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Unusual radiating Josephson vortex

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Received April 2, 2024
Revised June 13, 2024
Accepted July 31, 2024

Traveling vortices in a Josephson junction embedded in a decelerating media are studied. It is shown that in such a system, in addition to the usual Josephson vortices whose velocity is limited by the Swihart velocity, there can exist unusual vortices having a large limiting velocity. The conditions for the parameters of the Josephson junction and the external medium where these vortices can radiate electromagnetic waves into the medium due to the Vavilov−Cherenkov effect have been established. The characteristic frequencies of these waves fall in the terahertz region and are smoothly tunable, which is interesting for the use of unusual vortices in the design of compact superconductor devices.

Keywords: Josephson junction, Cherenkov radiation, terahertz radiation.

DOI: 10.61011/TP.2024.10.59351.109-24

Introduction

Terahertz radiation in a general sense is now commonly associated with the frequency range of electromagnetic waves extending from ~ 0.1 to $\sim 10\,\text{THz}$ [1,2]. The search for sources, detectors, amplifiers, and filters to be used in this range has been ongoing for several decades. Solid-state terahertz physics is an important research direction in this field [3,4]. Devices utilizing the Josephson effect (specifically, flux-flow oscillators [5–9] and devices with synchronized arrays of Josephson junctions [10] or those serving as flexible waveguides for redirecting or splitting electromagnetic waves [11]) may be of interest in low-temperature applications.

Traveling vortices in long Josephson junctions, which emit electromagnetic waves due to the Vavilov−Cherenkov effect, are a promising terahertz radiation source. For example, Cherenkov radiation of vortices in a system consisting of two Josephson junctions was examined in [12–13], and Cherenkov radiation in the tail of a vortex traveling along a junction formed by bulk superconductors was characterized in [14].

An enquiry into the possibility of emission of Cherenkov radiation into the external medium from a junction in the sandwich geometry is a natural evolution of the idea of Cherenkov radiation of Josephson vortices. The intensity of this radiation may exceed the one inside the junction [15]. For this to be feasible, the velocity of a Josephson vortex should exceed the speed of light in the surrounding medium. It is not easy to fulfill such a condition for usual vortices. It is demonstrated below that an unusual Josephson vortex may exist in the sandwich geometry. The maximum velocity of this unusual vortex may exceed significantly the Swihart velocity in a sandwich, relaxing the requirements to the speed of light in a decelerating medium surrounding the sandwich.

1. Main equations

Let us consider a long Josephson sandwich surrounded by a decelerating medium with permittivity ϵ_m (see the figure). A sandwich is formed by planar superconducting electrodes occupying regions $-d-L<x<-d$ and $d<x<d+L$, which are separated by a thin tunnel layer with width $2d$. Let us assume that the magnetic field has only one component ($\mathbf{H}=H\mathbf{e}_y$), while the electric field has two components: $\mathbf{E}=E_x\mathbf{e}_x+E_z\mathbf{e}_z$. It is assumed that the fields and phase difference $\varphi(z,t)$ of the superconducting order parameter across the Josephson junction are independent of coordinate y.

Having solved Maxwell's equations in all five regions of the structure with account for the boundary conditions at $x=\pm d$ and $\pm d\pm L$, one may write an integro-differential equation characterizing the evolution of the phase difference. It takes the following form for a vortex traveling with velocity $v>0$ [16]:

