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Suppression of the flow instability by random fluctuations of the rotational velocity

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The possibilities of suppression of the flow instability in a spherical gap by the noise — random fluctuations of the internal sphere rotation velocity in time — have been investigated experimentally. Fluctuations with zero mean value were added to the constant mean rotation velocity. It is found that noise can suppress instability in the form of running azimuthal waves, with a transition to a long-lasting stationary flow after the noise is turned off. Significant differences were found in the interaction of azimuthal modes in suppressing instability by periodic and random in time fluctuations of rotation speed.

Keywords: noise, instability control, spherical Couette flow.

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Much attention is currently being paid to the study of methods for suppressing instabilities in various physical processes [1–5]. Flows may be stabilized under the influence of a magnetic field [2,3], acceleration in a shear flow [4], and distributed injection [5]. Suppression of hydrodynamic instability in flows with rotation may be of use in technological processes (e.g., growth of crystals from liquid melt by the Czochralski method [6,7]). In this case, instability suppression provides an opportunity to rectify the heterogeneity of the crystallization front and, consequently, make the structure of single crystals more homogeneous [8]. The possibilities of suppressing the instability of flows with rotation were investigated numerically [6,7]. Impulse [6] and resonant [7] methods for controlling the rotation velocity, the mean value of which was maintained at a constant level, were used. The same condition was fulfilled in experiments on instability suppression in a spherical Couette flow (SCF) under the influence of time-periodic modulation of the rotation velocity of the internal sphere [9]. SCF is a flow of viscous incompressible fluid that forms in a gap between coaxially located spheres under the influence of their rotation. In SCF, periodic modulation of the rotation velocity [10] and noise in the form of random fluctuations with zero mean [11,12] may induce the same effects (e.g., mean flow generation). Suppression of instability by periodic modulation has already been observed in experiments [9], but it remains unclear whether instability suppression by noise is possible. The aim of the present study is to clarify this issue.

Flow was induced by rotation of the internal sphere only. Noise was added to its constant angular velocity Ω_0 in the form of fluctuations random in time with zero mean. Consequently, it spreads throughout the entire flow. The spheres are optically transparent; the radius of the internal one is $r_1 = 0.075$ m, and the radius of the external sphere is $r_2 = 0.150$ m. The gap is filled with silicone oil with kinematic viscosity $\nu = 5 \cdot 10^{-5}\,\mathrm{m^2/s}$ at an oil temperature of 22°C. Aluminum powder was added to the oil in order to visualize the flow. The spheres were introduced into an optically transparent thermostat. The oil temperature in it was maintained constant in each experiment (with an accuracy of $\pm 0.05^\circ$C) and measured with a sensor positioned at the equator of the external sphere. Azimuthal component u_φ [m/s] of the flow velocity was measured with a laser anemometer at mid-latitudes at a point located 0.078 m away from the equatorial plane and 0.105 m away from the rotation axis. The time-averaged value of the set angular velocity of rotation was maintained by a digital control system with an error no greater than 0.02%. A rotation velocity sensor on the drive shaft produced a phase signal. Instantaneous angular velocity values were calculated as a time derivative of the phase signal and compared with the set value to generate an actuating signal for the drive. Noise was produced by adding normalized disturbances from by a random number generator to the velocity signal. In accordance with the results reported in [11], noise inducing the greatest change in flows was chosen (white noise in the frequency range from 0.01 to 1 Hz); at higher frequencies, exponential amplitude decay is observed in the spectrum (see Fig. 2 in [11]). The time step of the control system was 0.04 s. To obtain the required noise spectrum, rotation velocity disturbances were induced at every tenth step. The limit values of disturbance amplitude are determined by the limit acceleration that is related to the change in rotation rate $\Omega(t)/2\pi$ $(0.2\ \mathrm{s}^{-2})$. Dimensionless noise amplitude N was defined as $N = \frac{1}{\Omega_0}\sqrt{\frac{1}{K-1}\sum_{i=1}^{K}\left(\Omega(t_i) - \Omega_0\right)^2}$, where $\Omega(t_i)$ is the instantaneous rotation velocity value and K is the time sample length.

