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Theoretical study of self-oscillations in the RFTES detector

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Hypotheses for the self-oscillation in the RFTES detector are considered. The model of competition between heating current and critical current in the film microbridge at the interface with the electrodes, and a model of local suppression of superconductivity due to hot spot, are analyzed. The simulation results qualitatively confirm the experimental data.

Keywords: RFTES bolometer, high-Q resonator, superconducting microbridge, electron gas, self-oscillation, critical current.

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1. Introduction

Detectors based on superconducting materials are currently of particular interest for astronomical observations. This is attributable to the low temperature and high nonlinearity of such detectors, which is a prerequisite for detection of low-energy photons. Such detectors have a number of advantages, and today there are several varieties of practical superconducting sensors. The most studied are the Transition Edge Sensor, (TES) [1], which requires an ultra-low-noise current amplifier (SQUID amplifier) to measure the resistance of a TES thermometer at direct current, and Microwave Kinetic Inductance Detector (MKID) [2–3], which uses the reaction of a high-quality factor microwave resonator to a change in the concentration of superconducting carriers in the current antinode. Another experimentally tested technology is the Hot Electron Direct Detector (HEDD) [4–5], which employs the effect of heating an electron gas when the electron-phonon relaxation time becomes long enough ($\tau_{e-ph} \gg \tau_{e-e}$), and the electronic subsystem can be considered thermally insulated from the lattice (from interaction with phonons). The technology of Cold Electron Bolometer (CEB) is also being developed [6–8].

The detectors listed above have their advantages and disadvantages. Another solution was proposed in Ref. [9–11], in which a superconducting thin-film microbridge is incorporated into a high-Q quarter-wave resonator located in a cryostat at temperatures near a superconducting transition of the order of hundreds millikelvins. Such a detector has the abbreviation RFTES (Radio Frequency Transition Edge Sensor) — a bolometer on the edge of a superconducting transition with high-frequency reading. Such a sensor allows getting away from the reading circuit with a SQUID amplifier, as in a TES detector, and, unlike MKID, it operates at temperatures close to critical, which allows using an active impedance component that controls the Q-factor of the resonator by heating/cooling a superconducting bridge under the impact of high-frequency power, which corresponds to an increase/reduction of its impedance. The reading is performed by measuring the depth of the resonant curve of the microwave scattering coefficient S_{21} (power transmission coefficient from port *1* to port *2*, see Figure 1). Two methods can be used for heating of the superconducting thin film: by terahertz photons directly through the antenna, or by pumping photons supplied via a coaxial cable to the chip input and then to the bridge in the resonator. It is important that the heat exchange processes in the absorber are much slower than the pumping period, so its effect can only be characterized by amplitude, which is similar to the effect of direct current.

Experimental studies demonstrated that self-oscillations may occur in the RFTES detector in certain modes [12], manifested as modulation of the microwave carrier with a frequency of $\sim 10\,\text{kHz}$, which depend on both the power of the carrier and the power of THz radiation supplied to the antenna input of the detector. This is possible if these fluctuations are temperature fluctuations. At the same time, the experimental detector retains high sensitivity to optical radiation, that is, the presence of self-oscillations does not result in its obvious degradation. Phenomenologically, this mode is associated with the „crater“ on the resonance curve of the RFTES detector [12,13].

The condition for the occurrence of relaxation oscillations may be the transition of a superconducting film to a normal state through a critical current. In a transition mode, an increase of impedance always results in a mismatch with the pump source, a sharp decrease of the amplitude of the microwave current and further cooling of the film, as a result of which it becomes possible to restore the superconducting

