05.1

Quantized nature of abrupt plastic deformation

© L.B. Zuev

Institute of Strength Physics and Materials Science, Siberian Branch, Russian Academy of Sciences, Tomsk, Russia
E-mail: lbz@ispms.ru

Received January 29, 2024
Revised February 29, 2024
Accepted March 9, 2024

The nature of the elastoplastic invariant of plastic flow, which connects the characteristics of the elastic and plastic components of deformation, is considered, and its connection with fundamental physical constants is established. It is shown that abrupt plastic deformation can be considered as a macroscopic quantum effect associated with the discreteness of the crystal lattice.

Keywords: metals, localization, abrupt deformation, autowaves.

DOI: 10.61011/TPL.2024.06.58479.19877

Plastic (irreversible) deformation is commonly characterized as a result of motion and multiplication of dislocations [1] that are accompanied by a change in the structure of a deformed medium, which consists in accumulation of lattice defects. In recent years, plastic flow has increasingly been viewed as an autowave process [2,3], since it turned out to be difficult to characterize plasticity on the basis of dislocation theory only. Such attempts, which employed certain concepts from the theory of non-equilibrium processes (synergetics), have also been undertaken earlier [4–7].

Autowave representations of the nature of plastic flow have been verified experimentally and grounded theoretically. This approach relies on the notion of paramount importance of localization of plastic deformation and generation of self-excited autowave processes, which are related to the self-organization of a plastically deformed active medium, in the course of plastic flow. A special technique for *in situ* observation of autowaves of localized plastic flow with the use of speckle photography has been developed [3]. Autowaves are characterized by their length λ and propagation velocity V_{aw}; notably, the wavelength is only weakly dependent on the material properties and assumes a value of $\sim 10^{-2}$ m in virtually all cases, while the velocity depends on loading rate and falls within the range of $10^{-5} \leqslant V_{aw} \leqslant 10^{-4}$ m/s. Owing to a vast difference in scales, autowaves of localized plasticity cannot be related directly to the parameters of dislocation substructures or dislocation deformation mechanisms of the Frank–Read kind. However, larger-scale plastic flow features, such as Lüders bands and fronts [8], are regarded as variants of autowave plasticity modes.

The aim of the present study is to interpret the key equations of the autowave plasticity theory and establish their relation with certain physical constants. As was demonstrated in [3], the development of localized autowave plastic deformation of a medium is governed by invariant relation

$$\frac{\lambda V_{aw}}{\chi V_t} = \hat{Z} \approx \frac{1}{2}, \tag{1}$$

where χ is the interplanar distance and V_t is the velocity of transverse elastic waves. The importance of expression (1) consists in the fact that it relates the characteristics of plastic (λV_{aw}) and elastic (χV_t) material deformation.

The physical nature of relation (1) has been discussed in [2,3]. One more possible explanation, which was conceived following the publication of [9], is presented below. It has been demonstrated in [9] that extreme values of physical characteristics of materials may be estimated on a scale set by the Hartree system of units, which allows one to express the coefficients of key relations in a more physically meaningful way.

Specifically, the length scale in the Hartree system of units is set by the Bohr radius of a hydrogen atom:

$$a_0 = \frac{\hbar^2}{me^2} = 5.291 \cdot 10^{-11}\,\text{m}. \tag{2}$$

Having applied this expression and relations $V_t \approx (G/\rho)^{1/2}$ and $V_t \approx \omega_D a_0$ for the transverse sound velocity [10], the author of [9] managed to demonstrate that

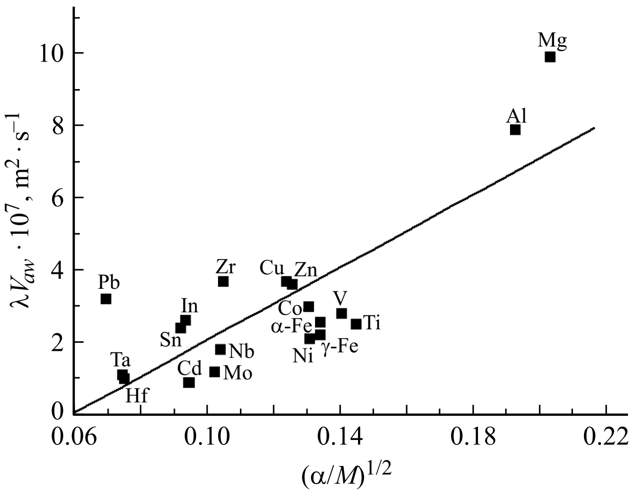
$$V_t \approx \frac{e^2}{\hbar}\left(\frac{m}{2M}\right)^{1/2}, \tag{3}$$

