11.2

Chaotic oscillations in a system of two coupled self-oscillators with dedicated inertia

© L.A. Morozova, S.V. Savel'ev

Fryazino Branch, Kotel'nikov Institute of Radio Engineering and Electronics, Russian Academy of Sciences, Fryazino, Moscow oblast, Russia
E-mail: mila-morozova.ludmila@yandex.ru

Received May 20, 2024
Revised May 20, 2024
Accepted May 31, 2024

Using the example of a generator model with dedicated inertia, a theoretical study of two coupled self-oscillators with capacitive coupling, their sequential single-frequency synchronization, chaos and two-frequency synchronization with an adiabatic change in the magnitude of the coupling between partial self-oscillators was carried out. The parameters of self-oscillators and the values of the coupling coefficient at which the specified operating modes of coupled self-oscillators exist are determined. The results of numerical studies, illustrating the conditions for excitation of single-frequency, chaotic and dual-frequency oscillations in a system of coupled self-oscillators are presented.

Keywords: generator with dedicated inertia, coupling value, system of two coupled self-oscillators.

DOI: 10.61011/TPL.2024.09.59164.19998

Chaotic oscillation regimes of systems of coupled self-oscillators have been examined in a number of studies (see, e.g., [1,2]). They have always attracted research attention due both to the great variety of oscillatory processes and to the quality of generated chaotic oscillations (see, e.g., [3–9]).

Coupled systems allowing for chaotic dynamics of partial self-oscillators are of particular interest, since they feature the largest set of oscillatory regimes, which includes both regular and chaotic oscillations with multi-frequency dynamics [3,4]. However, the authors of the overwhelming majority of studies focused their attention on systems of coupled self-oscillators with vastly different natural frequencies and control parameters and introduced additional elements and external signals to facilitate the generation of chaos in these systems.

Specifically, the influence of a low-pass filter, which alters the phase of common oscillations to expand the region of chaos on the plane of control parameters, on synchronization of chaotic oscillations of a pair of unidirectionally coupled generators of a chaotic signal was examined in [5,6].

A system of coupled Kislov–Dmitriev self-oscillators with non-identical control parameters was studied in [7,8], and it was noted that the primary scenario of oscillations in transition to chaos is the destruction of the quasi-periodic regime.

The synchronization of chaotic oscillations in a system of two mutually coupled non-identical Rössler oscillators in the helical chaos regime was investigated in [9]. The fundamental role played by the difference in parameters of partial self-oscillators in the transition to well-developed chaotic oscillations was noted.

It follows from this brief review that systems of coupled self-oscillators with equal partial frequencies of components remain virtually unstudied. Thus, it is of interest to determine which oscillation regimes may be inherent in systems of two coupled self-oscillators with identical partial frequencies and identify the possible scenario of development of an oscillatory process in such a system in transition to chaos.

In the present study, we report the results of numerical analysis of a system of two coupled self-oscillators with dedicated inertia and practical equality of partial frequencies.

A mathematical model of a generator with dedicated inertia (GDI) was proposed in [10]. This model is noteworthy for the fact that it characterizes adequately the dynamics of an amplifier stage with a high-power bipolar transistor in large-signal operation. The parameters of the generator model correspond to the actual parameters of systems based on high-power transistors and may be used in calculation of real circuit designs (see [11,12]). Chaotic oscillations of such systems have a near-normal probability density distribution and a wide frequency range, which provides an opportunity to solve real-world problems on construction of chaos generators with a high energy potential. Therefore, a system of coupled GDIs may be regarded as the most instructive prototype model for studying the complex dynamics of coupled oscillators with close frequencies.