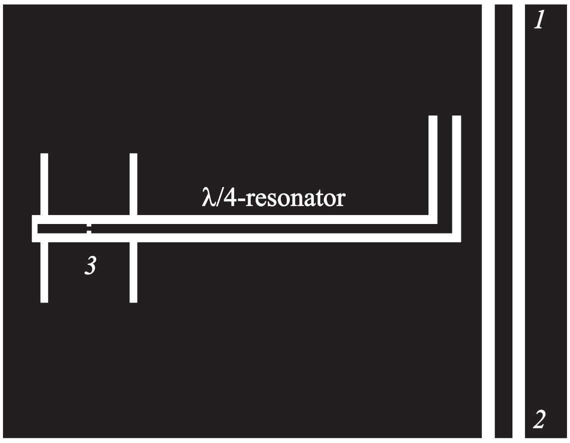

Figure 1. Conceptual topology of the RFTES bolometer [12]. The numbers indicate the following: *1* and *2* — input and output of the chip (transmission line port for the EM model), port *3* — superconducting micro bridge, coupled simultaneously with a planar double-slot antenna and a quarter-wave resonator.

state. This is similar to the unstable thermal hysteresis observed with a direct current when a voltage source is used. Our experimental studies demonstrated that the intrinsic noise of the RFTES detector can be measured not only as a dispersion of the carrier power at the detector output, as in Ref. [13], but also as a dispersion of the frequency of self-oscillations, i. e. by measuring the width of the spectral line of oscillations. Two models of self-oscillation are proposed. The first model is relevant to the aging of the sample, in which the oxidation process and the occurrence of a weak link at the bridge-electrode interface are possible. A competition between the heating current and the critical current may occur in such model in the area of the Hf/Nb contact (bridge/resonator). The superconducting junction is accompanied in this case by an abrupt increase of resistance in the bridge circuit, which leads to a decrease of the current, and gradual cooling of the bridge. The amplitude of the current in the bridge circuit decreases to a value below the critical value after a certain time interval, determined by the inertia of the resonator (relaxation time), as well as the thermal inertia of the bridge, after which the superconducting channel is restored. The second relevant model is a local destruction of superconductivity by current in a micro bridge, resulting in the appearance of a hot spot [14–17]. The spot can change its size, which determines the resistance of the bridge, which, in turn, can result in a reversal of the electrothermal feedback [13]. This model is based on the effect of the occurrence of Abrikosov vortices because of a local decrease of the energy barrier preventing their penetration into the film [18]. Such a movement of vortices takes place across the film, and this can result in the destruction of the superconducting state and the appearance of additional resistance, which will lead to a change of the amplitude of the microwave pump current, similar to the first model of weak coupling at the interface between the bridge and the supply electrodes.

2. Principle of operation of the RFTES detector

The concept of the RFTES detector is that a thin-film superconducting microbridge with a critical temperature of T_c^b is incorporated in a high-quality superconducting microwave resonator with a critical temperature of T_c^{res} (moreover, $T_c^{\mathrm{res}} \gg T_c^b$). The transmission coefficient of the chip S_{21} is controlled not by the kinetic energy of Cooper pairs, as in MKID, but by the nonlinear active impedance of a superconducting thermometer/absorber near its critical temperature, which, unlike MKID, is responded not in the shift of the resonant frequency, but in the depth of the resonant dip S_{21}.

The heat transfer in the microbridge of the RFTES detector at temperatures in the range of hundreds of millikelvins is described by the hot electron gas model, which is characterized by slow thermalization between electrons and phonons and rapid heating of electrons. Heating is described by two processes: heating due to the absorption of high-energy photons coming from the THz antenna, and heating with a microwave bias current I_{bias} at the resonator oscillation frequency (carrier frequency). The bias current creates low-energy photons and plays the heating role when setting the operating temperature of the electronic subsystem, at which the temperature coefficient of resistance (TCR) $\mathrm{TCR}(T) = dR/dT$ of the microbridge, characterizing the steepness $R(T)$, has an optimal value. The frequency of the current used to shift the electronic temperature of a hafnium microbridge from $T_c \approx 400$ mK, as previously determined in Ref. [10], affects the temperature dependence $\mathrm{TCR}(T)$. The theory of Mattis–Bardin [19] allows calculating such a temperature dependence, which is important for determining the thermal dynamics of the bridge. The results of modeling of the impedance of a hafnium microbridge at two frequencies of 1.5GHz and 4.2GHz predict that the TCR significantly decreases when the resonator frequency changes, and there is practically no such effect in case of usage of thinner films with a lower critical temperature of the order of hundreds of millikelvins, which is favorable for improving the sensitivity of the bolometer in case of deeper cooling [20].

