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Justification of empirical constitutive equations of a material state under quasi-static deformation within the framework of the acoustoplastic effect

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A modified model of the acoustoplastic effect is proposed. Within its framework, the processes of elastic and plastic deformation of materials are considered. The conditions under which it leads to widely used empirical models for the dependence of stress on deformation (Johnson–Cook, Voce, and Hollomon models) are analyzed. The features of using these empirical models are revealed. The relationship between the constants used in these empirical models and such material parameters as internal friction stress, activation volume of defects, their relaxation time and their equilibrium concentration, as well as with the parameter characterizing the degree of interaction of defects, is determined.

Keywords: deformation, mechanical stress, polycrystalline structures, activation mechanism, defects.

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1. Introduction

Analysis of stress behavior in a sample due to deformation set by loading test machines, is widely used to determine a set of important mechanical properties of the material. Experimentally, these dependences are obtained using loading machines, which specify a certain strain rate and record a value of applied stress corresponding to the deformation [1,2]. Such experiments execution in a region of elastic and plastic deformations of the material ensures determination of its Young's modulus, as well as yield stress and parameters characterizing its deformation strengthening. Transition from elastic deformation to plastic deformation is accompanied by complex physical processes of formation, interaction and movement of defects.

To describe properties of materials in the region of plastic deformations a set of empirical dependences was suggested and is widely used. Using them the material behavior in the region of plastic deformations is quantitatively characterized by a set of parameters, physical meaning of which is frequently unclear. At the same time, in paper [3] it was shown that under the acoustoplastic effect it is possible to describe the material behavior during transition from the region of elastic deformations to the region of plastic deformations. At that, it was necessary to set the stress-strain law in the plastic region basing on some a priori considerations. In paper [4] we showed that this dependence, in principle, can be determined from consideration of relaxation properties of defects considering their interaction as per activation mechanism.

Due to this the main objective of the present paper is studying possibility to obtain the known empirical relations linking the stress in a sample with deformations in the plastic region, based on the approaches used to explain acoustoplastic effects.

2. Model description

Under the acoustoplastic effect the description of the dynamics of stress behavior σ in the sample at its nonstationary deformation is based on the following equation [3–5]

$$\frac{1}{E}\frac{\partial\sigma}{\partial t} = \dot{\varepsilon} - \dot{\varepsilon}_{\mathrm{p}}, \tag{1}$$

where E is the Young's modulus of material, $\dot{\varepsilon}$ is the object total strain rate driven externally, $\dot{\varepsilon}_{\mathrm{p}}$ is the plastic strain rate of the material.

In order to determine the plastic strain rate $\dot{\varepsilon}_{\mathrm{p}}$, it is usually assumed that defects in the material are generated as per an activation law of Arrhenius, and it can be found from the relationship

$$\dot{\varepsilon}_{\mathrm{p}} = \dot{\varepsilon}_{v} \exp\left(\frac{\Omega(\sigma - \sigma_{\mathrm{f}} - \sigma_{\mathrm{p}}(\varepsilon))}{k_{\mathrm{B}}T}\right), \tag{2}$$

where σ_{f} is stress due to internal friction presence for defects; $\sigma_{\mathrm{p}}(\varepsilon)$ is stress in the sample, associated with defects generation in it; the factor $\dot{\varepsilon}_{v}$ describes material strain rate due to dislocation movement, and generally is supposed to be constant; Ω is the activation volume of a defect; k_{B} is Boltzmann constant; T is the sample temperature.

Determination of dependence $\sigma_{\mathrm{p}}(\varepsilon)$ on strain requires special consideration. In major cases the dependence is selected empirically in a form of slowly changing function of strain. In particular, in paper [3] stress $\sigma_{\mathrm{p}}(\varepsilon)$ was assumed proportional to $\sqrt{\varepsilon}$. The analysis of this approach shows that it is limited and does not allow one to obtain a relationship between stress and strain in the form widely used in empirical relationships, for example, those proposed in [6–8]. Thus, more detailed consideration of this issue concerning the dependence of stress $\sigma_{\mathrm{p}}(\varepsilon)$ on strain is nessesary. In paper [4] we determined the behavior of defects concentration in material during deformation under relaxation approximation. It was shown that, taking into account the change in the activation energy of defects due to their interaction, defect concentration is determined by the equation

$$n(\varepsilon) = n_{\mathrm{r}}\bigl(1 - \exp(-(\varepsilon/\dot{\varepsilon}_v \tau)^{\beta})\bigr), \tag{3}$$

where n_{r} is the equilibrium concentration of defects, τ is the time of defects relaxation, β is the coefficient characterizing degree of defects interaction [4,9].