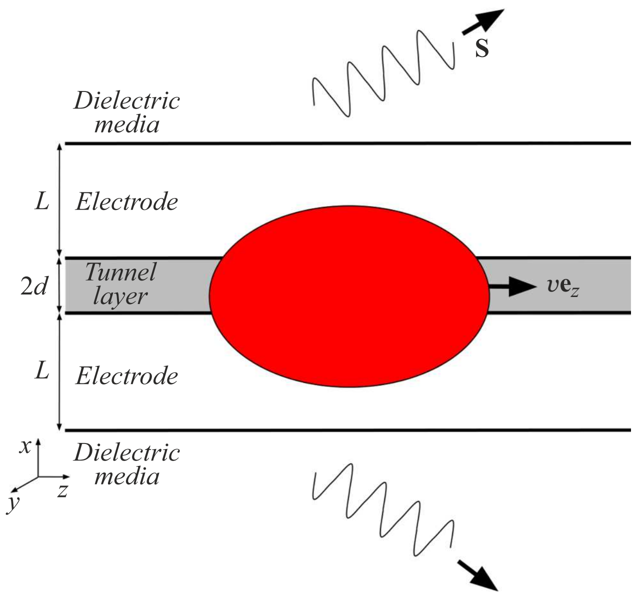
$$\omega_J^2\sin\psi(\xi)+v^2\psi''(\xi)=v_S^2\frac{d}{d\xi}\int d\xi' q(\xi-\xi')\,\frac{d\psi(\xi')}{d\xi'},\tag{1}$$

where ω_J is the Josephson plasma frequency, $\psi(\xi)=\varphi(z,t)$, $\xi\equiv z-vt$, and v_S is the Swihart velocity in the case of bulk electrodes when $L=\infty$. The Fourier transform of the $q(\xi)$ kernel is written as

$$q(k)\equiv \operatorname{th}\left(\frac{L}{\lambda}\right)\frac{R(k)-\operatorname{cth}(L/\lambda)}{R(k)-\operatorname{th}(L/\lambda)},\tag{2}$$

where λ is the London penetration depth of the magnetic field into superconducting electrodes, $R(k)\equiv c_m^2\kappa/\lambda v^2k^2$, $c_m\equiv c/\sqrt{\epsilon_m}$ is the speed of light in the external dielectric,

$$\kappa\equiv\sqrt{|1-v^2/c_m^2|}\,|k|\left[\theta(c_m-v)-i\,\theta(v-c_m)\operatorname{sgn}k\right],$$



Transverse section of a Josephson sandwich in the xOz plane. An elementary vortex (shown schematically as an ellipse) travels along axis Oz with velocity $v\mathbf{e}_z$. The magnetic field is oriented along the Oy axis. Cherenkov waves propagating from the side surfaces into the external dielectric medium are shown outside of the sandwich. Arrows indicate the direction of radiation energy flux density **S**.

and $\theta(\cdot)$ is the Heaviside function. Relationship (2) is written in the limit

$$\lambda|k| \ll 1, \qquad (3)$$

which corresponds to the examination of vortices with their characteristic spatial scale of variation along axis Oz being significantly greater than λ.

If the phase difference corresponding to a specific vortex is known, one may determine the structure of the electromagnetic field in the entire system. Specifically, the magnetic field inside the tunnel layer induced by a traveling vortex has the following form:

$$H_J(\xi) = -\frac{\phi_0}{4\pi\lambda}\int d\xi'\, q(\xi-\xi')\,\frac{d\psi(\xi')}{d\xi'},$$

where ϕ_0 is the magnetic flux quantum. At $x = \pm d \pm L$ (i.e., the outer sandwich boundaries), the magnetic field is

$$H_s(\xi) = \int d\xi'\, h(\xi-\xi')H_J(\xi'), \qquad (4)$$

and the Fourier transform of the h kernel is given by

$$h(k) \equiv \frac{1}{\mathrm{sh}\,(L/\lambda)}\,\frac{1}{\mathrm{cth}\,(L/\lambda) - R(k)}. \qquad (5)$$

Outside the sandwich, the Fourier components of fields depend on coordinate x as $\exp[-\kappa(|x|-x_s)]$, where $x_s \equiv d + L$. Owing to this, the fields of subluminal vortices with $v < c_m$ and κ is real decrease with distance as one moves deeper into the dielectric. In turn, the fields of superluminal vortices with $v > c_m$ and κ being purely imaginary take the form of waves propagating from the sandwich.