The dynamics of the electronic temperature of a hot electron gas in a hafnium bridge can be described by the following differential equation

$$C_e(T_e)\,\frac{dT_e}{dt} = P_{\mathrm{bias}}(f, T_e) + P_{\mathrm{opt}} - \int\limits_{T_{\mathrm{cr}}}^{T_e} G(T)dT, \qquad (1)$$

where $P_{\mathrm{bias}}(f, T_e) = P_{\mathrm{in}}|S_{31}(Z_s(f), R_b(f_0, T_e))|P_{\mathrm{in}}$ — bias power supplied to the chip, $S_{31}(Z_s(f), R_b(f_0, T_e))$ — coefficient of transmission of bias power to the absorber (according to Figure 1 from port *1* to port *3*), $Z_s(f)$ — impedance of the equivalent current source heating the bridge, f_0 — carrier frequency (resonator frequency), $G(T_e)$ — heat dissipation coefficient (in first approximation constant G_0), T_{cr} — cryostat temperature (temperature of the substrate), $C_e(T_e)$ — the electronic heat capacity of a superconducting bridge (in first approximation, the constant C_{e0}).

It can be seen from the differential equation that the relaxation time of the electron gas depends on the ratio of the thermal conductivity G to the electronic heat capacity C_e of the material, and the change of the active impedance $R_b(f_0, T_{\text{state}})$ at a certain pump level P_{in}can be calculated from the stationary heat balance equation by calculating T_{state}.

$$P_{\text{bias}}(f, T_{\text{state}}) + P_{\text{opt}} - \int_{T_{\text{cr}}}^{T_{\text{state}}} G(T)dT = 0. \tag{2}$$

The dynamics of heat release in a nonlinear resistance depends on the ratio of the current value $R_b(T)$ under the condition $0 < R_b(T) < R_n$, and the impedance of the heating source with an internal resistance $Z_S(f)$.

The stability of the RFTES bolometer, as for the classical TES bolometer, is associated with the presence of alternating electrothermal feedback. The RFTES bolometer is a linear power converter: the bias increment at the output ΔP corresponds to the optical signal P_{opt} received by an antenna, with a coefficient that has a sense of a gain. The dimensionless power gain can be calculated using the heat balance equation

$$Gain = \frac{\Delta P}{P_{\text{opt}}} = \frac{P_{\text{in}}\Delta S_{21}}{\Delta P_b - P_{\text{in}}\Delta S_{31}} = \frac{P_{\text{in}}(dS_{21})/dT}{G_0 - P_{\text{in}}(dS_{31})/dT}, \tag{3}$$

where the numerator — the change of the shift power, and the denominator — the signal power on the bridge, which consists of the total increment minus the power of the Electro Thermal Feedback (ETF). The ETF effect is caused by the change of the resistance of the bridge when it is heated by the signal. The right part of the ratio shows that the physical meaning of the ETF is an addition to the heat loss or gain, which depends on the sign dS_{31}/dR_b. ETF changes its sign depending on which of the relations, $R_b(T_e) < R_S$ or $R_b(T_e) > R_S$ is fulfilled for the selected detector operation mode (positive, PETF, near the superconducting state of the bridge or negative, NETF, respectively). Thus, it is possible to determine the transition temperature from PETF mode (positive ETF) to NETF (negative ETF) mode. It is important to note that the stability condition does not coincide with the condition for changing the sign of the ETF, which follows from (3): $G_0 - P_{\text{in}}(dS_{31})/dT > 0$, i. e. related to TCR($T$).