A wide variety of instability types may be observed in SCF: when only the internal sphere rotates, instability

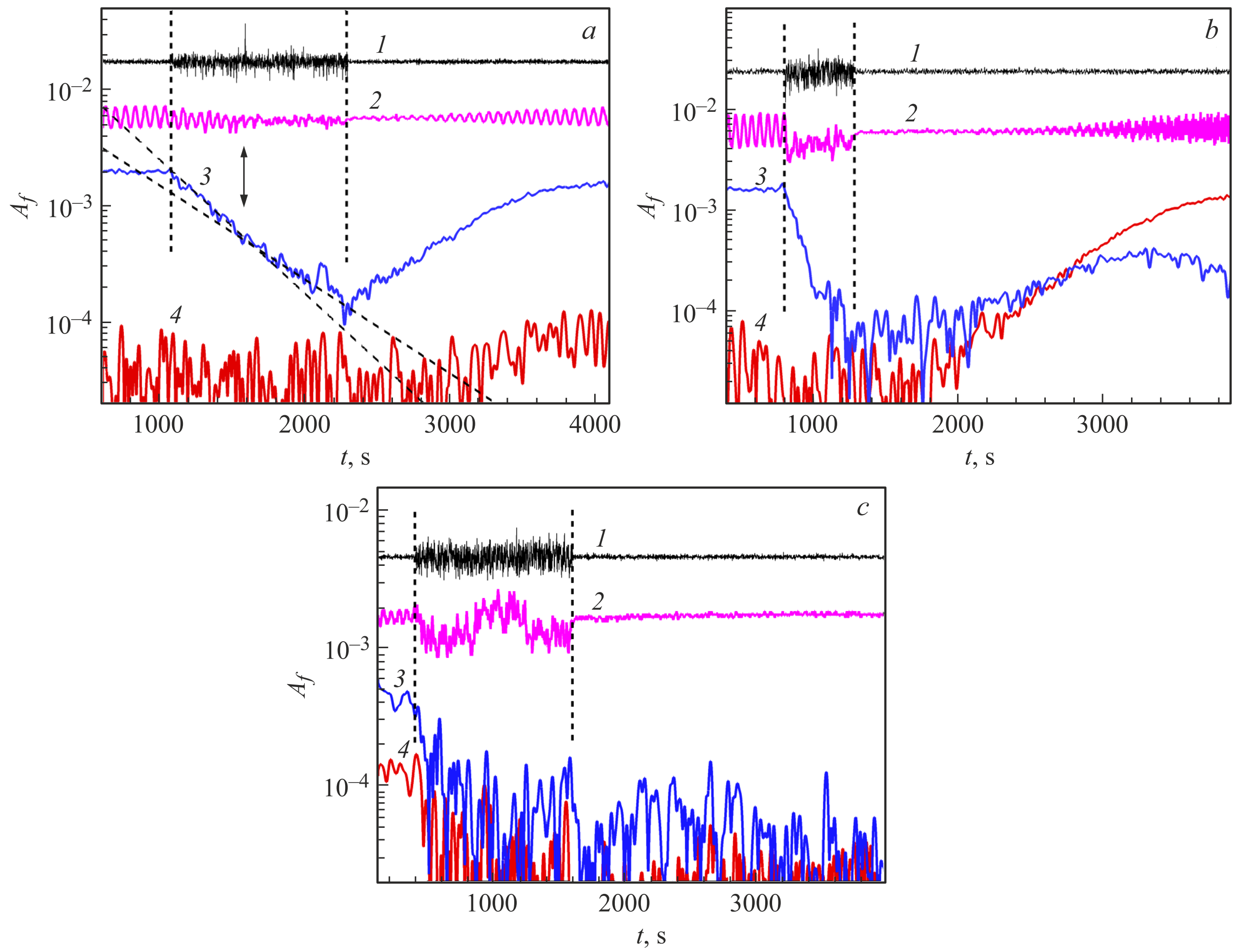

Figure 1. Dependences of internal sphere rotation rate $\Omega_1(t)/2\pi$ [Hz] (*1*), measured flow velocity $u_\varphi(t)$ [m/s] (*2*), and amplitudes $A_f(t)$ [m/s] of initial mode $m = 3$ (blue curves *3*) and secondary mode $m = 4$ (red curves *4*) on time t at $\mathrm{Re}_1/\mathrm{Re}_c = 1.0046$. *a* — $N = 0.045$, *b* — $N = 0.0648$, and *c* — $N = 0.0702$. For clarity, dependences *1* and *2* are presented with an increased time interval between adjacent points. Vertical dashed lines denote the moments of time when additional noise was turned on and off. Inclined dashed lines in panel *a* represent exponential approximations of amplitude A_{f3}, and the vertical arrow indicates the moment of decrement change. A color version of the figure is provided in the online version of the paper.

develops, depending on layer thickness $\delta = (r_2 - r_1)/r_1$, in the form of either stationary Taylor vortices or running azimuthal waves [13]. In the layer with thickness $\delta = 1$ examined in our experiments, unstable flow assumes, depending on the initial conditions, the form of running azimuthal waves with wave numbers $m = 3$ or $m = 4$ [13] propagating in the direction of rotation of the internal sphere. The frequencies of azimuthal modes are $f_3 = 0.3{-}0.32$ Hz and $f_4 = 0.4{-}0.42$ Hz for $m = 3$ and $m = 4$, respectively [13,14]. This is exactly the case in which not only instability was suppressed in [9] via modulation of the rotation velocity, but also