Equation (3) describes a law of defect accumulation during deformation that differs from the purely exponential law. Coefficient β is in range $0 \le \beta \le 1$. Case $\beta = 1$ corresponds to low concentration of defects and absence of their interaction. From further consideration it follows, that the coefficient β for actual materials generally are in the range of 0.2 to 0.5. Within the framework of our proposed model β is considered to be a constant determined from comparison of the theoretical results and the experimental data.

Knowledge of the behavior of defect concentration during deformation ensures stress $\sigma_{\mathrm{p}}(\varepsilon)$ determination. For this it is possible to use the expression

$$\sigma_{\mathrm{p}}(\varepsilon) \simeq \sigma_{\mathrm{f}}\Omega n(\varepsilon) - n(\varepsilon)e_{\mathrm{p}}, \tag{4}$$

where e_{p} is the energy of plastic deformation per one defect.

Regarding Eq. (4) it is necessery to note the following. Generally it is assumed that defect formation in the material leads to the occurrence of the additional stress $E\Omega n$ [10]. But, when considering material behavior in the region of plastic deformations we consider as more correct the use of σ_{f} instead of E. The last term in Eq. (4) reflects the stress change in sample due to the energy release e_{p} near the defect.

Use of Eqs. (3) and (4) and transition from integration over time to integration over strain ensures determination of the solution of equation (1) in form

$$\sigma(\varepsilon) = E\varepsilon - \frac{k_{\mathrm{B}}T}{\Omega}\ln\Biggl[1 + \frac{\Omega E}{k_{\mathrm{B}}T}\dot{\varepsilon}_v \int\limits_0^{\varepsilon} d\varepsilon'$$
$$\times \frac{1}{\dot{\varepsilon}'}\exp\biggl(\frac{\Omega(E\varepsilon' - \sigma_{\mathrm{f}} - \Omega\sigma_{\mathrm{f}} n(\varepsilon') + n(\varepsilon')e_{\mathrm{p}}(\varepsilon'))}{k_{\mathrm{B}}T}\biggr)\Biggr]. \tag{5}$$

The integration region in Eq. (5) can be divided into two parts. The first part integration is the strain range of 0 to ε_{e}, where ε_{e} is the maximum strain in the elastic region. In this region the integral term is significantly less than 1, and behavior of stress $\sigma(\varepsilon)$ corresponds to elastic part. Integration over the range from ε_{e} to ε corresponds to zone of plastic deformation. As empirical relations are used for the analysis of experimental data exactly in this section, then further behavior of $\sigma(\varepsilon)$ is analyzed at $\varepsilon \ge \varepsilon_{\mathrm{e}}$. In this region the second term under the sign of logarithm becomes above 1. Besides, during integration we can consider that main contribution to the integral is made by the first term in the exponent. Then the factor with the rest terms in the exponent can be taken out from under the integral at $\varepsilon' = \varepsilon$. According to numerical calculations the maximum error of such approximation takes place in the plastic region at low β, and does not exceed 0.1%. Considering the said facts for stress behavior in the zone of plastic deformations, we obtain the following result from Eq. (5):

$$\sigma(\varepsilon_{\mathrm{p}}) \simeq -\frac{k_{\mathrm{B}}T}{\Omega}\ln\frac{\dot{\varepsilon}_v}{\dot{\varepsilon}_{\mathrm{p}}} + \sigma_{\mathrm{f}} + \Omega\sigma_{\mathrm{f}} n(\varepsilon_{\mathrm{p}}) - n(\varepsilon_{\mathrm{p}})e_{\mathrm{p}}. \tag{6}$$