3. Weak coupling model at the bridge-electrodes interface

Let us assume that there is a proximity effect at the Nb/Hf contact that slightly increases the gap potential of hafnium (increases its T_c) and forms a superconducting weak link, which, unlike the bridge, has its own critical current $I_c^{\text{int}}(T_e)$. The exact nature of the weak link (tunnel contact, microshort, normal interlayer, etc.) does not matter in our consideration. We slowly increase the amplitude of the microwave pump current when establishing the operating mode of the RFTES bolometer. At some point, the pumping amplitude is compared with the critical current of the interface ($I_b \geq I_c^{\text{int}}$), and a resistance jump takes place. Let us take the moment of transition to the normal state as the starting point of the oscillatory cycle. After that, heat will be released in two sources — in the bridge, as before, and on the additional resistance. It is possible to combine the bridge into a single heat source with a resistance $R_{\text{eff}}(T_e) = R_b(T_e) + \Delta R_n$ because of its small size and to consider that all the released heat is concentrated in the volume of the bridge. At the same time, it is intuitively clear that the entire system can relax to a stationary state with a different temperature $T_{\text{state}}^{\text{eff}}$ and current $I_b(T_{\text{state}}^{\text{eff}})$. It should be noted that for the nonlinear resistance $R(T)$, the temperature of the new stationary state can both increase and decrease. This depends on a change of the transmitted power ΔP_{in}, or, equivalently, on a new matching condition between $R_{\text{eff}}(T)$ and R_s. If the jump occurred at $R_b < R_s$, then the matching of the bridge with the source R_s will also improve by a jump under the condition of $R_{\text{eff}} < R_s^2/R_b$, but if $R_{\text{eff}} > R_s^2/R_b$, then the power delivered to the bridge-interface system will decrease, and the stationary temperature will decrease. If the jump occurred at $R_b > R_s$, then the thermal power after the jump will always be lower.

It should be noted here that a high-quality resonator, which is a current source for the bridge, due to its inertia, cannot change the current instantly and smoothly damped current fluctuations affect the increased resistance of the system, creating a thermal pulse (temporary additional heating) with a power of $dP(t) = \left(I_b(t)\right)^2 \Delta R_n$ with an initial power of $dP(0) = (I_c^{\text{int}})^2 \Delta R_n$. Such a pulse can keep the system in a normal state for some time, since the critical current always decreases with the increase of the heating of the electronic subsystem. The critical current remains for some time lower than the interface current because of such heating, and the system remains in a resistive state until the moment when the interface current, relaxing to a stationary state R_{eff}, reaches the value of I_c^{int}. If the current of such a stationary state $I_b(T_{\text{state}}^{\text{eff}})$ turns out to be lower than the value $I_c^{\text{int}}(T_{\text{state}}^{\text{eff}})$, then superconductivity will be restored, and the amplitude of the current in the resonator will begin to increase again to the stationary value I_b (T_{state}), which is determined by the pump level P_{in}. This increase will occur until the system returns to the starting point of the oscillatory cycle — to transition to a normal state through the critical interface current $I_b = I_c^{\text{int}}$. If the new stationary state $I_b(T_{\text{state}}^{\text{eff}}) \geq I_c^{\text{int}}(T_{\text{state}}^{\text{eff}})$ after the jump, then superconductivity will not recover, that is, the system will switch to the state R_{eff} and remain in it (stick).

Based on the above, we assume that it is possible to determine the period of such oscillations $\tau_{\text{osc}} = \tau_r + \tau_s$ (where τ_r — time of the resistive state R_{eff}, τ_s — time of the superconducting state R_b) in the relaxation oscillation mode $\left(I_b(T_{\text{state}}) \geq I_c^{\text{int}}(T_{\text{state}}),\ I_b(T_{\text{state}}^{\text{eff}}) < I_c^{\text{int}}(T_{\text{state}}^{\text{eff}})\right)$, for the known values of pump level P_{in}, the optical power P_{opt} and the characteristic relaxation times of the temperature of the electronic subsystem and resonator.