2. Non-radiating unusual Josephson vortex

Let us first consider subluminal vortices with velocities lower than c_m. In the $R(k) \gg \mathrm{cth}\,(L/\lambda)$ limit, we then obtain $q(\xi) \simeq \mathrm{th}(L/\lambda)\,\delta(\xi)$, and Eq. (1) characterizes a usual Josephson vortex in the case of electrodes of a finite thickness:

$$\psi = 4\,\mathrm{arctg}\,(\exp(-k_J\xi)), \qquad (6)$$

where $k_J \equiv \omega_J/\sqrt{V_S^2 - v^2}$ and $V_S \equiv v_S \mathrm{th}^{1/2}(L/\lambda)$ [17,18]. Note that the above constraint on $R(k)$ and inequality (3) require that the vortex velocity not be too close to c_m and V_S. In accordance with (4) and (5), the magnetic field at the sandwich boundary is $\sim R(k_J)\,\mathrm{sh}(L/\lambda) \gg 1$ times weaker than the field inside the tunnel layer.

The other limit with $R(k)$ being small (namely, $R(k) \ll \mathrm{th}\,(L/\lambda)$) may also be examined.

In this case, $q(\xi) \simeq \mathrm{cth}(L/\lambda)\,\delta(\xi)$ and Eq. (1) characterizes an unusual Josephson vortex. The shape of this vortex is also governed by expression (6), which, however, features quantity $K_J \equiv \omega_J/\sqrt{U_S^2 - v^2}$, where $U_S \equiv v_S\,\mathrm{cth}^{1/2}(L/\lambda)$, instead of k_J. The degree of accuracy of expression (6) as applied to an unusual vortex is discussed in the Appendix.

It follows from the analysis of conditions $\lambda K_J \ll 1$ and $R(K_J) \ll \mathrm{th}\,(L/\lambda)$ that an unusual vortex may travel with velocities up to c_m at $c_m < U_S$. If $c_m = U_S$, the vortex velocity may reach $v_R = U_S\left[1 - \mathcal{O}((\lambda/\lambda_J)^2)\right]$. When c_m increases to a certain limit value, the vortex velocity is also limited from above by v_R. If the c_m value exceeds this limit, it becomes impossible to fulfill both inequality (3) and the condition of smallness of $R(k)$; i.e., an unusual vortex is infeasible.

Thus, an unusual vortex may travel with velocities as high as c_m. The maximum unusual vortex velocity is $\simeq U_S$, which is $U_S/V_S = \mathrm{cth}(L/\lambda) > 1$ times higher than V_S. This difference is especially noticeable in the case of relatively thin electrodes.

Another important property of an unusual vortex is the ratio of magnetic fields inside the Josephson junction and at the sandwich boundaries: $H_S \simeq H_J\,\mathrm{ch}^{-1}(L/\lambda)$. Contrary to the case of a usual vortex, the decay of the field inside the electrodes is governed by a factor that may be on the order of unity (if the electrodes are thin). The energy of the electromagnetic field is distributed mostly in the external dielectric, and it can be said that a strong coupling is established between the fields inside the Josephson junction and outside the sandwich. Note that the indicated ratio of magnetic fields translates into the fact that, similar to a usual vortex in a Josephson junction with bulk electrodes, the considered unusual vortex carries a single magnetic flux quantum.

3. Radiating unusual Josephson vortex

The possibility of Cherenkov radiation of Josephson vortices has long been studied both theoretically and experimentally. The motion of the unusual Josephson vortex characterized above may also induce Cherenkov radiation if its velocity exceeds c_m. If, as before, the value of $R(k)$ is considered to be small

$$|R(k)| \ll \operatorname{th}(L/\lambda), \quad (7)$$

the kernel $q(k)$ may be presented as a sum of $\operatorname{cth}(L/\lambda)$ and an imaginary term proportional to $|R(k)|$. This term characterizes the field of Cherenkov radiation produced by the vortex.

