# 09 <br> Influence of nonlinear noise correlation on transmission range 

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#### Abstract

It has been established that the physical mechanism for increasing the operating range of coherent communication systems with digital compensation for linear signal distortions in the receiver is to reduce the correlation of nonlinear interference noise from neighboring spans in the absence of physical chromatic dispersion compensators. It is shown that changing the correlation coefficient $\varepsilon$ from 1 (a characteristic value for a fiber-optic link with full physical chromatic dispersion compensation) to 0 (the minimum value for a fiber-optic link without physical chromatic dispersion compensation) leads to a several times increase in the operating range. The optimal relationship between the gains of erbium fiber amplifiers and optical power losses in spans adjacent to the amplifier has been determined, ensuring maximum operating range of lines with an arbitrary value of the nonlinear noise correlation coefficient.


Keywords: DWDM-FOCL, OSNR required, EDFA BER, ASE-noise, nonlinear noise, Gaussian noise, coherent data transmission, multi-span FOCL with/without chromatic dispersion compensators.

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## Introduction

A rapid increase in transmitting capacity and operating range of communication networks on all levels is needed for sustaining the growth of newly produced data, scaling of existing data centers and construction of new ones, development of cloud services, and transition to a new generation of mobile networks. A high transmitting capacity of modern fiber-optic communication lines (FOCLs) is achieved by combining the dense wavelength division multiplexing (DWDM) technology with coherent detection and the use of multilevel modulation formats with a high symbol efficiency [1]. A considerable enhancement of the range of non-regeneration FOCL operation (to several thousand kilometers) is achieved through the use of erbiumdoped fiber amplifiers (EDFAs) positioned between spans of fiber with a length on the order of 100 km .

In addition to amplifiers, chromatic dispersion compensators are installed after each span in traditional noncoherent FOCLs. In high-speed coherent systems with digital signal processing, chromatic dispersion is compensated in the course of digital signal processing in the receiver, which makes it unnecessary to use physical dispersion compensators [2-6]. The very first experiments have
revealed that the rate of accumulation of nonlinear distortion in communication systems with digital chromatic dispersion compensation is significantly lower than the one in communication systems with physical chromatic dispersion compensators. The correlation of nonlinear interference noise from adjacent spans is characterized by dimensionless coefficient $\varepsilon$ that is determined experimentally $[7,8]$. The relation between correlation coefficient $\varepsilon$ and the presence or lack of physical chromatic dispersion compensators was examined in $[7,9,10]$, but the relation between the maximum operating range of FOCLs and $\varepsilon$ has not been investigated.
It is demonstrated in the present study that the suppression of correlation of nonlinear interference noise from adjacent spans without the use of physical chromatic dispersion compensators is the physical mechanism of enhancement of the operating range of coherent communication systems with digital compensation of linear signal distortion in the receiver. It is shown that the variation of correlation coefficient $\varepsilon$ from 1 (a value typical of FOCLs with full physical chromatic dispersion compensation) to 0 (the minimum value for FOCLs without physical chromatic dispersion compensation) corresponds to a 2.8 -fold enhancement of the transmission range. The optimum relation between EDFA gain factors and optical power losses in spans,
which provides the maximum FOCL operating range, is determined for arbitrary $\varepsilon$ values.

## 1. Models of OSNR evolution in multi-span communication lines

### 1.1. Model of additive summation of nonlinear noise (IGN model)

The model with Gaussian noise (GN model) is the one used most often to characterize nonlinear distortion in coherent systems without dispersion compensation. This model provides a fine fit to experimental data for long communication lines (with more than five spans) and is based on the assumption that nonlinear distortion is Gaussian noise, which is additive to spontaneous emission noise [10-13]. The fundamentals of the nonlinear noise model have been verified experimentally and via numerical modeling $[8,14-18]$. It is assumed that the bit error rate (BER) depends only on total noise (the sum of linear (amplified spontaneous emission, ASE) and nonlinear ( NL ) noise) and is independent of the ratio between the contributions of each noise type to total noise. This assumption provides an opportunity to construct a simple phenomenological model of communication lines that is easy to use in practice. In what follows, the absolute values of quantities are denoted with capital letters, while the corresponding logarithmic values are denoted with lowercase letters: $p=10 \lg P$. Total noise in the GN model is written as

$$
P_{\Sigma}=P_{\mathrm{ASE}}+P_{\mathrm{NL}} .
$$

Dividing both parts of this expression by signal power $P$ at the beginning of a span, we find [19-21]

$$
\begin{equation*}
\frac{1}{O S N R_{\mathrm{BER}}}=\frac{1}{O S N R_{\mathrm{L}}}+\frac{1}{O S N R_{\mathrm{NL}}} \tag{1}
\end{equation*}
$$

Quantity $O S N R_{\mathrm{BER}}=P / P_{\Sigma}$ is the ratio between the signal power and the total noise power and defines the BER value for a communication line. The relation between $O S N R_{\text {BER }}$ and BER in the considered model is unambiguous and is characterized by the transponder calibration curve.