3.1. Modeling and comparison with experiment

We use the model dependence $R(T)$ for modeling the relaxation oscillations in an RFTES detector which is calculated on the basis of the Mattis-Bardin theory, according to which the microwave current in a superconducting microbridge produces of Joule heat, and calculate the current $I_b(T_{\text{state}})$ required to heat the bridge from the temperature of the cryostat T_{cr} to a steady temperature T_{state}. It is reasonable to assume that the current destruction of superconductivity at the Nb/Hf interface will take place because of the suppression of the order parameter by hafnium. Let us suppose that the temperature dependence of the critical current of the interface $I_c^{\text{int}}(T_e)$ is proportional to the critical current $I_c(T_e)$ in a hafnium microbridge. The critical current density for a superconducting material is expressed by the following ratio [14]:

$$j_c(T) = \frac{8\pi^3}{7\xi(3)} \frac{\gamma}{3\sqrt{3}} eN(0)\Delta(0) \left(\frac{k_{\text{B}} T_c D}{h}\right)^{1/2} \times \left(1 - (T/T_c)^2\right)^{3/2} \left(1 + (T/T_c)^2\right)^{1/2}, \quad (4)$$

where w — width, d — thickness, $\gamma = 0.577$ — Euler constant, $\xi(3) = 1.202$ — Aperi constant, $N(0)$ — density of states at the Fermi level at 0 K, $\Delta(0)$ — energy gap of a superconducting material at 0 K, D — diffusion coefficient.

Next, we determine the stationary values for the interface current and the critical current at a given supplied power P_{in}, these dependencies are shown in Figure 3.

The relaxation oscillations in the model described above can be found by solving a system of two interconnected differential equations with additional imposed conditions that are associated with the competition of the interface current and the critical current

If $I(t) < I_c\left(T_e(t)\right)$

$$\begin{cases} \dfrac{dT_e}{dt} = \dfrac{1}{\tau_{\text{rel}}}\left(T_e\left(I(t)\right) - T_e(t)\right), \\ \dfrac{dI}{dt} = \dfrac{1}{\tau_{\text{res}}\left(T_e(t)\right)}\left(I(T_{\text{state}}) - I(t)\right). \end{cases} \quad (5)$$

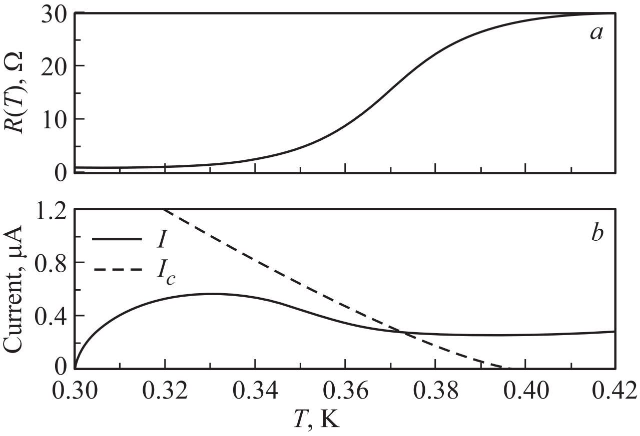

Figure 2. The temperature dependences used in calculations for a superconducting hafnium film at a frequency of 1.5 GHz and $T_c = 0.4$,K: (*a*) active component of the microwave impedance used in the calculations, (*b*) the critical current of the interface. $I_c^{\text{int}}(T) = 0.1 I_c(T)$ (dotted line) and the heating current through the interface for the source impedance $R_s = 3$ Ohm (solid).

If $I(t) \geq I_c\left(T_e(t)\right)$

$$\begin{cases} \dfrac{dT_e}{dt} = \dfrac{1}{\tau_{\text{rel}}}\left(T_e\left(I(t)\right) - T_e(t)\right) + \dfrac{1}{\tau_{i-m}} I^2(t)\Delta R_n, \\ \dfrac{dI}{dt} = \dfrac{1}{\tau_{\text{res}}\left(T_e(t)\right)}\left(I(T_{\text{state}}^{\text{eff}}) - I(t)\right). \end{cases} \quad (6)$$

These systems of differential equations describe the temporal evolution of current, critical interface current, and temperature. In formulas (5)–(6) τ_{rel} — relaxation time of the electronic subsystem, $\tau_{\text{res}}(T_e)$ — relaxation time of the resonator, depending on the nonlinear temperature of the impedance of the superconducting bridge, τ_{i-m} — characteristic time of heat transfer from the interface to the bridge when additional resistance occurs ΔR_n, $T_e\left(I(t)\right)$ — dependence of temperature on current, as in Figure 2, *b*. Temperatures T_{state}, $T_{\text{state}}^{\text{eff}}$ — these are stationary values depending on the pumping level P_{in} and optical power P_{opt}, i. e., stationary states without taking into account additional resistance ΔR_n and taking into account the additional resistance, respectively.