If the following condition is satisfied

$$|R(k)| \ll \operatorname{sh}(2L/\lambda)(1 - v^2/U_S^2)/2, \quad (8)$$

the reverse effect of radiation on the vortex may be neglected. As a first approximation, we then assume that the radiating unusual vortex has the same shape (see (6)) as before with characteristic spatial scale K_J^{-1}. Inequality (3), the condition of smallness of R (7), and the condition of smallness of radiation losses (8) may always be fulfilled simultaneously at vortex velocities close to c_m.

If magnetic field H_s (4) induced by the vortex on the sandwich surface is known, one may determine the field of a Cherenkov wave in the surrounding dielectric from Maxwell's equations:

$$H(x,\xi) = H_s\left(\xi + \sqrt{\frac{v^2}{c_m^2} - 1}(|x| - x_s)\right) = \frac{\phi_0 K_J}{2\pi\lambda \operatorname{sh}(L/\lambda)}$$

$$\times \operatorname{sch}\left(K_J\left(\xi + \sqrt{\frac{v^2}{c_m^2} - 1}(|x| - x_s)\right)\right), \quad |x| > x_s. \quad (9)$$

The radiation power (per unit length along axis Oy) corresponding to such a wave is equal to the integral along axis Oz of the x-component of the energy flux density calculated over both side surfaces of the sandwich:

$$P = \frac{\phi_0^2 \omega_J}{4\pi^3\lambda^2}\mathscr{P}, \quad (10)$$

where

$$\mathscr{P} \equiv c_m\sqrt{v^2 - c_m^2} \,/\, \operatorname{sh}^2(L/\lambda) v \sqrt{U_S^2 - v^2}$$

is a dimensionless parameter specified by the electrode thickness and the proximity of the vortex velocity to c_m and U_S. Factor $(U_S^2 - v^2)^{-1/2}$ emerges in the expression for power due to the fact that, by virtue of (9), all areas of the sandwich surfaces corresponding to characteristic size $\sim K_J^{-1}$ of the unusual vortex contribute to the radiation flux.

With a slight excess over the speed of light in the external dielectric, the power depends on velocity as $\propto \sqrt{v - c_m}$, and radiation is directed at small angle $\simeq \sqrt{2(v - c_m)/c_m}$ to the sandwich surface. The power grows monotonically with increasing unusual vortex velocity, while radiation deviates more and more from the electrode surfaces and the radiation pattern expands.

Conclusion

Let us summarize briefly the properties of an unusual Josephson vortex.

First, an unusual Josephson vortex may be detected experimentally in the „standard" Josephson sandwich geometry. It is assumed that a planar superconducting sandwich of this kind is surrounded by a decelerating dielectric medium. Depending on the ratio between the vortex velocity and the speed of light in the external medium, this vortex may be either non-radiating or radiating.

Second, the shape of an unusual Josephson vortex is similar to the shape of a usual Josephson vortex (2π-kink). Their important difference is in the characteristic spatial scales: $\sim \sqrt{U_S^2 - v^2}/\omega_J$ and $\sim \sqrt{V_S^2 - v^2}/\omega_J$, respectively. These scales are governed by different limit velocities of unusual and usual Josephson vortices: U_S and V_S. In the case of thin electrodes ($L \ll \lambda$), these velocities may differ greatly: U_S is $\operatorname{cth}(L/\lambda) \gg 1$ times higher than V_S.

Third, an unusual vortex makes it easier to experiment with the emission of electromagnetic waves into the external medium induced by the Vavilov–Cherenkov effect. This radiation is produced when the vortex velocity exceeds the speed of light in the external medium. The velocity of an unusual vortex is limited from above by the U_S value; therefore, the prerequisite for the emergence of Cherenkov radiation is written as $U_S > c_m$, which is equivalent to $\epsilon_m > \epsilon(\lambda/d)\operatorname{th}(L/\lambda)$, where ϵ is the permittivity of the tunnel layer. In typical Josephson junctions with a tunnel layer width of several nanometers, ϵ of several units, and $\lambda \sim 100$ nm, this condition for electrodes with thickness $\gtrsim \lambda$ is fulfilled at $\epsilon_m \sim 100$. The ϵ_m requirement may be relaxed in the case of thin electrodes when the inequality is satisfied at significantly lower values of ϵ_m. Specifically, $\epsilon_m > 32$ at $\epsilon = 4$, $d = \lambda/25$, and $L = \lambda/3$.

