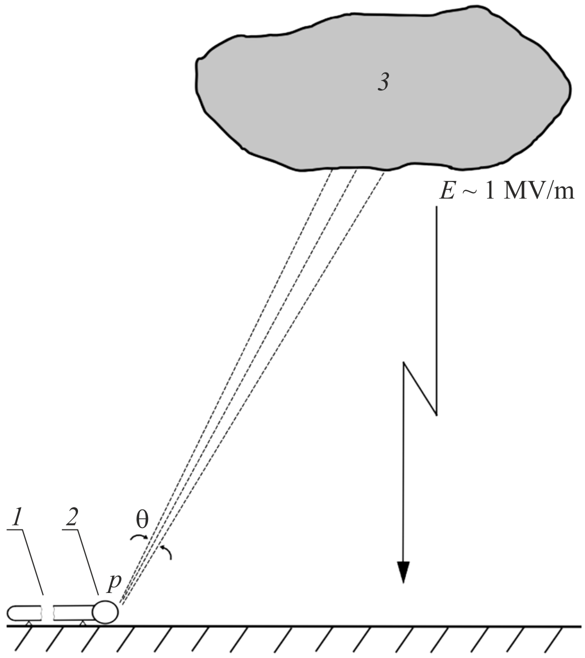

Figure 1. Diagram of generation of atmospheric electrical discharges: *1* — linear accelerator of medium-energy protons; *2* — deflecting magnets at the accelerator output; *3* — rain cloud; *p* — proton beam; and θ — angle of multiple scattering and deflection of protons in the cloud field.

a linear accelerator of medium-energy protons [6] generates a beam of protons with an energy sufficient to penetrate the atmosphere and enter the cloud. Irradiation time τ is set so as to produce an electric field near the cloud within which breakdown occurs under actual atmospheric conditions. It follows from the experimental data collected in rocket sounding of thunderclouds and flights of specially equipped laboratory aircraft through such clouds that the electric field strength does not exceed 1 MV/m within them and is 0.1–0.8 MV/m in their immediate vicinity [1,2]. The maximum recorded electric field strength of a thundercloud ($E_{\max} \approx 1$ MV/m) is used in the present study as an estimate of the level at which an atmospheric electrical discharge is guaranteed to occur.

Let us evaluate the optimum parameters of a rain cloud at which protons with a given initial energy enter the cloud positioned at a given altitude and remain within it.

2. Parameters of a rain cloud satisfying the experimental conditions

A rain cloud at an altitude of approximately 500 m is a two-phase system consisting of air and water droplets. The density of such a system is written as [7]

$$\rho_{cloud} = \rho_{water} a + \rho_{air}(1 - a), \tag{1}$$

where a is the volume fraction of water droplets in a cloud:

$$a = n \cdot V_{drop} = \frac{n\pi d^3}{6}, \tag{2}$$

$\rho_{water} = 10^3$ kg/m^3 is the density of water, $\rho_{air} = 1.29$ kg/m^3 is the density of air, n is the concentration of water droplets in a cloud, V_{drop} is the volume of a spherical droplet, and d is the droplet diameter.

The water content of clouds is defined as mass m_{water} of condensed water per unit volume V_{cloud} [8] of a cloud:

$$\omega = \frac{m_{water}}{V_{cloud}} = \frac{n \cdot m_{water}}{N_{drop}} = \frac{n\rho_{water} V_{drop} N_{drop}}{N_{drop}} = \frac{n\rho_{water}\pi d^3}{6}, \tag{3}$$

where N_{drop} is the number of droplets in a cloud.

Combining (2) and (3), one may present the volume fraction of droplets in the following form:

$$a = \frac{\omega}{\rho_{water}}. \tag{4}$$

The precipitation rate is written as [8]

$$I = \frac{\pi}{6} d^3 n(v_{drop} - u), \tag{5}$$

where v_{drop} is the velocity of a falling droplet, [m/s]; u is the updraft velocity, [m/s].