The solution of the time evolution of this system of differential equations is shown in Figure 4. The parameters in the system of equations were chosen to best match the experimental data provided in Figure 5. The calculated spectra obtained are shown in Figure 6 which shows their qualitative agreement with the experiment.

4. Hot spot model

The model is based on the well-known effect when a hot spot appears in the film after photon absorption, which is a limited area with an increased concentration of quasiparticles. In this case, a local increase of the electron temperature occurs, which affects the redistribution of the critical

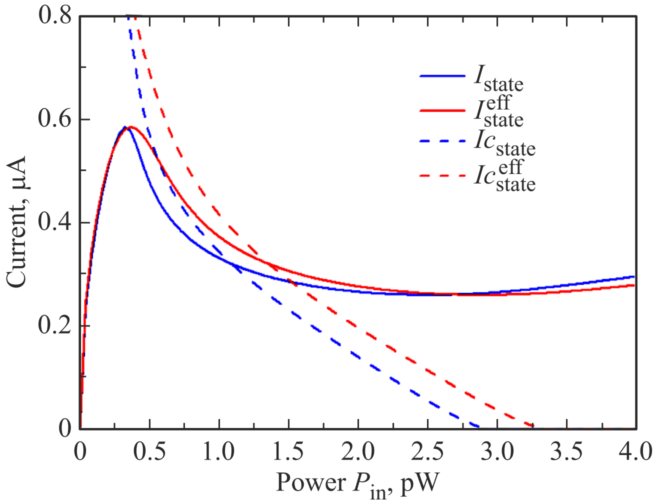

Figure 3. Graphical representation of stationary solutions of currents I_{state} and $I^{\text{eff}}_{\text{state}}$ and critical currents at the interface Ic_{state} and $Ic^{\text{eff}}_{\text{state}}$ depending on the level of pumping/heating P_{in} without additional resistance ΔR_n (taking into account $\Delta R_n = 5\,\text{Ohm}$).

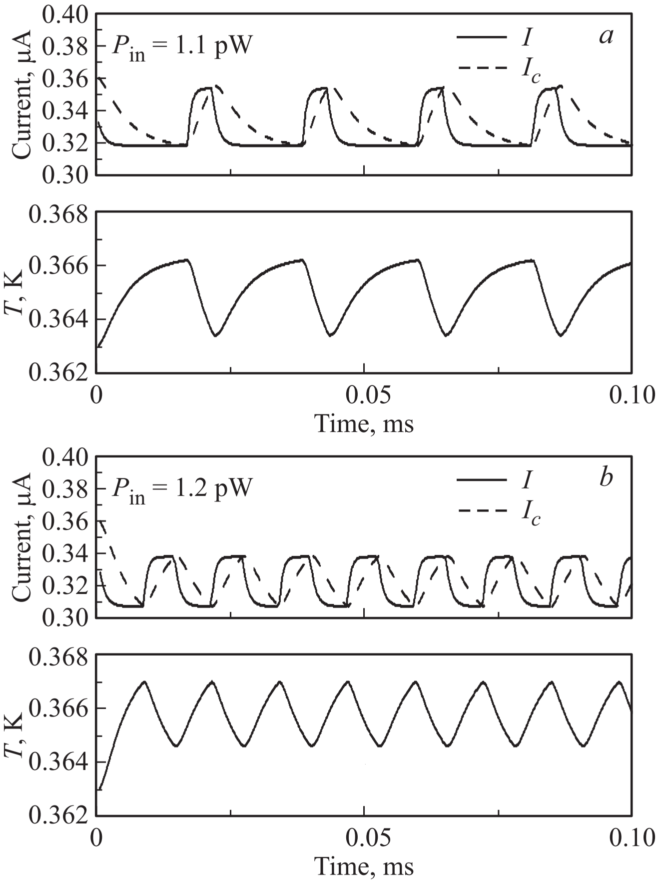

Figure 4. Time evolution of current (solid), critical interface current (dashed line) and temperature at different pump levels P_{in}: (*a*) 1.1 pW, (*b*) 1.2 pW. Following the substitution principle such differences can be caused by an increase of the optical power of 0.1 pW.