while the Debye cutoff frequency is

$$\omega_D \approx \frac{E}{\hbar}\left(\frac{m}{M}\right)^{1/2}. \tag{4}$$

In accordance with the notation adopted in [9], $\hbar = h/2\pi$ in Eqs. (2)–(4) is the reduced Planck constant, e and m are the charge and the mass of an electron, and M is the atom mass. The deformed medium is characterized in this case by shear modulus G, density ρ, and binding energy E. The meaning and the procedure of evaluation of this quantity were not specified in calculations performed in [9].

The critical step taken in the present study consists in performing the $\chi \to a_0$ substitution in Eq. (1) in accordance



Dependence of the plasticity parameter of metals on the mass of atoms. The correlation coefficient is ~ 0.8.

with [9]. This substitution yields

$$\lambda V_{aw} = \frac{\chi V_t}{2} \approx \frac{\hbar}{2(mM)^{1/2}} \approx \frac{\xi}{M^{1/2}} \sim M^{-1/2}, \qquad (5)$$

where coefficient $\xi = \hbar/2\sqrt{m} \approx 5.5 \cdot 10^{-20}$ J $\cdot$ s $\cdot$ kg$^{-1/2}$. Dependence (5) allows for experimental verification, since the values of the plasticity parameter for 19 different metals have already been determined in [2,3]. The data presented in the figure in coordinates λV_{aw}–$(\alpha/M)^{1/2}$ ($\alpha = 1.66 \cdot 10^{-27}$ kg is the atomic mass unit) confirm the linearity of relation (5), and the corresponding estimate is $\xi^{exp} \approx 5.4 \cdot 10^{-20}$ J $\cdot$ s $\cdot$ kg$^{-1/2}$; i.e., $\xi \approx \xi^{exp}$. In addition, the value of $\lambda V_{aw} \approx 10^{-7}$ m^2/s determined in studies into plastic flow of materials [11] matches the minimum value of viscosity for a number of possible motions in a condensed medium that were analyzed in [9]. Thus, although experimental data have an evident spread due to the complexity of observation of localized plasticity regions in certain cases, the obtained dependence is suitable for further analysis.

In discussing this result, we note that plastic flow is commonly regarded now as a discrete sequence of relaxation jumps of stress and deformation related to the thermally activated crossing of local barriers of various nature by dislocations [12,13] and their subsequent quasi-viscous motion between barriers or over them. In view of the macroscopic nature of localized plasticity autowaves, it is hard to specify the type of barriers and the way of overcoming them; we may thus limit ourselves to the semi-empirical approach to thermally activated plasticity mechanisms [13].

It is assumed that the jump-like mechanism of deformation is general in nature and should be observed in any deformation regime. Therefore, it should be supposed that a smooth plastic flow curve consists of numerous jumps, which, however, may in certain cases be indiscernible to recording instruments. At the same time, macroscopic manifestations of abrupt deformation have been examined in detail [14–16].

The presence of the Planck constant in Eq. (5) suggests an analogy between abrupt deformation and physical phenomena the quantum nature of which illustrates their macroscopic scale. As is known [10,11], these phenomena are superfluidity, superconductivity, and the quantized Hall effect. Their key equations include combinations of fundamental constants, which necessarily contain $\hbar$ [17], as coefficients. Specifically, combination $\hbar c/e$ (c is the speed of light) specifies the magnetic flux quantum for vortices in a superconductor. In superfluid He, ratio $\hbar/M_{\mathrm{He}}$ sets the vortex rotation velocity [17]. In the quantized Hall effect, combination $e^2/\hbar$ is the quantum of conductance of two-dimensional electronic systems [18].

This is the reason why coefficient $\xi = \hbar/2\sqrt{m}$ of Eq. (5) written in the form of a combination of fundamental constants $\hbar$ and m may be regarded as a sign of possibility of quantum-mechanical interpretation of the nature of macroscopic abrupt plastic deformation. This belief is strengthened by a comparison of the data on all mentioned macroscopic quantum phenomena (see a brief summary in the table).

The interpretation of plasticity as a quantum effect, which is guided by the form of expression $\xi_4 = \hbar/2\sqrt{m}$, may appear unusual for researchers working in the field of plasticity physics, since the characteristic spatial scale of macroscopic plasticity phenomena exceeds considerably the scales on which quantum mechanics is applied traditionally. Specifically, the ratio of autowave λ and dislocation b (Burgers vector) scales is $\lambda/b \approx 10^8$.