Relying on the results from [10], one may present the system of equations of two coupled self-oscillators with dedicated inertia and capacitive coupling in the following

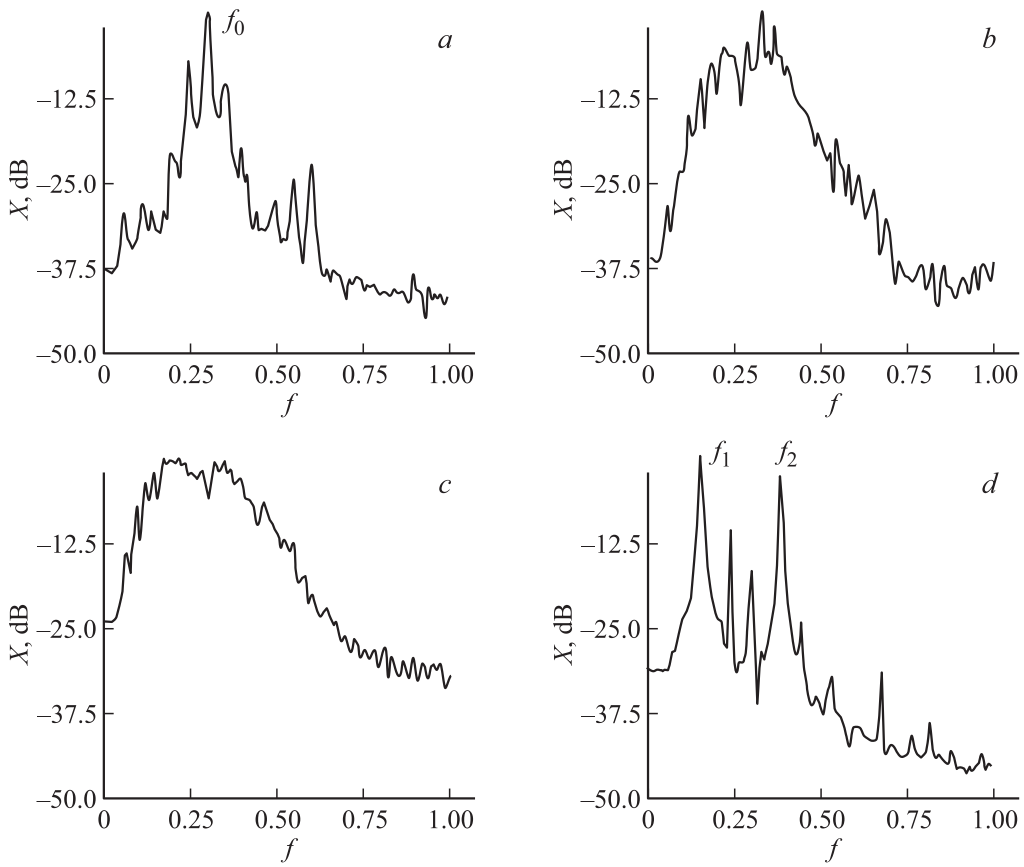

Figure 1. Dynamics of the oscillatory process of a system of two coupled GDIs with variation of the coupling coefficient between partial oscillators: $k = 0.1$ (a), 0.42 (b), 0.53 (c) and 0.61 (d).

form:

$$
\begin{gathered}
\dot{X}_i = Y_i + (m_{1i} - m_{2i})X_i - X_i Z_i + kX_j, \quad X_i \le q_i, \\
\dot{X}_i = Y_i - m_{2i}X_i - q_i Z_i, \quad X_i > q_i, \\
\dot{Y}_i = -X_i, \\
\dot{Z}_i = -g_i Z_i + g_i F_i(2X_i - m_{2i}W_i)(2X_i - m_{2i}W_i)^2, \\
F_i(a) = \begin{cases} 1, \ a \ge 0, \\ 0, \ a < 0, \end{cases} \\
\dot{W}_i = X_i - m_{2i}W_i,
\end{gathered} \tag{1}
$$

where i, $j = 1$, 2; $i \ne j$; k is the coupling coefficient; X, Y, Z, and W are the dimensionless voltage at the nonlinear amplifier input, current in the feedback circuit, voltage at the output of a half-wave inertial converter, and current in the input circuit, respectively; m_1, m_2, q, and g are the excitation, dissipation, limit, and inertia parameters; and $F(a)$ is the Heaviside step function.

System (1) was solved with a slight detuning in inertia parameters $g_1 = 0.045$, $g_2 = 0.05$ and equal values of the remaining parameters of partial self-oscillators, which corresponded to the values used in [10]: $m_1 = 1.6$, $m_2 = 0.2$, and $q = 1$. The condition of equality of partial frequencies was thus satisfied.