Equations (3) and (5) yield the following relation between the water content and the precipitation rate:

$$\omega = \frac{I\rho_{water}}{v_{drop} - u}. \tag{6}$$

Inserting (4) into (1), we obtain a dependence of the density of a cloud on its water content:

$$\rho_{cloud} = \rho_{air} + \omega\left(1 - \frac{\rho_{air}}{\rho_{water}}\right). \tag{7}$$

As was demonstrated in [8,9], the fall velocity of droplets is proportional to their diameter (Table 1), and the diameter of droplets and the precipitation rate are proportional to the updraft velocity (Figs. 2, 3).

Since protons will be distributed along the surface of water droplets within a cloud, its water content should be maximized to make the charging process more efficient. In the natural environment, rain clouds commonly form at continuous updraft velocities $u = 1$ m/s [9]. The parameters of such clouds are presented in Table 1 and Figs. 2 and 3: $d = 4 \cdot 10^{-3}$ m; $v_{drop} = 8.83$ m/s; $I = 1.3 \cdot 10^{-4}$ m/s.

According to (6), (7), their maximum water content and density are

$$\omega = 0.017\ \text{kg/m}^3,$$
$$\rho_{cloud} = 1.31\ \text{kg/m}^3. \tag{8}$$

For simplicity, we perform calculations for a spherical cloud with radius R that consists of air only and is positioned at altitude $h = 500$ m above the ground. Let us

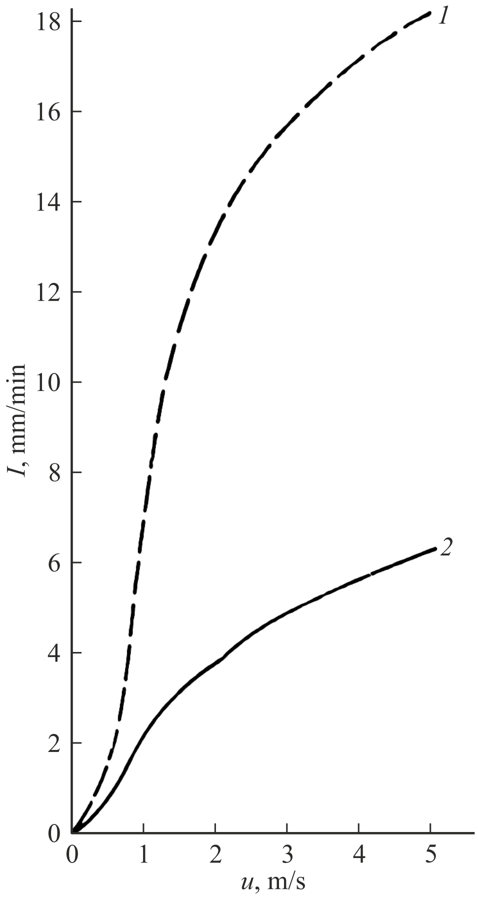

Figure 2. Maximum precipitation rate at different updraft velocities in clouds [9]: *1* — with continuous updraft; *2* — with updraft ceasing after the onset of precipitation.

Table 1. Dependence of the terminal velocity of a droplet on its size [8]

Diameter, mm	Fall velocity, m/s	Diameter, mm	Fall velocity, m/s
0.1	0.27	2.6	7.57
0.2	0.72	2.8	7.82
0.3	1.17	3.0	8.06
0.4	1.62	3.2	8.26
0.5	2.06	3.4	8.44
0.6	2.47	3.6	8.60
0.7	2.87	3.8	8.72
0.8	3.27	4.0	8.83
0.9	3.67	4.2	8.92
1.0	4.03	4.4	8.98
1.2	4.64	4.6	9.03
1.4	5.17	4.8	9.07
1.6	5.65	5.0	9.09
1.8	6.09	5.2	9.12
2.0	6.49	5.4	9.14
2.2	6.90	5.6	9.16
2.4	7.27	5.8	9.17