However, one may make a number of compelling arguments in favor of the proposed interpretation. First of all, deformation quantization appears to be perfectly natural at the dislocation level, since Burgers vector $b \approx 10^{-10}$ m specifies the minimum possible slip in a crystal lattice and may be regarded as a quantum of shear deformation [19]. In addition, an analogy between dislocations and quantized vortices in superfluid helium or quantized currents in type II superconductors has already been noted [20]. The generalized dispersion law of plastic flow autowaves for phonons and autolocalizons [2] also bears a similarity to the dispersion curve for phonons and rotons in superfluid helium [21]. This similarity may be explained if one takes the view proposed in [22] regarding the analogy between plastic flow and superfluidity. According to this interpretation, elements of a deformable medium are akin to atoms of a superfluid liquid in being simultaneously engaged in slow motions of separate volumes in the process of macroscopic shape change of a body as a whole and in motion of dislocations with high velocities (potentially up to $V_{disl} \approx V_t$ [1]) that supports this shape change. Two viscosity coefficients of a deformable medium corresponding to these motions differ by several orders of magnitude [23].

The evident physical reason behind the manifestation of quantum effects in macroscopic plastic deformation

Comparison of macroscopic quantum phenomena

№	Phenomenon	Quantized parameter	Coefficient (quantum)
1	Superconductivity [10]	Magnetic flux $\Phi = \xi_1 i$	$\xi_1 = \frac{\pi\hbar c}{e}$
2	Superfluidity [10]	Vortex rotation rate $v = \xi_2 r^{-1} i$	$\xi_2 = \frac{\hbar}{M_{\mathrm{He}}}$
3	Quantized Hall effect [11]	Hall conductance $G_H = \xi_3 i$	$\xi_3 = \frac{e^2}{\hbar}$
4	Abrupt plastic deformation	Elongation on jump $\delta L = \xi_4 (V_{aw}\sqrt{M})^{-1} i$	$\xi_4 \approx \frac{\hbar}{2\sqrt{m}}$

Note. r is the vortex radius, M_{He} is the mass of a He atom, and $i = 1, 2, 3 \ldots$.

processes is the close connection between the autowave plasticity mechanism and the lattice characteristics, which is defined distinctly by elastoplastic invariant (1) [2]. Indeed, the generation of autowaves of localized plastic flow, which acts as the mechanism of self-organization of a deformable medium, is effected in a crystalline medium with its very existence and properties governed by quantum-mechanical laws [10]. It is evident that the processes of transformation of elastic and plastic deformation fields in a medium capable of self-organization, which form the basis for the autowave mechanism of plastic flow and the generation of localized plasticity autowaves, are subject to the quantum nature of interparticle bonding in crystals.

The macroscopic abruptness is easy to explain within this interpretation if one makes a natural assumption that an integer number $i = 1, 2, 3 \ldots$ of autowaves with length λ should fit within sample length L (i.e., $L = \lambda i$). This is equivalent to quantization of a localized plasticity autowave. Let us rewrite Eq. (5) in the form

$$\lambda = \frac{\hbar}{2(mM)^{1/2}} \frac{1}{V_{aw}} = \frac{\xi}{M^{1/2} V_{aw}} \tag{6}$$

and, taking relation $\lambda \approx \delta L/i$ into account, determine the sample elongation from Eq. (6):

$$\delta L \approx \frac{\hbar}{2(mM)^{1/2}} \frac{i}{V_{aw}} = \xi \frac{i}{M^{1/2} V_{aw}} = \frac{\xi}{\kappa} i. \tag{7}$$

Coefficient $\kappa = V_{aw}\sqrt{M}$ in Eq. (7) characterizes the type of deformed material and the deformation conditions, since $V_{aw} \approx 10 V_{mach}$ [2], where V_{mach} is the rate of motion of the movable gripper of a testing machine.

A numerical order-of-magnitude estimate of δL obtained in accordance with Eq. (7) for an Al sample at $i = 1$ and characteristic velocity $V_{aw} \approx 1.8 \cdot 10^{-4}$ m/s of a localized plasticity autowave [3] is $\delta L \approx 10^{-4}$ m, which corresponds to a deformation increment $\sim 10^{-3}$. This estimate agrees with the parameters of an individual deformation jump measured experimentally in studies into abrupt deformation [15,16].