Figure 1 presents spectral patterns that allow one to track the development of an oscillatory process in system (1) under variation of coupling parameter k. With an initially weak coupling ($k = 0.1$), periodic motion (Π) is established in the system in the form of a stable limit cycle based on frequency f_0 equal to the frequency of autonomous oscillations of partial self-oscillators (Fig. 1, a).

The periodic motion regime persists in the system until the coupling coefficient reaches the level of $k = 0.4$; in this context, an enhancement of coupling induces an interchange of periodic motion regimes of different multiplicity. At $k = 0.4$, a complex oscillatory process starts to develop in the system, which eventually leads to the emergence of chaotic oscillations based on frequency f_0 (Fig. 1, b).

Further variation of parameter k results in transition from the regime of a strange attractor based on a single frequency (SA_1) to the regime of generation of chaotic oscillations based on dual-frequency motion (SA_2), which is manifested in the spectral domain as a double-humped spectral characteristic of variable X_1 (Fig. 1, c). The next stage of evolution of oscillation regimes is presented in Fig. 1, d. When the level of $k = 0.56$ is exceeded, the SA_2 regime gives way to dual-mode regular motion (T_2) based on frequencies f_1 and f_2, $f_1 < f_0 < f_2$. An increase in the coupling coefficient leads to structural rearrangements

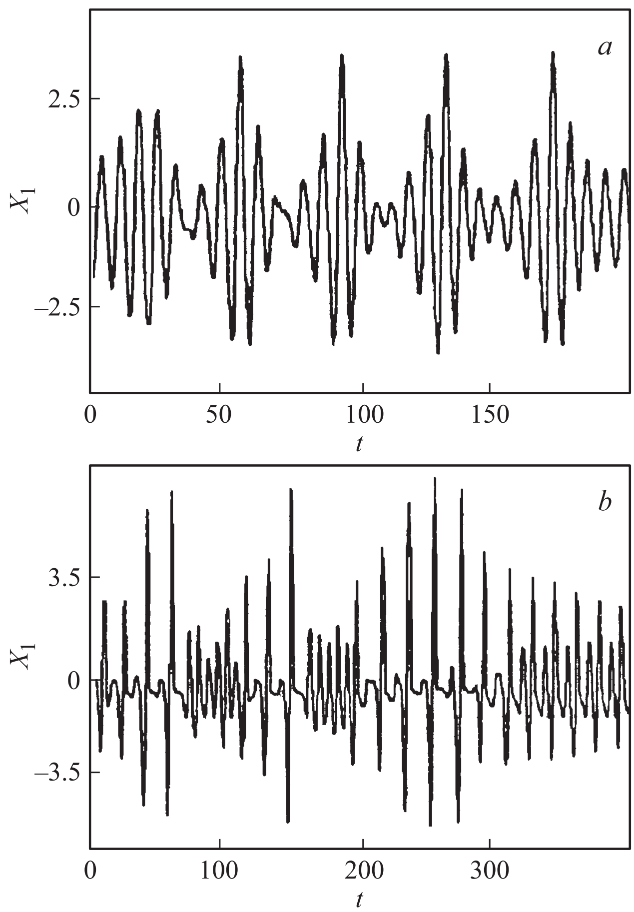

Figure 2. Time realizations of X_1 corresponding to oscillations of a system of coupled GDIs at $k = 0.42$ (SA_1) (*a*) and $k = 0.56$ (SA_2) (*b*).

of resonant tori in the phase space of the system; notably, a larger coupling coefficient corresponds to a smaller number of spectral components in the oscillation power spectrum of the system. The dual-frequency system dynamics is indicative of the emergence of additional regions of synchronization in the system of coupled GDIs at high coupling coefficient values.

Let us examine the time realizations of oscillations to analyze the processes proceeding in the system under study. The variation of X_1 for SA_1 and SA_2 is presented in Figs. 2, *a* and *b*, respectively. These data offer the most insight into the mechanism of the SA_1-SA_2 transition.