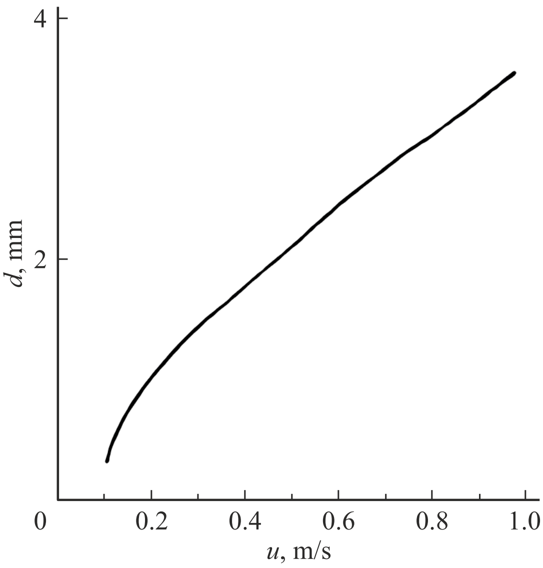

Figure 3. Dependence of the diameter of droplets in the initial period of rainfall on the updraft velocity in the cloud [9].

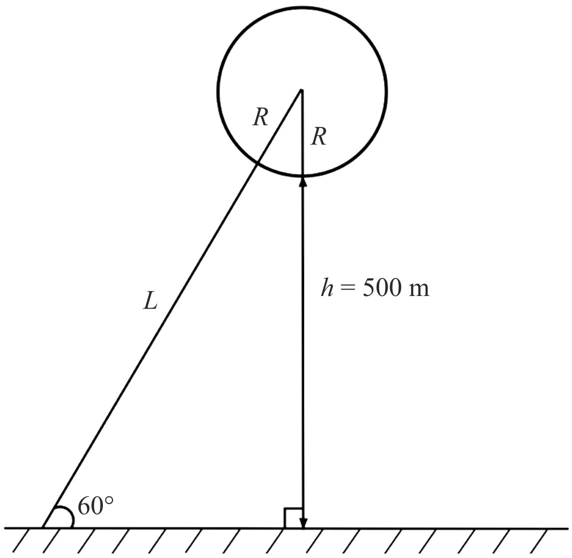

Figure 4. Geometric characteristics of the considered model.

assume that protons with energy $W = 400\,\text{MeV}$ leave the accelerator at an angle of 60° to the horizon (Fig. 4).

The range of a proton in air at this energy is $\lambda_{air} = 726\,\text{m}$ [10].

The following condition must be satisfied for protons to remain within the cloud:

$$L < \lambda_{air} \leq L + 2R, \quad (9)$$

where L is the distance traveled by a proton in the atmosphere and R is the cloud radius.

Parameters L and R may be expressed from the geometric characteristics of the model (Fig. 4):

$$\sin 60^\circ = \frac{500 + R}{L + R}. \quad (10)$$

Conditions (9) and (10) specify number k of cloud diameters D that fit within the distance traveled by a proton in air:

$$\begin{cases} L + 2R = 726 \\ \frac{500+R}{L+R} = \frac{\sqrt{3}}{2} \end{cases} \Rightarrow \begin{cases} L = 588\,\text{m} \\ R = 69\,\text{m} \end{cases} \Rightarrow k = \frac{\lambda_{air}}{D} = 5.26.$$

The proton range in air (in kg/m^2) per cloud diameter is then

$$\frac{\rho_{air}\lambda_{air}}{k} = 178.2\,\text{kg/m}^2,$$

the distance traveled by a proton in the cloud with density ρ_{cloud} obtained in (8) is

$$\lambda_{cloud} = \frac{\rho_{air}\lambda_{air}}{k\rho_{cloud}} = 136\,\text{m},$$

and the condition under which particles remain within the cloud with diameter D takes the form

$$0 < \lambda_{cloud} \leq D. \quad (11)$$

3. Multiple proton scattering

Propagating through the atmosphere, protons undergo numerous collisions, which alter their trajectory. The theory of multiple scattering allows one to characterize this process [11,12]. Since all parameters in the present study are estimated in order of magnitude, scattering off nitrogen atoms, which constitute 80% of the atmosphere and naturally produce the greatest contribution to the scattering process, will be examined:

- atomic mass of the scattering particle (nitrogen atom) $A = 14\,\text{Da}$;
- charge of the scattering particle (nitrogen atom) $Z = 7$;
- proton charge $z = 1$;
- distance traveled by a proton in air $l = L\rho_{air} = 758\,\text{kg/m}^2$;
- kinetic energy of a proton $W = 400\,\text{MeV}$;
- speed of light $c = 2.99 \cdot 10^8\,\text{m/s}$;
- proton mass $m_p = 1.67 \cdot 10^{-27}\,\text{kg}$.