The required $O S N R$ value providing a critical bit error rate prior to the application of forward error correction (FEC) is designated as $O S N R_{\mathrm{BTB}}$. Parameter $O S N R_{\mathrm{L}}=P / P_{\text {ASE }}$ is the ratio between the signal power and the ASE noise power. The amplifier noise (at the end of a span) is given by $h \nu B(G F-1) \approx h \nu B G F$, where $h v$ is the energy of a quantum, $B$ is the normalized bandwidth, $G$ is the amplifier gain factor, and $F$ is the amplifier noise factor. Parameter $O S N R_{\mathrm{NL}}=P / P_{\mathrm{NL}}$ is the ratio between the signal power and the NL noise power. The NL noise magnitude in the GN model depends on signal power $P$ as $P_{\mathrm{NL}}=\eta P^{3}$, where $\eta$ is the nonlinearity factor [22]. Nonlinearity factors are calculated by finding the best fit to experimental data on the transponder signal spectrum,
the signal band, and the electrical filter band. It follows that [19-21]

$$
\begin{equation*}
\frac{1}{O S N R_{\mathrm{NL}}}=\eta P^{2} \tag{2}
\end{equation*}
$$

The operability condition for a line is given by the following inequality [21]:

$$
\begin{equation*}
O S N R_{\mathrm{BER}}>O S N R_{\mathrm{BTB}} \tag{3}
\end{equation*}
$$

which may be rewritten in the form

$$
\frac{1}{O S N R_{\mathrm{L}}}<\frac{1}{O S N R_{\mathrm{BTB}}}-\frac{1}{O S N R_{\mathrm{NL}}}
$$

The quantity in the right-hand part of this expression is the required OSNR, which is defined as

$$
\begin{equation*}
\frac{1}{O S N R_{\mathrm{R}}}=\frac{1}{O S N R_{\mathrm{BTB}}}-\frac{1}{O S N R_{\mathrm{NL}}} \tag{4}
\end{equation*}
$$

In other words, $O S N R_{\mathrm{R}}$ is the minimum $O S N R_{\mathrm{L}}$ value in a line that guarantees its operability. In the back-to-back configuration, we have $O S N R_{\mathrm{R}}=O S N R_{\mathrm{BTB}}$. In an actual line, $O S N R_{\mathrm{R}}>O S N R_{\mathrm{BTB}}$ due to nonlinear signal distortion.

The OSNR margin is defined as [21]

$$
O S N R_{\mathrm{M}}=\frac{O S N R_{\mathrm{L}}}{O S N R_{\mathrm{R}}}
$$

When a communication line is being designed, a certain $O S N R$ margin (service margin, $A_{\mathrm{M}}$ ) is factored in. It is assumed that a line is operable as per the design if the following condition is satisfied:

$$
\frac{A_{\mathrm{M}}}{O S N R_{\mathrm{L}}}+\frac{1}{O S N R_{\mathrm{NL}}} \leq \frac{1}{O S N R_{\mathrm{BTB}}}
$$

The service margin is often set to $2\left(A_{\mathrm{M}}=2\right.$; the corresponding decibel value is $a_{\mathrm{M}}=3 \mathrm{~dB}$, which corresponds to a range enhancement of 15 km at kilometric attenuation $\left.\alpha_{0}=0.2 \mathrm{~dB} / \mathrm{km}\right)$ at the design stage. All calculations below are performed with the service margin factored in.

The model of additive (incoherent) summation of nonlinear noise (incoherent Gaussian noise model, IGN model) is used for DWDM lines consisting of several spans and lacking physical dispersion compensators [19-21]:

$$
\begin{equation*}
\frac{1}{O S N R_{\mathrm{NL}}}=\sum_{i} \frac{1}{O S N R_{\mathrm{NLi}}} \tag{5}
\end{equation*}
$$

This model requires the determination of nonlinearity factors $\eta$ of individual spans that are calculated based on the obtained experimental data on the transponder signal spectrum, the signal band, and the electrical filter band.