If we consider that in order to move a dislocation it is necessary to overcome an energy barrier e_{p}, then in quasistatic conditions we can determine the relation between $\dot{\varepsilon}_{\mathrm{p}}$ and $\dot{\varepsilon}_v$ from relationship $\exp\bigl(-\frac{e_{\mathrm{p}}}{k_{\mathrm{B}}T}\bigr) \simeq \frac{\dot{\varepsilon}_{\mathrm{p}}}{\dot{\varepsilon}_v}$. Then Eq. (6) is converted to the form

$$\sigma(\varepsilon_{\mathrm{p}}) = (\sigma_{\mathrm{f}} + \Omega\sigma_{\mathrm{f}} n)\biggl(1 + \frac{k_{\mathrm{B}}T}{\Omega\sigma_{\mathrm{f}}}\ln\frac{\dot{\varepsilon}_{\mathrm{p}}}{\dot{\varepsilon}_v}\biggr). \tag{7}$$

In the Johnson–Cook model [6–8] for stress–strain curve for a material at permanent temperature the relationship is used

$$\sigma(\varepsilon_{\mathrm{p}}) = (A + B\varepsilon_{\mathrm{p}}^{m})(1 + C\ln\dot{\varepsilon}_{\mathrm{p}}^{*}), \tag{8}$$

where A, B, C and m are material constants, $\dot{\varepsilon}_{\mathrm{p}}^{*}$ is the dimensionless strain rate normalized to $1.0\,\mathrm{s}^{-1}$.

If condition $\varepsilon_{\mathrm{p}} \le \dot{\varepsilon}_v \tau$ is met, than for the concentration n from Eq. (3) we obtain

$$n(\varepsilon_{\mathrm{p}}) \simeq n_{\mathrm{r}}(\varepsilon_{\mathrm{p}}/\dot{\varepsilon}_v \tau)^{\beta}. \tag{9}$$

After the substitution Eq. (9) in Eq. (7) and transformation of logarithm $\ln\frac{\dot{\varepsilon}_{\mathrm{p}}}{\dot{\varepsilon}_v} = \ln\dot{\varepsilon}_{\mathrm{p}}^{*} + \ln\frac{\dot{\varepsilon}_v}{\dot{\varepsilon}_0}$ (where $\dot{\varepsilon}_0 = 1\,\mathrm{s}^{-1}$), Eq. (7) looks like Johnson-Cook relationship with following values of material constants:

$$A = \sigma_{\mathrm{f}}\biggl(1 + \frac{k_{\mathrm{B}}T}{\Omega\sigma_{\mathrm{f}}}\ln\frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_v}\biggr),\ B = \frac{\Omega\sigma_{\mathrm{f}} n_{\mathrm{r}}}{(\dot{\varepsilon}_v\tau)^{\beta}}\biggl(1 + \frac{k_{\mathrm{B}}T}{\Omega\sigma_{\mathrm{f}}}\ln\frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_v}\biggr),$$

$$C = \frac{k_{\mathrm{B}}T}{\Omega\sigma_{\mathrm{f}}}\biggl(1 + \frac{k_{\mathrm{B}}T}{\Omega\sigma_{\mathrm{f}}}\ln\frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_v}\biggr)^{-1},\ m = \beta.$$

The papers [11,12] showed that for Al and Cu at temperature 273 K the behavior of stress on deformation is well approximated by the relationship

$$\sigma(\varepsilon_{\mathrm{p}}) = \sigma_0 + \sigma_1\biggl[1 - \exp\biggl(-\frac{\varepsilon_{\mathrm{p}}^{m}}{\varepsilon_c}\biggr)\biggr], \tag{10}$$

where σ_0, σ_1 and ε_c are the material constants depending on temperature.

Comparison of Eqs. (7) and (10) shows that their forms coincide if condition $\dot{\varepsilon}_\mathrm{p} \simeq \kappa\dot{\varepsilon}_v$ is met, where κ is the proportionality coefficient, and substitution for $n(\varepsilon)$ from Eq. (3). Then, for the material constants included in Eq. (10) we obtain

$$\sigma_0 = \sigma_\mathrm{f} + \frac{k_\mathrm{B}T}{\Omega}\ln\kappa,\ \ \sigma_1 = n_\mathrm{r}(\Omega\sigma_\mathrm{f} + k_\mathrm{B}T\kappa),$$

$$\varepsilon_c = (\dot{\varepsilon}_v\tau)^\beta,\ \ m = \beta.$$

Note that at $m = \beta = 1$ Eq. (10) corresponds to Voce's law first suggested in [13] (see also [14]), and at $\varepsilon_\mathrm{p} < \dot{\varepsilon}_v\tau$ corresponds to Hollomon's law [15,16]

$$\sigma(\varepsilon_\mathrm{p}) = \sigma_0 + K\varepsilon_\mathrm{p}^m, \tag{11}$$

where

$$\sigma_0 = \sigma_\mathrm{f} + \frac{k_\mathrm{B}T}{\Omega}\ln\kappa,\ \ K = \frac{n_\mathrm{r}(\Omega\sigma_\mathrm{f} + k_\mathrm{B}T)}{(\dot{\varepsilon}_v\tau)^m},\quad m = \beta.$$