The multiple scattering angle in [12] is determined as

$$\theta_{scat} = x_w \chi_c \sqrt{B}, \quad (12)$$

where

$$B - \ln B = b, \quad (13)$$

Table 2. Values of B and $\sqrt{B}$ at different parameters b [12]

b	B	$\sqrt{B}$	b	B	$\sqrt{B}$
3.2	4.760	2.182	8.8	11.217	3.349
3.6	5.260	2.294	9.2	11.656	3.414
4.0	5.749	2.398	9.6	12.093	3.477
4.4	6.229	2.496	10.0	12.528	3.539
4.8	6.702	2.589	10.4	12.962	3.600
5.2	7.170	2.678	10.8	13.395	3.660
5.6	7.632	2.763	11.2	13.827	3.718
6.0	8.091	2.844	11.6	14.257	3.776
6.4	8.545	2.923	12.0	14.678	3.832
6.8	8.997	2.999	12.4	15.116	3.888
7.2	9.446	3.073	12.8	15.544	3.943
7.6	9.892	3.145	13.2	15.971	3.996
8.0	10.336	3.215	13.6	16.397	4.049
8.4	10.777	3.283	14.0	16.823	4.102

and b is given by

$$b = \ln\left[2730\left(\frac{(Z+1)Z^{\frac{1}{3}}z^2 l}{A\beta^2}\right)\right] - 0.1544 = 12.83, \quad (14)$$

where

$$\beta = \frac{v}{c} = \sqrt{1 - \frac{1}{\left(1 + \frac{W}{m_p c^2}\right)^2}} = 0.72.$$

Equation (13) was solved for different values of b in [12]. The results of these calculations are listed in Table 2.

The value of $b = 12.83$ obtained in (14) corresponds to $B = 15.54$ and $\sqrt{B} = 3.94$.

Parameter χ_c characterizes the angle at which an average of just one collision occurs at $\chi > \chi_c$. In [11,12], this parameter was expressed as

$$\chi_c = \sqrt{0.157\left(\frac{Z(Z+1)z^2}{A}\right)\left(\frac{l}{(pv)^2}\right)}. \quad (15)$$

In the case of relativistic velocities [10],

$$pv = \sqrt{W\beta(W + 2m_p c^2)} = 809.72\,\text{MeV}. \quad (16)$$

Inserting (16) into (15), we obtain

$$\chi_c = 0.0085\,\text{rad}.$$

The dependence of parameter χ_w on B is presented in Fig. 5. The value of $B = 15.54$ corresponds to $x_w = 0.96$.

Equation (12) then yields the multiple proton scattering angle:

$$\theta_{scat} = 0.032\,\text{rad} = 1.84^\circ. \quad (17)$$

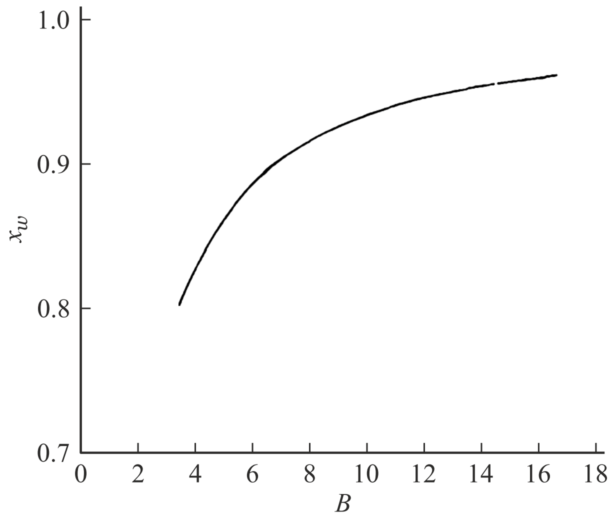

Figure 5. Variation of x_w with parameter B [12].