### 1.2. Model of superlinear summation of nonlinear noise ( $\varepsilon$-model)

The inverse linear $O S N R_{\mathrm{L}}$ and the nonlinear $O S N R_{\mathrm{NL}}$ are calculated in different ways. The inverse linear $\operatorname{OSNR} n$ of
span $n$ multiplied by the margin for a line consisting of $N$ spans is written as [20]

$$
\begin{equation*}
\frac{A_{\mathrm{M}}}{O S N R_{\mathrm{L} n}}=A_{\mathrm{M}} \frac{h \nu B A_{n} F_{n}}{P_{n}}, \tag{6}
\end{equation*}
$$

where $A_{\mathrm{M}}=2$ is the design OSNR margin, $v$ is the carrier frequency, $B=12.5 \mathrm{GHz}$ is the normalized bandwidth, $A_{n}=10^{\alpha_{n} L_{n} / 10}$ is the loss factor $(A \geq 1), F_{n}$ is the noise factor of an amplifier at the end of span $n$, and $P_{n}$ is the power at the optical span inlet.

The overall inverse linear $O S N R$ with an allowance made for the margin is then given by

$$
\begin{equation*}
\frac{A_{\mathrm{M}}}{O S N R_{\mathrm{L}}}=A_{\mathrm{M}} \sum_{n=1}^{N} \frac{1}{O S N R_{\mathrm{L} n}}=A_{\mathrm{M}} \sum_{n=1}^{N} \frac{h v B A_{n} F_{n}}{P_{n}} \tag{7}
\end{equation*}
$$

As was demonstrated in [20,21], the accumulation of nonlinear noise in a multi-span line is, in the general case, superlinear ( $\propto N^{1+\varepsilon}$, where $N$ is the number of spans). With (2) taken into account, superlinear summation of noise yields the following expression for the inverse nonlinear OSNR [20,21]:

$$
\begin{align*}
\frac{1}{O S N R_{\mathrm{NL}}} & =\left[\sum_{n=1}^{N}\left(\frac{1}{O S N R_{\mathrm{NL} n}}\right)^{1 /(1+\varepsilon)}\right]^{1+\varepsilon} \\
& =\left[\sum_{n=1}^{N}\left(\eta_{n} P_{n}^{2}\right)^{1 /(1+\varepsilon)}\right]^{1+\varepsilon} \tag{8}
\end{align*}
$$

where $0 \leq \varepsilon \leq 1$ is the dimensionless correlation coefficient that is determined experimentally. In an idealized FOCL with dispersion compensators in each span, $\varepsilon=1$. In the other class of idealized FOCLs, where digital signal processing (DSP) in the receiver is used instead of physical dispersion compensators, a minimum value of $\varepsilon=0$ may be achieved. In real-world DWDM lines without physical dispersion compensation, $\varepsilon$ varies from 0.1 to 0.3 ; in lines with dispersion compensation, $\varepsilon$ exceeds 0.8 . The precise $\varepsilon$ value depends on the fiber type, the transmission format (QPSK, 16QAM, 64QAM), the symbol signaling rate, and other parameters. General expression (8) turns into formula (5), which characterizes a special case, at $\varepsilon=0$.

According to (1), the design inverse $O S N R_{\text {BERP }}$ with the influence of adjacent channels being neglected is

$$
\begin{equation*}
\frac{1}{O S N R_{\mathrm{BER} P}}=A_{\mathrm{M}} \sum_{n=1}^{N} \frac{h \nu B A_{n} F_{n}}{P_{n}}+\left[\sum_{n=1}^{N}\left(\eta_{n} P_{n}^{2}\right)^{1 /(1+\varepsilon)}\right]^{1+\varepsilon} \tag{9}
\end{equation*}
$$

Let us introduce the following designations: $h \nu B A_{n} F_{n} A_{\mathrm{M}}=C_{n}$ and $\frac{1}{O S N R_{\mathrm{BTB}}}=b$. Expressions (3) and (9) then take the form

$$
\begin{equation*}
\sum_{n=1}^{N} \frac{C_{n}}{P_{n}}+\left[\sum_{n=1}^{N}\left(\eta_{n} P_{n}^{2}\right)^{1 /(1+\varepsilon)}\right]^{1+\varepsilon} \leq b \tag{10}
\end{equation*}
$$

One of the design objectives is to minimize the left-hand part with respect to $\mathbf{P}=\left(P_{1}, P_{2}, \ldots, P_{n}\right)$ while preserving the inequality with respect to $b$.