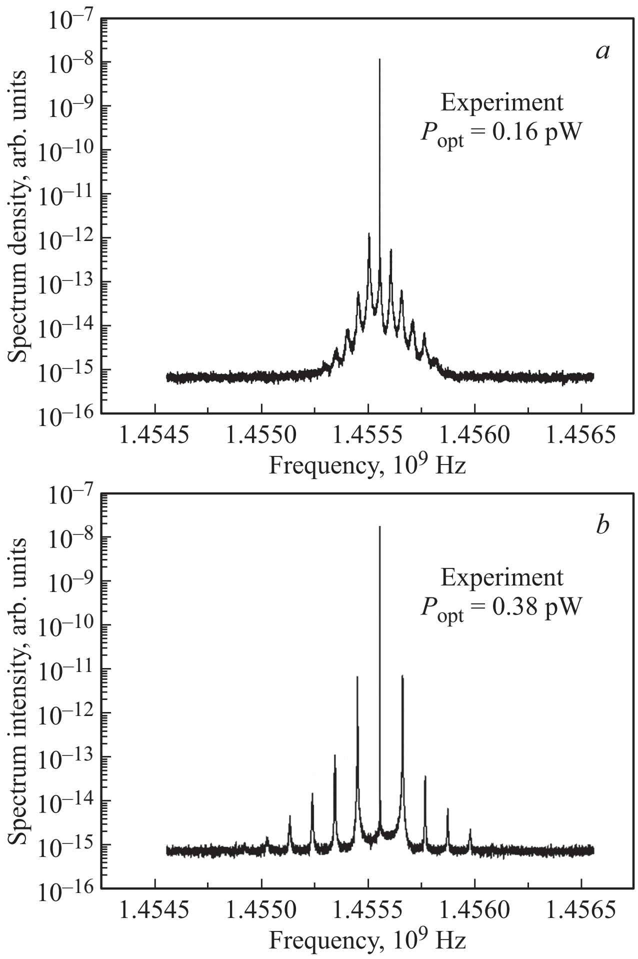

Figure 5. Experimentally obtained spectra when an absorber is heated with a power of P_{opt}: (*a*) 0.16 pW, (*b*) 0.38 pW.

current density in the bridge. A local phase transition to a normal state may take place here, initiated either by a critical temperature or a critical current density in the local region. It is known that the emergence of a hot spot is also associated with film defects that may appear during the aging.

The evolution of the electron temperature in the hot spot model can be described by the following thermal conductivity equation for the one-dimensional case [15]:

$$\frac{dT_e}{dt} = D\frac{d^2T_e}{dx^2} + \frac{P^+ - P^-}{C_e}, \tag{7}$$

where P^+ and P^- — supplied and delivered power on the bridge. The evolution of the size of the hot spot can be predicted using equation (7). It is possible to determine the temperature distribution in the bridge at the specified parameters (heat capacity, relaxation of the resonator, supplied power) by equating the left and right sides to zero and setting the correct boundary conditions for simplicity. We are interested in the dynamics of the hot spot after the appearance of the final resistance, which leads to a noticeable change in the pump power transfer coefficient to the bridge.