4. Combined influence of multiple scattering and deflection by the electrostatic field of a cloud on the proton trajectory

As the cloud gets charged, the electrostatic field it produces (E) contributes to the deflection of protons from their original trajectory. We use the approach proposed in [13] to take into account the simultaneous influence of multiple scattering of protons and their deflection by the macroscopic cloud field. This influence is characterized by the following system of equations:

$$\begin{cases} \frac{d\mathbf{p}}{dt} = -e\mathbf{E} - g(w)\frac{\mathbf{p}}{p}, \\ \frac{d\eta}{dt} = \frac{e\sqrt{\eta^2-1}}{p}\left(E_\perp + \sqrt{\eta^2-1}E_\parallel\right) + v \cdot f(W), \\ \mathbf{v} = \frac{c\cdot\mathbf{p}}{\sqrt{p^2+m_p^2c^2}}, \\ \frac{d\mathbf{s}}{dt} = \mathbf{v}, \end{cases} \tag{18}$$

where $\mathbf{p}$ is the momentum of a proton, $\mathbf{v}$ is its velocity, $\mathbf{s}$ is the radius vector of a proton drawn from the point where it left the accelerator, and η is the inclination coefficient establishing a correspondence between distance ds traveled along the proton trajectory and its projection dx onto the axis going through the center of the cloud (Fig. 6):

$$ds = \eta dx$$

$$\eta = \frac{1}{\cos\theta}, \tag{19}$$

where θ is the overall angle of deflection of protons from the initial trajectory with account for the simultaneous influence of their multiple scattering and the electrostatic field of the cloud.

The first equation in (18) characterizes the motion of a proton in electric field $\mathbf{E}$ with account for energy losses,

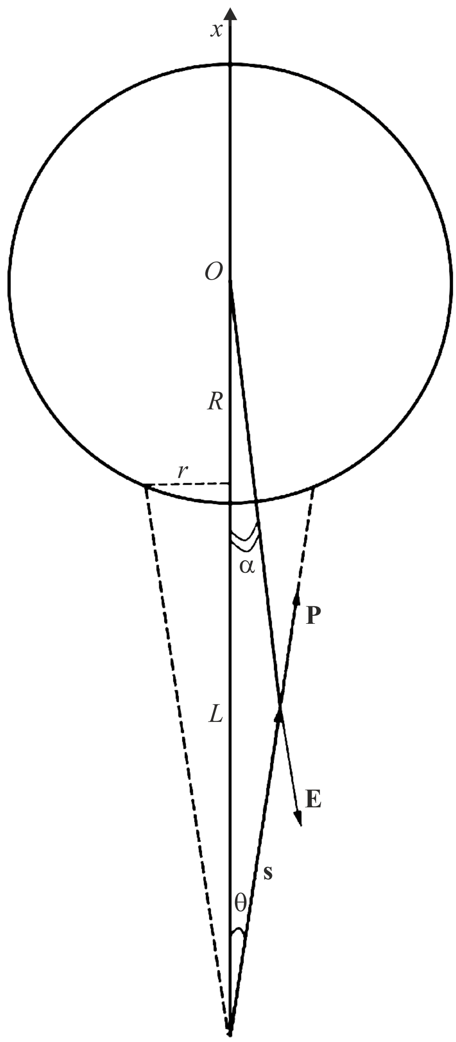

Figure 6. Multiple scattering and deflection of protons in the cloud field.

where $g(W) = -\frac{dW}{ds}$ is the energy loss per unit length for a proton propagating through air. The second equation characterizes the rate of variation of the inclination coefficient as a result of deflection by the electrostatic field of the cloud and multiple scattering, where $2f(W) = d\overline{\theta^2_{scat}}/ds$ is the mean square of the multiple proton scattering angle per unit path length [13].

This summation of angles is made possible by the fact that multiple scattering of protons is a statistical process [11,12] and the effect of deflection by the macroscopic field of the cloud persists throughout the entire length L of the ground-to-cloud trajectory of a proton.