If all spans are the same $\left(C_{n}=C\right)$ and input powers are also the same for all spans $\left(P_{1}=P_{2}=\ldots=P_{N}=P\right)$, (7) and (8) take the form

$$
\begin{equation*}
\frac{A_{\mathrm{M}}}{O S N R_{\mathrm{L}}}=N \frac{C}{P} \tag{11}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{1}{O S N R_{\mathrm{NL}}}=N^{1+\varepsilon} \eta P^{2} \tag{12}
\end{equation*}
$$

A transmission system with $N$ identical spans is operable as per the design if

$$
\begin{equation*}
N \frac{C}{P}+N^{1+\varepsilon} \eta P^{2} \leq b \tag{13}
\end{equation*}
$$

This inequality allows one to determine number $N$ of spans characterizing the maximum transmission range of a FOCL and the corresponding value of power P. In subsequent derivations performed for boundary parameters of a system, the equality sign is used in (13) instead of the inequality sign.

## 2. DWDM line with spans of equal length

### 2.1. Single-span line; determination of $P_{\text {min }}$ and $\boldsymbol{P}_{\text {max }}$

The operability condition for a single-span line is obtained by inserting $N=1$ into (13):

$$
\begin{equation*}
\frac{C}{P}+\eta P^{2} \leq b \tag{14}
\end{equation*}
$$

Substituting the inequality sign with the equality sign, we find an equation for critical values of $P$. This equation has one non-physical negative solution and two positive solutions, which are designated as $P_{\min }$ and $P_{\text {max }}$. Condition (14) is satisfied if the power input into a span is $P_{\text {min }} \leq P \leq P_{\text {max }}$.

### 2.2. Approximation of additive summation of nonlinear noise ( $\varepsilon=0$ )

At $\varepsilon=0$, nonlinear noise from different spans is summed additively. Relation (13) then takes the form

$$
\begin{equation*}
N \frac{C}{P}+N \eta P^{2} \leq b \tag{15}
\end{equation*}
$$

Following the transformation of expression (15) and the transition to logarithmic units $p([\mathrm{dBm}])=10 \lg (P[\mathrm{~mW}])$, $\operatorname{osnr}[\mathrm{dB}]=10 \lg (O S N R)$, we find

$$
\begin{equation*}
n=p-\text { osnr }_{\mathrm{BTB}}-10 \lg \left(\text { OSNR }_{\mathrm{M}} h v B A F+\eta 10^{0.3 p}\right) \tag{16}
\end{equation*}
$$

A set of dependences of the maximum line length on power input into a span for a line with $N$ spans 100 km in


Figure 1. Dependence of the maximum line length on power input into a span for identical spans. The modeling parameters are as follows: the span length is $100 \mathrm{~km}, N F=10 \lg F=5 \mathrm{~dB}$, $o s n r_{\mathrm{BTB}}=12 \mathrm{~dB}, \alpha_{0}=0.2 \mathrm{~dB} / \mathrm{km}, \eta=140 \cdot 10^{-6} \mathrm{~mW}^{-2}, \varepsilon=0$, $p_{\text {min } \mathrm{BER}}=0.86 \mathrm{dBm}, p_{\mathrm{G}}=1 \mathrm{~dB}, p_{\text {max }}=12.26 \mathrm{dBm}$.
length and different values of $o s n r_{M}$ is shown in Fig. 1. In addition, it presents the calculated values of

$$
\begin{gathered}
P_{\min \mathrm{BER}}=\left(\frac{h v B A F}{2 \eta}\right)^{1 / 3}[\mathrm{~W}], \\
p_{\min \mathrm{BER}}=10 \cdot \lg \left(P_{\min \mathrm{BER}} \cdot 1000\right)[\mathrm{dBm}]
\end{gathered}
$$

- power providing the minimum BER;

$$
\begin{gathered}
P_{\mathrm{G}}=\left(\frac{h v B A F}{\eta}\right)^{1 / 3}[\mathrm{~W}], \\
p_{\mathrm{G}}=10 \lg \left(P_{\mathrm{G}} \cdot 1000\right)=p_{\min \mathrm{BER}}+1[\mathrm{dBm}], \\
P_{\max \mathrm{M}}=\left(3 N \eta O S N R_{\mathrm{BTB}}\right)^{-1 / 2}[\mathrm{~W}], \\
p_{\max \mathrm{M}}=10 \lg \left(P_{\max \mathrm{M}} \cdot 1000\right)[\mathrm{dBm}]
\end{gathered}
$$

- power providing the maximum $O S N R$ margin.