Fourth, the characteristic radiation frequencies of an unusual Josephson vortex may be estimated as vK_J. Since typical Josephson plasma frequencies are on the order of hundreds of gigahertz, radiation frequencies $\sim \omega_J/\sqrt{(U_S/v)^2 - 1}$ of an unusual vortex fall into the terahertz range. Note that the radiation frequency depends on the vortex velocity and may be adjusted smoothly.

In conclusion, let us estimate the power of Cherenkov radiation of an unusual vortex. The following power of radiation from the entire height L_y of a sandwich along axis Oy is derived from (10) at the values of $\epsilon = 4$, $d = \lambda/25$, and $\lambda = 100$ nm used above: $PL_y \simeq \mathscr{P}(L_y/\lambda_J)10^{-3}$ W, where $\lambda_J \equiv \omega_J v_s$ is the Josephson length. The condition of smallness of vortex radiation losses (8) yields the following upper estimate: $\mathscr{P} \ll (\lambda/\lambda_J)\operatorname{cth}^{1/2}(L/\lambda)$. Since typical values $\lambda_J \sim 100\lambda$, these estimates indicate that the radiation power is within the microwatt range. To estimate the value of $\mathscr{P}$, we assume that $L = \lambda/3$, $U_S = 1.2c_m$, $\lambda/\lambda_J = 0.01$, and the vortex velocity is sufficiently close to the Cherenkov radiation threshold, namely $(v/c_m) - 1 = 6 \cdot 10^{-7}$. The obtained estimate is $\mathscr{P} \simeq 0.014$; therefore, the radiation power is $\simeq 14\,\mu$W for a sandwich with height $L_y = \lambda_J$.

These characteristics of an unusual vortex indicate that it may serve as the basis for a compact solid-state source of terahertz radiation with tunable frequency.

Appendix

To determine the correction to the phase difference of an unusual vortex in the $R(k) \ll \operatorname{th}(L/\lambda)$ limit, we present the Fourier transform of the $q(\xi)$ kernel as $q(k) = \operatorname{cth}(L/\lambda) + \delta q(k)$, where

$$\delta q(k) \equiv \operatorname{sh}^{-2}(L/\lambda)\left[R^{-1}(k) - \operatorname{cth}(L/\lambda)\right]^{-1}.$$

Zero denominator of the $\delta q(k)$ answers to a subluminal surface wave. The introduction of a slight attenuation of this wave provides an opportunity to calculate the contribution from the pole. We seek the phase difference across the Josephson junction in the form $\psi(\xi) \simeq 4\operatorname{arctg}(\exp(-K_J\xi)) + \delta\psi(\xi),$, where $\delta\psi(\xi)$ is the first-order correction in $\delta q(\xi)$. It then follows from Eq. (1) that

$$\delta\tilde{\psi}''(\xi) - \left(1 - \frac{2}{\operatorname{ch}^2 \xi}\right)\delta\tilde{\psi}(\xi) = -\frac{4v_S^2 K_J^2 R(K_J)}{\pi\omega_J^2 \operatorname{sh}^2(L/\lambda)}\, I\left(\frac{2\xi}{\pi}, p\right), \quad \text{(A1)}$$

where $\delta\tilde{\psi}(\xi) \equiv \delta\psi(\zeta)$, $\xi \equiv K_J\zeta$, $p \equiv R(K_J)\operatorname{cth}(L/\lambda)/2$, $I(b, p) \equiv I_1(b, p) + I_2(b, p)$,

$$I_1(b, p) \equiv \text{v.p.}\int_0^\infty du\, \frac{u}{u - \pi p}\, \frac{\sin(bu)}{\operatorname{ch} u},$$

$$I_2(b, p) \equiv -\pi^2 p\, \frac{\cos(\pi p b)}{\operatorname{ch}(\pi p)}. \quad \text{(A2)}$$

The maximum absolute values of $I_1(b, p)$ are on the order of unity, and the $I_2(b, p)$ contribution from the pole is much lower than unity, since $p \ll 1$.