At energy $W = 400$ MeV:

- $g(w) = 4.2 \cdot 10^{-14}$ J/m [10];
- $f(w) = 8.7 \cdot 10^{-7}$ rad^2/m by virtue of (12) and (17).

Projecting the first and third equations from (18) onto the momentum direction and the fourth equation onto the direction parallel to the axis going through the center of the spherical cloud (Figs. 6 and 7), we transform system (18)

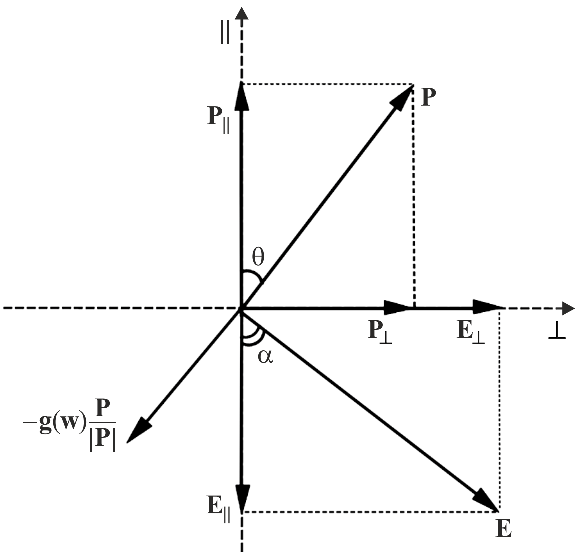

Figure 7. Vector diagram of forces acting on a moving proton, where || — direction parallel to axis X that goes through the center of the cloud (Fig. 6); ⊥ — direction perpendicular to axis X.

to the form

$$\begin{cases} \frac{dp}{dt} = eE\left(\sin\alpha \cdot \sqrt{1 - \frac{1}{\eta^2}} - \frac{\cos\alpha}{\eta}\right) - g(w), \\ \frac{d\eta}{dt} = \frac{eE}{p} \cdot \sqrt{\eta^2 - 1}\left(\sin\alpha + \cos\alpha\sqrt{\eta^2 - 1}\right) + v \cdot f(W), \\ v = \frac{c \cdot p}{\sqrt{p^2 + m_p^2 c^2}}, \\ \frac{dx}{dt} = \frac{v}{\eta}, \end{cases} \tag{20}$$

where α is the angle of inclination of electric-field vector $\mathbf{E}$ to the axis going through the center of the cloud, $e = +1.6 \cdot 10^{-19} C$ is the proton charge, and x is the distance traveled along the axis going through the center of the cloud.

The initial conditions are as follows:

$$\begin{cases} \eta(0) = 1, \\ p(0) = 6 \cdot 10^{-19}\, \frac{\text{kg}\cdot\text{m}}{s}, \\ x(0) = 0\,\text{m}. \end{cases} \tag{21}$$

Solving system (20) numerically for a cloud with radius $R = 90$ m at time point $t = 2.95 \cdot 10^{-6}$ s when protons reach the cloud, we find

$$\eta = 1.0078,$$
$$p = 3.87 \cdot 10^{-19}\, \frac{\text{kg} \cdot \text{m}}{s},$$
$$x = 634.54\,\text{m},$$

which yields, when inserted into (19), the following value of overall angle θ of deflection of protons from the initial trajectory:

$$\theta = 0.1242\,\text{rad} = 7.12°.$$

The proton spot radius on the cloud is then

$$r = x(t) \cdot \text{tg}\,\theta \simeq 80\,\text{m}. \tag{22}$$

In further analysis, we consider a cloud with radius $R = 90$ m.

5. Beam current limitation by volume charge

When a charged particle beam is being transported, the current density is limited by the accumulation of charge in the head of the beam that causes a redistribution of potential and the emergence of a „virtual anode (cathode)," which is a potential barrier reflecting a fraction of particles with insufficient energy. The Boguslavsky–Langmuir law specifies the maximum current that may be passed through a gap. To evaluate the influence of this effect on a scattered proton beam with a conical shape, we consider two limit cases of planar and spherical geometries.