Thus, the jump-like nature of plastic deformation may be regarded as a corollary of discretization of elongation of a deformed sample; according to Eq. (7), the presence of jumps is obligatory. The specifics of evolution of abrupt deformation suggest that this phenomenon is of a quantum nature.

The presented data support the existence of a certain similarity between abrupt deformation and macroscopic quantum phenomena in condensed media. We managed to reveal a clear connection between the elastoplastic invariant of plastic flow and physical constants, namely, the Planck constant ($\hbar$) and the rest mass of an electron (e). There emerge reasons to believe that macroscopic localization of plastic flow may be regarded as a quantum effect underpinned by the discreteness of a crystal lattice.

Funding

This study was carried out under the state assignment of the Institute of Strength Physics and Materials Science of the Siberian Branch of the Russian Academy of Sciences (project No. FWRW-2021-0011).

Conflict of interest

The author declares that he has no conflict of interest.

References

[1] D. Hull, D.J. Bacon, *Introduction in dislocations* (Elsevier, Oxford, 2011).
[2] L.B. Zuev, S.A. Barannikova, V.I. Danilov, V.V. Gorbatenko, Prog. Phys. Met., **22** (1), 3 (2021). DOI: 10.15407/ufm.22.01.003
[3] L.B. Zuev, Yu.A. Khon, V.V. Gorbatenko, *Fizika neodnorodnogo plasticheskogo techeniya* (Fizmatlit, M., 2024) (in Russian).
[4] V.E. Panin, V.E. Egorushkin, Phys. Mesomech., **18** (4), 377 (2015). DOI: 10.1134/S1029959915040104.
[5] A.I. Slutsker, Phys. Solid State, **47** (5), 801 (2005). DOI: 10.1134/1.1924836.
[6] O.B. Naimark, Fiz. Mezomekh., **6** (4), 45 (2003) (in Russian).
[7] O.B. Naimark, Fiz. Mezomekh., **18** (3), 71 (2015) (in Russian).
[8] J. Pelleg, *Mechanical properties of materials* (Springer, Dordrecht, 2013).
[9] V.V. Brazhkin, Phys. Usp., **66** (11), 1154 (2023). DOI: 10.3367/UFNe.2022.11.039261.

[10] R.E. Newnham, *Properties of materials* (University Press, Oxford, 2005).
[11] L.B. Zuev, Ann. Phys., **10** (11-12), 965 (2001). DOI: 10.1002/andr.200610233
[12] V.L. Indenbom, A.N. Orlov, Yu.Z. Estrin, *Elementarnye protsessy plasticheskoi deformatsii kristallov* (Nauk. Dumka, Kiev, 1978), pp. 93–113 (in Russian).
[13] D. Caillard, J.L. Martin, *Thermally activated mechanisms in crystal plasticity* (Elsevier, Oxford, 2003).
[14] A.C. Iliopoulos, N.S. Nikolaidis, E.C. Aifantis, Physica A, **438**, 509 (2015). DOI: 10.1016/j.physa.2015.07.007
[15] A.A. Shibkov, M.F. Gasanov, M.A. Zheltov, A.E. Zolotov, V.I. Ivolgin, Int. J. Plast., **86** (8), 37 (2016). DOI: 10.1016/j.ijplas.2016.07.014
[16] M.A. Lebyodkin, D.A. Zhemchuzhnikova, T.A. Lebedkina, E.C. Aifantis, Results Phys., **12** (8), 867 (2019). DOI: 10.1016/j.rinp.2018.12.067
[17] L.P. Pitaevskii, Sov. Phys. Usp., **9** (6), 888 (1967). DOI: 10.1070/PU1967v009n06ABEH003230.
[18] K. von Klitzing, *The quantized Hall effect. Nobel lecture* (Stockholm, 1985).
[19] M.O. Katanaev, Phys. Usp., **48** (7), 675 (2005). DOI: 10.1070/PU2005v048n07ABEH002027.
[20] V.L. Indenbom, *Sovremennaya kristallografiya* (Nauka, M., 1979), Vol. 2, pp. 297–341 (in Russian).
[21] N.B. Brandt, V.A. Kul'bachinskii, *Kvazichastitsy v fizike kondensirovannogo sostoyaniya* (Fizmatlit, M., 2007) (in Russian).
[22] A.S. Balankin, Pis'ma Zh. Tekh. Fiz., **15** (22), 15 (1989) (in Russian).
[23] D. Blaschke, D. Motolla, E. Preston, Phil. Mag. A., **100** (10), 571 (2020). DOI: 10.1080/14786435.2019.1696484

Translated by D.Safin