The contribution to the solution of Eq. (A1) associated with term I_1 on the right-hand side has the form

$$\frac{2v_S^2 K_J^2 R(K_J)}{\pi\omega_J^2 \operatorname{sh}^2(L/\lambda)}\left[f_1(\xi)\int_0^\xi d\xi' I_1\left(\frac{2\xi'}{\pi}, p\right) f_2(\xi') + f_2(\xi)\int_\xi^\infty d\xi' I_1\left(\frac{2\xi'}{\pi}, p\right) f_1(\xi')\right], \quad \text{(A3)}$$

where $f_1(\xi) \equiv \operatorname{sech}\xi$, $f_2(\xi) \equiv \operatorname{sh}\xi + \xi\operatorname{sech}\xi$. The maximum absolute values of the quantity in square brackets are on the order of unity. Since there is a small parameter in front of the square bracket, the corresponding contribution to (A3) provides a small correction to the phase difference of an unusual vortex.

The contribution from the pole associated with function $I_2(b, p)$ yields an even smaller correction to the expression for $\delta\tilde{\psi}(\xi) \propto R^2(K_J)\cos(2p\xi)$. The smallness of this correction part indicates that the surface wave does not exert a significant influence on an unusual vortex.

Expression (2) is written in limit (3); i.e., it characterizes such distributions of the phase difference that vary on scales larger than λ. A more general expression for the kernel with account for the so-called spatial nonlocality may be written. To obtain such a kernel, one needs to make a consistent allowance for the term containing the second derivative of the magnetic field with respect to coordinate z when solving the London equation in superconducting electrodes. The resulting expression is [19]

$$q(k) \equiv \frac{\lambda(k)}{\lambda}\operatorname{th}\left(\frac{L}{\lambda(k)}\right)\frac{\lambda(k)R(k)/\lambda - \operatorname{cth}(L/\lambda(k))}{\lambda(k)R(k)/\lambda - \operatorname{th}(L/\lambda(k))}, \quad \text{(A4)}$$

where $\lambda(k) \equiv \lambda/\sqrt{1 + \lambda^2 k^2}$. In the case of $R(k) \ll \operatorname{th}(L/\lambda)$, the corrections to the kernel associated with both R and spatial nonlocality may be taken into account additively. The effect of the first correction was discussed above. Let us examine the influence of terms $\propto \lambda^2 k^2$ on an unusual vortex. To do this, we write the following approximate equality:

$$q(k) \simeq \operatorname{cth}\left(\frac{L}{\lambda}\right) - \frac{\lambda^2 k^2}{2}\operatorname{cth}\left(\frac{L}{\lambda}\right)\left[1 + \frac{2L}{\lambda\operatorname{sh}(2L/\lambda)}\right]. \quad \text{(A5)}$$

It can be seen that the introduction of spatial nonlocality leads tentatively to the emergence of a small negative quadratic correction to the Fourier transform of the kernel. It is known that the introduction of such a correction has two corollaries [14,15]. First, the shape of a soliton obtained by solving the sine-Gordon equation is deformed weakly. This change is $\propto \lambda^2$ and vanishes at infinity. Second, a Swihart wave, which also has a small amplitude ($\propto \lambda^2$), emerges at the tail of a vortex. If the study of Cherenkov radiation of a Swihart wave is not the goal, one may assume that spatial nonlocality has little effect on an unusual vortex in the $\lambda K_J \ll 1$ limit.

The corrections to the phase difference mentioned above indicate that Eq. (1) may have unusual solutions that can be sought by abandoning the use of approximate expressions for kernel (2) or its modifications. The search for such solutions is a separate topic of interest for advancing the electrodynamics of Josephson junctions.

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by D.Safin