flow stationarity was maintained due to the interaction of azimuthal modes after switching off the control input. Experiments were conducted in accordance with the following procedure. Velocity Ω_0 was first increased to levels at which Reynolds number $\mathrm{Re}_1 = (\Omega_0 r_1^2)/\nu$ exceeded the critical value corresponding to the flow stability limit: $\mathrm{Re}_c = 460 \pm 2$ [13]. The value of $\Omega_{0c}/2\pi$ corresponding to critical Re_c was kept constant during each experiment and, depending on the oil temperature in the layer, ranged from 0.6521 to 0.6582 Hz. Experiments were carried out at $\mathrm{Re}_1/\mathrm{Re}_c = 1.0031 \pm 0.0004$ and, as in [9], at $\mathrm{Re}_1/\mathrm{Re}_c = 1.0046 \pm 0.0004$. As instability develops, the amplitudes of both modes increase. Then amplitude of one of the modes (we call it the initial one) then reaches a constant level. The amplitude of the other mode (secondary one) decreases after reaching its maximum. The mode choice is governed by past history of flow evolution [13] and noise amplitude N [14]. The recording of u_φ measurements began at the moment when the flow became unstable; this moment and wave number m were determined from the flow visualization. After this, noise was fed into the velocity signal (in a stepwise manner, from zero to the chosen value). The N value remained constant throughout the experiment and fell within the $0.095 > N > 0.044$ range. The duration of exposure to noise (τ) varied from 400 to 1500 s. If the visualization and the nature of temporal variation of u_φ in the experiment made it clear that instability was suppressed,