The case of SA_1 is characterized by irregular intermittency of trains of oscillations of different periods. An increase in coupling parameter k leads to a sequential change of states of the system in the form of stable limit cycles with their oscillation periods increasing gradually by unity. The system of coupled GDIs demonstrated additive growth of the oscillation period multiplicity in unit steps in the transition from stable periodic motion with a period of n/f_0 to periodic motion $(n+1)/f_0$, where $n = 1, 2, \ldots$. With each subsequent transition to a stable cycle with a unit increase in the oscillation period, the distance between the critical values of variable parameter k decreased. In the numerical experiment, the maximum value is $n = 5$ at $k = 0.39$.

In the dual-frequency chaotic SA_2 regime, a competition of interacting modes is observed, which is manifested in the fact that oscillations with frequencies f_1 and f_2 alternate chaotically in the system (Fig. 2, *b*). Frequency components of partial self-oscillators do not compete in the case under consideration; it is the system modes that are competing, and the examined system of coupled self-oscillators acts as a unified system with its unique properties. Additional synchronization regions are established in the system of equivalent self-oscillators, which is manifested in the dual-frequency oscillation regime.

The probability density distribution was calculated to reveal the statistical properties of chaotic oscillations in the SA_2 regime. The calculation of the histogram showed that the probability density distribution of oscillations is close to a normal Gaussian one at $k = 0.53$.

The scenario of development of oscillations on exit from the SA_2 regime is a sequential change of the number of combination components with the $(f_2 - f_1)/h$ arrangement, where $h = 4, 3, 2$; i.e., the transition from dual-frequency chaos to the resonant tori regime was characterized by a sequential reduction in the number of combination components with an increase in coupling coefficient k in accordance with the law inverse to the natural series.

Thus, the numerical experiment revealed the emergence of secondary nonlinear resonances and chaotization of oscillations in the system of coupled GDIs as a result of transition from the single-frequency interaction to the dual-frequency one. The examined $\Pi-SA_1-SA_2-T_2$ oscillation scenario demonstrates that the transition to chaotic oscillations in the studied system of coupled GDIs with practically equal partial frequencies is accompanied by mode competition and intermittency. The dynamics of the system is characterized by a pattern typical of pulling and switching of modes in the chaos region, which manifests itself as a transition from a regime based on single-frequency oscillations to a regime based on dual-frequency oscillations.

Funding

This study was carried out under the state assignment of the Kotel'nikov Institute of Radio Engineering and Electronics of the Russian Academy of Sciences.

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] B.E. Zhelezovskii, É.V. Kal'yanov, Tech. Phys. Lett., **35** (7), 647 (2009). DOI: 10.1134/S1063785009070177

[2] Yu.P. Emel'yanova, A.P. Kuznetsov, Tech. Phys., **56** (4), 435 (2011). DOI: 10.1134/S106378421104013X

[3] A.Yu. Nemets, D.M. Vavriv, Vopr. At. Nauki Tekh., **98** (4), 282 (2015) (in Russian).
[4] V.A. Buts, D.M. Vavriv, Vopr. At. Nauki Tekh., **116** (4), 213 (2018) (in Russian).
[5] A.A. Prokhorov, E.S. Mchedlova, Tech. Phys. Lett., **33** (2), 170 (2007). DOI: 10.1134/S106378500702023X
[6] A.A. Prokhorov, E.S. Mchedlova, Tech. Phys., **53** (11), 1463 (2008). DOI: 10.1134/S106378420811011X
[7] A.P. Kuznetsov, V.I. Paksyutov, Tech. Phys., **53** (4), 518 (2008). DOI: 10.1134/S1063784208040221
[8] A.P. Kuznetsov, Yu.V. Sedova, N.V. Stankevich, Tech. Phys., **67** (14), 2174 (2022). DOI: 10.21883/TP.2022.14.55214.145-21
[9] A.V. Makarenko, Tech. Phys., **61** (2), 265 (2016). DOI: 10.1134/S106378421602016X
[10] S.V. Savel'ev, Zh. Radioelektron., № 6 (2017). http://jre.cplire.ru/jre/jun17/10/text.pdf
[11] S.V. Savel'ev, L.A. Morozova, Tech. Phys., **65** (12), 2056 (2020). DOI: 10.1134/S1063784220120245
[12] S.V. Savel'ev, L.A. Morozova, Tech. Phys. Lett., **49** (4), 48 (2023). DOI: 10.21883/TPL.2023.04.55877.19138.

Translated by D.Safin