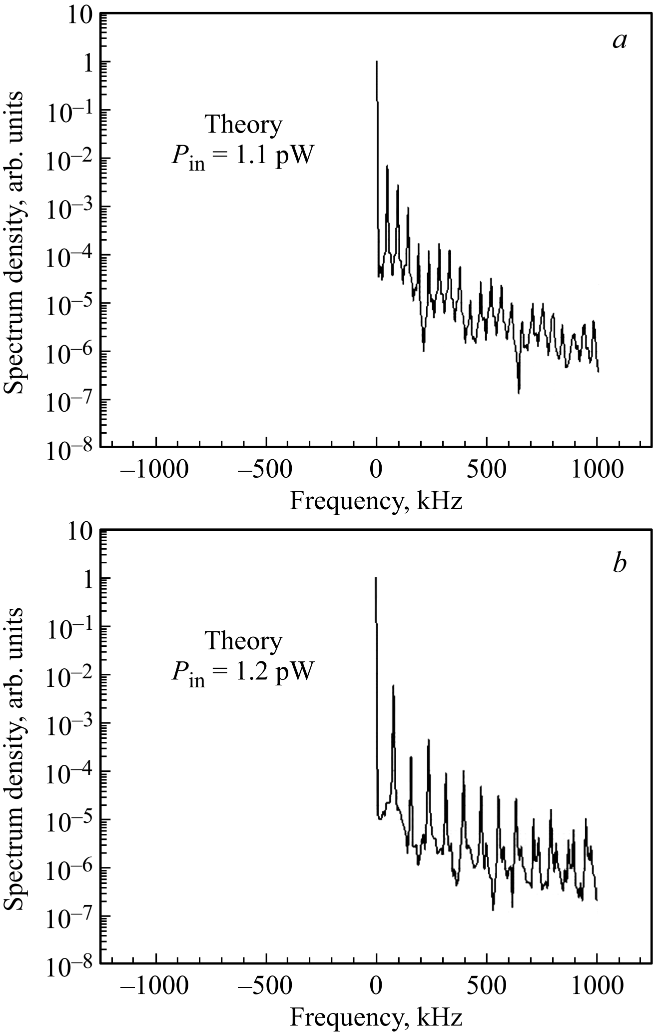

Figure 6. Relaxation oscillation spectra similar to those observed in the experiment (Figure 5) obtained from the weak-link model at the Nb/Hf interface at different pumping levels P_{in}: (*a*) 1.1 pW, (*b*) 1.2 pW, which is very close to the parameters of the experiment.

It is known that a change of the heating power can occur both in the direction of heating and in the direction of cooling with a decrease of the size of the hot spot, which depends on the current size of the spot, that is again a normalized ratio of the form $\big(R(T_e) + \Delta R_n\big)/R_s$, which was used earlier and, apparently, remains relevant for the hot spot model the only difference being that the dependency $\Delta R_n(T)$ will appear. The normalized ratio also determines the sign of the electrothermal feedback, and the process described by the oscillatory equation is probably possible. The reduction of the size of the normal region can result in the restoration of the superconducting channel, and in the continuation of the cyclic process. The similarity and difference of the process with the destruction of superconductivity in the interface and the details of the hot spot model will be discussed later.

5. Conclusion

A theoretical model of self-oscillations was developed. These self-oscillations were previously detected experimentally in the RFTES detector, and no simple physical explanation was previously found for them. These oscillations are registered as carrier modulation with a frequency increasing proportionally to the supplied bias power. It was shown that the weak coupling model at the Nb/Hf interface, associated with the competition of the heating current and the critical current of the interface, can be consistent with experimental data by fitting the parameters of such a model. Perhaps a new effect was discovered that makes it possible to establish an exact relation between the detected power and the frequency of such self-oscillations. The detailed development of a model of such oscillations, the sensitivity of which is based on the temperature dependence of the critical current and inertia of a high-Q resonator, will allow measuring attowatt signals using a frequency meter, which possibly can increase their accuracy compared to other existing technologies of superconducting detectors. It seems feasible to study and, if possible, develop a special manufacturing technology for such power−frequency converters.

Approaches to explaining fluctuations using the hot spot model were also evaluated. Such a model is based on a change of the size of the normal area of the bridge with the formation of vortices, and we plan to develop it in the near future, along with a more detailed comparison of the results of the experiment and both models being developed.

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Conflict of interest

The authors declare that they have no conflict of interest.

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