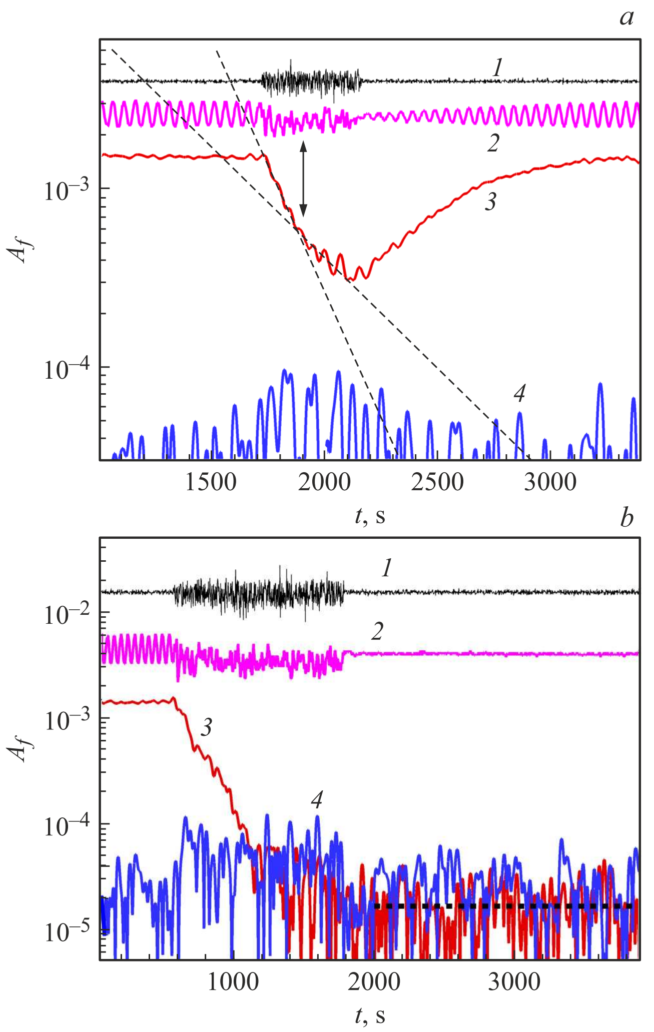

Figure 2. Dependences of internal sphere rotation rate $\Omega_1(t)/2\pi$ [Hz] (*1*), measured flow velocity $u_\varphi(t)$ [m/s] (*2*), and amplitudes $A_f(t)$ [m/s] of initial mode $m = 4$ (red curves *3*) and secondary mode $m = 3$ (blue curves *4*) on time t at $\mathrm{Re}_1/\mathrm{Re}_c = 1.0046$ and $N = 0.0648$. a — $\tau = 432$ s; b — $\tau = 1210$ s. For clarity, dependences *1* and *2* are presented with an increased time interval between adjacent points. Inclined dashed lines in panel a represent exponential approximations of amplitude A_{f4}, and the vertical arrow indicates the moment of decrement change. The horizontal dashed line in panel b represents the mean amplitude of the initial mode after turning off the noise. A color version of the figure is provided in the online version of the paper.

noise was turned off. The recorded u_φ values were used to determine the oscillation amplitudes of secondary flow modes A_f [9]: $A_f = |u_f(t) + i\mathrm{HT}(u_f(t))|$, where HT is the Hilbert transform and $u_f(t)$ is the result of filtering of velocity signal $u_\varphi(t)$ within frequency band $f_3 \pm \Delta f$ for $m = 3$ and $f_4 \pm \Delta f$ for $m = 4$, $\Delta_f = 0.005{-}0.01$ Hz. The Hilbert transform allows one to determine the temporal variation of the signal amplitude at a given frequency [15].

Figure 1 presents the scenarios of response of an unstable flow with initial mode $m = 3$ to an increase in the noise amplitude (N increases from Fig. 1, a to Fig. 1, c). When