In the planar case, this limitation may be written as [14]

$$j_b^{\max} = 54.6 \cdot \frac{\varphi^{3/2}}{L^2},$$

where φ is the proton-accelerating potential, [MV]; L is the accelerator–cloud distance, [m].

The maximum current density for a proton beam with energy $W = e\varphi = 400$ MeV and distance $L = 588$ m from the accelerator to the cloud is

$$j_b^{\max} = 1.26\, \frac{\text{A}}{\text{m}^2}. \tag{23}$$

In the spherical geometry [15],

$$j_b^{\max} = 54.6 \cdot \frac{\varphi^{3/2}}{\alpha^2\left(\frac{r_a}{r_c}\right) r_a^2},$$

where r_c is the cathode radius, [m]; r_a is the anode radius, [m]; and α is the Langmuir function.

We set cathode radius $r_c = 0.01$ m equal to the proton beam radius and anode radius $r_a = L = 588$ m equal to the distance to the cloud. With these parameters, the Langmuir function squared is $\alpha^2 \approx 8.5$ [16]. The maximum proton current density in the spherical case is then

$$j_b^{\max} = 0.15\, \frac{\text{A}}{\text{m}^2}. \tag{24}$$

As was demonstrated above (22), the proton beam radius near the cloud is $r = 80$ m, which corresponds to the following spot area on the cloud:

$$S = \pi r^2 = 2 \cdot 10^4\,\text{m}^2.$$

The beam current in accelerators of the examined type may reach $i_b = 0.1$ A [17,18]. The beam current density at the point of entry into the cloud is then

$$j_b = \frac{i_b}{S} \simeq 5 \cdot 10^{-6}\, \frac{\text{A}}{\text{m}^2}. \tag{25}$$

It follows from the comparison of Eqs. (23)–(25) that condition $j_b \ll j_b^{\max}$ is satisfied by a wide margin.

Thus, a beam of protons with an energy of 400 MeV has a current density low enough to cover a distance of 588 m from the ground to the cloud.

6. Estimation of parasitic ion current near the cloud

As protons propagate through the atmosphere, air molecules within the channel will get ionized. It was demonstrated in [19 (p. 94, Fig. 3.5)] that the time of electron attachment to neutral molecules in the lower layers of the atmosphere is on the order of 10^{-8} s; therefore, the primary contribution to conductivity is produced by positive and negative light ions. Having low mobility and a fairly long lifetime, light ions [20] induce a parasitic current under the influence of the electric field of a positively charged cloud. This current may interfere with the accumulation of positive charge on the cloud.

Without going into details regarding the complex kinetics of processes inside the ionized channel, we estimate the maximum strength of parasitic current based on the law of conservation of energy:

$$dA = dQ + d\Pi,$$

where A is the work of the accelerator beam current, Q is the Joule heating caused by parasitic currents, and Π are beam energy losses that are not associated with Joule heating.

$$P = \int_V \left([\sigma_1 + \sigma_2]E^2 + \frac{d\Pi}{dtdV} \right) dV, \quad (26)$$

where $P = \frac{dA}{dt} = i_b \frac{\gamma mc^2}{e} = 40\,\text{MW}$ is the accelerator beam power; σ_1 is the air conductivity in the region of cloud irradiation; $\sigma_2 = 10^{-14}\Omega^{-1} \cdot m^{-1}$ is the air conductivity outside the irradiation region that corresponds to the average conductivity of air under normal conditions [20]; and E is the strength of the electric field of the cloud.

Assuming that energy Π is unknown, we may present formula (26) as an inequality:

$$P > \int\limits_V \left([\sigma_1 + \sigma_2]E^2 \right) dV. \quad (27)$$

Let us consider the volume of a small layer of gas adjacent to the cloud:

$$\Delta V = S \cdot \Delta L,$$

where ΔL is the layer thickness chosen in such a way that the electric field strength remains approximately constant and equal to the maximum value $E_{\max} \simeq 10^6$ V/m recorded within a thundercloud under real atmospheric conditions (which is fulfilled within the accuracy of 10% in a layer with a thickness of 5 m).