3. Conclusion

The obtained results show that the use of the modified theory of the acoustoplastic effect allows us to obtain empirical laws that are widely used in physics and mechanics to describe the stress–strain relationship in the region of material plasticity. At that the obtained within framework of the acoustoplastic effect results for the stress dependence on deformation ensure linking of the values of parameters used in the empirical approaches with such material characteristics as yield stress, activation volume of defects participating in process, their equilibrium concentration and relaxation time, degree of interaction of defects. Besides, the obtained results show limits of applicability of the empirical models. Thus, the Johnson-Cook and Hollomon models are best used in the analysis of plastic deformations in materials with slow relaxation processes (condition $\varepsilon_\mathrm{p} \le \dot{\varepsilon}_v\tau$). On the other hand, models of type used in papers [11,12] are better suitable for analysis of plastic deformations in materials with not too slow relaxation processes (do not require meeting of the condition $\varepsilon_\mathrm{p} \le \dot{\varepsilon}_v\tau$). This result is indirectly confirmed by the fact that last model well operates during materials deformation at elevated temperatures [12], when the relaxation processes accelerate. At the same time the model of Voce type [13] to describe deformation processes at elevated temperatures requires modification [17]. For most actual materials the use of empirical relationships leads to parameter values m in range of 0.2 to 0.5. From the approach based on acoustoplastic effect it follows that for all empirical models $m = \beta$, at that the parameter β characterizes the interaction degree of defects (case $\beta = 1$ corresponds to interaction absence). So, the approach within the framework of the acoustoplastic effect shows that this range of values of parameter m corresponds to rather strong interaction of defects during plastic deformation of materials, and can be used as its quantitative characteristic.

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Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] J.E. Field, T.M. Walley, W.G. Proud, H.T. Goldrein, C.R. Siviour. Int. J. Impact Eng. **30**, *7*, 725 (2004).
[2] T. Bhujangrao, C. Froustey, E. Iriondo, F. Veiga, P. Darnis, F.G. Mata. Metals **10**, *7*, 894 (2020).
[3] G.A. Malygin. Phys. Solid State **42**, *1*, 72 (2000).
[4] A.L. Glazov, K.L. Muratikov. Phys. Solid State **66**, *3*, 345 (2024).
[5] A.V. Kozlov, S.I. Selitser. Mater. Sci. Eng. A **131**, *1*, 17 (1991).
[6] G.R. Johnson, W.H. Cook. Proceed. 7th Symposium on Ballistics. The Hague, The Netherlands (1983). P. 541–547.
[7] G.R. Johnson, W.H. Cook. Eng. Fracture Mechanics **21**, *1*, 31 (1985).
[8] T.J. Jang, J.-B. Kim, H. Shin. J. Comput. Design. Eng. **8**, *4*, 1082 (2021).
[9] K. Trachenko, A. Zaccone. J. Phys.: Condens. Matter **33**, *31*, 315101 (2021).
[10] A.M. Kosevich. Fizicheskaya mekhanika real'nykh kristallov. Naukova dumka, Kiev (1981). 328 s. (in Russian).
[11] N.G. Chinh, G. Horvath, Z. Horita, T.G. Langdon. Acta Materialia **52**, *12*, 3555 (2004).
[12] N.G. Chinh, J. Illy, Z. Horita, T.G. Langdon. Mater. Sci. Eng. A **410–411**, 234 (2005).
[13] E. Voce. J. Inst. Metals **74**, 537 (1948).
[14] C. Zhang, B. Wang. J. Mater. Res. **27**, *20*, 2624 (2012).
[15] J.H. Hollomon. Trans. Metallurg. Soc. AIME **162**, *2*, 268 (1945).
[16] R.K. Nutor, N.K. Adomako, Y.Z. Fang. Amer. J. Mater Synth. Processing **2**, *1*, 1 (2017).
[17] H. Teng, Y. Xia, C. Pan, Y. Li. Metals **13**, *5*, 986 (2023).

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