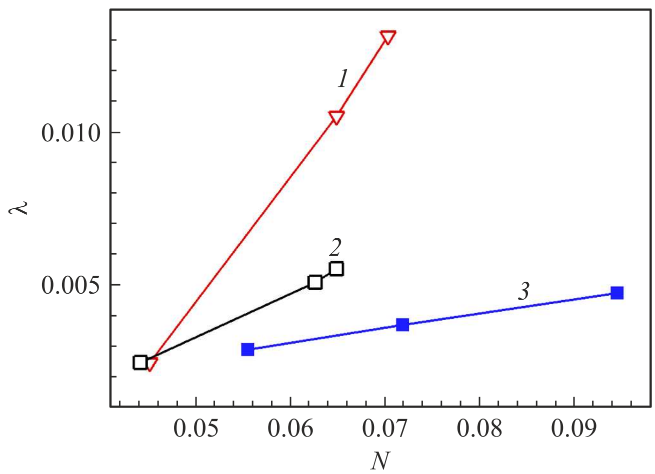

Figure 3. Dependence of damping decrement λ [m/s^2] on noise amplitude N. *1* — Initial mode $m = 3$, $\mathrm{Re}/\mathrm{Re}_c = 1.0046$; *2* — initial mode $m = 4$, $\mathrm{Re}/\mathrm{Re}_c = 1.0046$; *3* — initial mode $m = 4$, $\mathrm{Re}/\mathrm{Re}_c = 1.0031$.

noise is turned on, amplitude A_{f3} of the initial mode starts to decay exponentially; damping decrement λ does not remain constant and decreases once (Fig. 1, a). If the amplitude of the initial mode has exceeded the amplitude of the secondary one within the entire interval in which noise remained turned on, instability with the initial mode is restored (Fig. 1, a). There is a certain time interval in Fig. 1, b within which $A_{f3} \approx A_{f4}$. When noise is turned off, both modes start to grow, and $m = 3$ is replaced by $m = 4$. Note that the latter mode is dominant in the case of stationary rotation [13,14]. In Fig. 1, c, condition $A_{f3} \approx A_{f4}$ is satisfied within the greater part of the interval in which noise is turned on. When noise is turned off, the amplitudes of both modes do not increase, remain close in magnitude, and flow instability remains unrestored for a long time. Suppression of flow instability may be induced not only by an increase in N, but also by an increase in τ at the same level of N. This is illustrated in Fig. 2 for initial mode $m = 4$. Just as in the scenario presented in Fig. 1, a, the initial mode exceeds the secondary one when noise is turned on, and instability is restored after turning off the noise (Fig. 2, a). Damping decrement λ also does not remain constant and decreases at the moment when the secondary mode is maximized. As in the scenario presented in Fig. 1, c, an increase in τ (Fig. 2, b) leads to the equalization of mode amplitudes, and instability remains unrestored for a long time after turning off the noise. Three sections may be distinguished in the time dependence of A_{f4}. The first one is exponential decay with constant λ. When the amplitudes of two modes get equalized, $\partial A_{f4}/\partial t$ decreases (second section). In the third section, the average value of A_{f4} remains constant (indicated by the horizontal dotted line in Fig. 2, b). In contrast to the scenarios presented in Fig. 1, both scenarios in Fig. 2 involve a slight growth of the secondary mode. The dependence of the maximum values of damping decrement λ on N is shown in Fig. 3. At the same supercriticality, initial mode $m = 3$ decays faster than $m = 4$ (curves *1* and *2*). The greater the supercriticality is, the faster the initial mode decays ($m = 4$, curves *2* and *3*).

The presented data suggest that the convergence of amplitudes of competing azimuthal modes is the reason for both the suppression of instability by noise and the long-term retention of flow stationarity at supercritical values of the Reynolds number after the termination of noise. A similar result has been obtained earlier with periodic modulation of the rotation velocity [9]. However, there are certain differences. Specifically, periodic modulation led to a noticeable increase in the flow velocity; the attenuation of the initial mode (with a constant decrement) was accompanied by a significant enhancement of the secondary mode (see Fig. 1 in [9]). Long-term retention of stability after switching off the modulation was observed only for initial mode $m = 4$ (see Fig. 2 in [9]). The above features are not observed when noise is used. Specifically, the decreasing damping decrement of the initial mode (Figs. $1, a$ and $2, a$) is indicative of enhancement of the interaction between modes under the influence of noise [14], which leads (at any initial mode) to the suppression of instability by noise and to long-term retention of flow stability after turning off the noise.

The suppression of instability by noise, which was examined experimentally in the present work, is consistent with the results obtained for other systems in both numerical (suppression of instability of an inverted pendulum by noise [16]) and experimental (elimination of instability in a combustion chamber by added noise [17]) studies.

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Conflict of interest

The authors declare that they have no conflict of interest.

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