Inequality (27) assumes the following form for such a layer:

$$\delta P > \sigma_1 E^2 \Delta V_1 + \sigma_2 E^2 \Delta V_2, \quad (28)$$

where $\delta = \frac{\Delta L}{\lambda_{air}}$ is the fraction of beam energy lost in layer ΔL; $\lambda_{air} = 726$ m is the integral range of a proton in air; $\Delta V_1 = S_1 \Delta L$ is the volume of the gas layer in the irradiation region near the cloud; $S_1 = \pi r^2 = 2 \cdot 10^4\,\text{m}^2$ is the area of the spot on the cloud; $\Delta V_2 = S_2 \Delta L$ is the volume of the gas layer outside the irradiation region near the cloud; and $S_2 = S_{cloud} - S_1 = 8.2 \cdot 10^4\,\text{m}^2$ is the area of the cloud surface without the irradiation region.

Having performed certain transformations, we obtain the following from (28):

$$i_{par1} < \frac{P}{\lambda_{air} E} - i_{par2}.$$

The parasitic current outside the irradiation region is

$$i_{par2} = \sigma_2 E S_2 = 8.2 \cdot 10^{-4}\,\text{A}.$$

The current within the irradiation region is then

$$i_{par1} < 0.054\,\text{A},$$

and the following inequality is satisfied for the total parasitic current near the cloud:

$$i_{par} < 0.055\,\text{A}. \quad (29)$$

7. Cloud charging time

Let us determine the charge that should be imparted to a cloud with radius $R = 90$ m to guarantee a breakdown in a real-world environment. The potential near such a cloud should be

$$E_{\max} = \frac{\varphi}{R} \Rightarrow \varphi = 90\,\text{MV},$$

which corresponds to charge

$$\varphi = \frac{q}{4\pi\varepsilon_0 R} \Rightarrow q \simeq 0.9\,\text{C}.$$

According to (29), the minimum effective current charging the cloud is

$$i_{\text{eff}} = i_b - i_{\text{par}} = 0.045\,\text{A}.$$

The maximum time for charging the cloud with radius $R = 90$ m to potential $\varphi = 90$ MV is then

$$\tau = \frac{q}{i_{\text{eff}}} = 20\,\text{s}.$$

Conclusion

Calculations carried out for initial proton energy $W = 400$ MeV, an accelerator beam current of 0.1 A, and a spherical cloud positioned at altitude $h = 500$ m above the ground revealed the following:

1. With the energy losses, multiple scattering, and deflection by the electrostatic cloud field taken into account, the condition of all particles reaching a cloud with a radius of 90 m and remaining within it is satisfied.

2. The overall angle of deflection of protons from the initial trajectory under the simultaneous influence of multiple scattering and the electrostatic field of a charged cloud is 7.12°, and the radius of the spot on the cloud is 80 m.

3. The maximum density of a rain cloud at an updraft velocity of 1 m/s is 1.31 kg/m^3. The maximum mass of water for a cloud with a radius of 90 m is close to 52 t, which is within the carrying capacity of heavy transport aircraft.

4. Transport of high-energy protons through the atmosphere and their accumulation on the cloud to the point of a lightning discharge are physically feasible. The transfer of 0.9 C of charge to a cloud 90 m in radius is needed for this purpose. The corresponding charging time is 20 s, which will require 800 MJ of accelerator energy.

The obtained results provide a quantitative illustration of the proposed method for study of electrical atmospheric discharges and suggest that it should be applicable in practice not only at the exact parameters examined above, but also in a wider range of parameter values. For example, high-power linear medium-energy accelerators reach proton energies on the order of 1 GeV [17,18]. At this energy, the mean free path of a proton in air is approximately equal to 3 km [10], matching the average length of the lightning channel in the cloud-to-ground gap [1]. Thus, the proposed method opens the prospect of conducting full-fledged experiments with real lightning discharges and developing advanced means of lightning protection.

Conflict of interest

The authors declare that they have no conflict of interest

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