### 2.3. Maximum line length at an arbitrary correlation coefficient

In order to find the maximum number of spans (the transmission range is related linearly to it), one should substitute inequality (13) with the corresponding equality

$$
\begin{equation*}
N \frac{C}{P}+N^{1+\varepsilon} \eta P^{2}-b=0 \tag{17}
\end{equation*}
$$

The left-hand part of this expression is a function of $N$ and $P$ that reaches its maximum at

$$
\begin{gather*}
N=\left[\frac{4}{27} \frac{1}{O S N R_{\mathrm{BTB}}^{3}\left(A_{\mathrm{M}} h v B A F\right)^{2} \eta}\right]^{\frac{1}{3+\varepsilon}}, \\
P=\left(\frac{O S N R_{\mathrm{BTB}} h v B A F}{2 N^{\varepsilon} \eta}\right)^{1 / 3}=\left(\frac{O S N R_{\mathrm{BTB}} h v B A F}{2 \eta}\right)^{1 / 3} \\
\times\left[\frac{27}{4} O S N R_{\mathrm{BTB}}^{3}\left(A_{\mathrm{M}} h v B A F\right)^{2} \eta\right]^{\frac{\varepsilon}{3(3+\varepsilon)}} . \tag{18}
\end{gather*}
$$



Figure 2. Dependence of the maximum line length on power input into a span for different $\varepsilon$ values. The modeling parameters are as follows: the span length is $100 \mathrm{~km}, \quad N F=5 \mathrm{~dB}, \quad o s n r_{\mathrm{BTB}}=12 \mathrm{~dB}, \quad \alpha_{0}=0.2 \mathrm{~dB} / \mathrm{km}$, $\eta=140 \cdot 10^{-6} \mathrm{~mW}^{-2}, a_{\mathrm{M}}=3 \mathrm{~dB}$.

The maximum line length is the product of $N$ and the span length..

At $\varepsilon=0$, Eq. (17) takes the form

$$
\begin{equation*}
N \frac{C}{P}+N \eta P^{2}=b \tag{19}
\end{equation*}
$$

The critical minimum and maximum power levels for a single-span line are

$$
\begin{equation*}
P_{\min }=\frac{C}{b}, \quad P_{\max }=\left(\frac{b}{\eta}\right)^{1 / 2} . \tag{20}
\end{equation*}
$$

At $\varepsilon=1$, Eq. (17) takes the form

$$
\begin{equation*}
N^{2} \eta P^{2}+N \frac{C}{P}-b=0 \tag{21}
\end{equation*}
$$

The positive solution of this quadratic equation in $N$ is

$$
\begin{equation*}
N=\frac{\sqrt{C^{2}+4 b \eta P^{4}}-C}{2 \eta P^{3}} \tag{22}
\end{equation*}
$$

The critical minimum and maximum power levels are the same as in (20), since the specific value of $\varepsilon$ is of no importance for a single-span line. A set of dependences of $L$ on $P$ for a line with spans of equal length an different values of $\varepsilon$ is shown in Fig. 2.

It can be seen that the maximum transmission range achieved at $\varepsilon=0$ (lines without dispersion compensators) is 2.8 times higher than the range at $\varepsilon=1$ (lines with regular dispersion compensation).

## 3. Multi-span DWDM line with spans of different length

In the case of a multi-span DWDM line with spans of different length, dependences of the maximum transmission


Figure 3. Power diagram for span $k$. EDFA is an erbium-doped fiber amplifier.
range on power input into a span at arbitrary values of $\varepsilon$ are calculated in the same way as it was done in Section 2.3.

In order to determine the optimum gain factors, one needs to find the minimum of expression

$$
\begin{align*}
f\left(P_{1}, \ldots, P_{N}\right) & =\frac{1}{O S N R_{\mathrm{BER}}}=\frac{1}{O S N R_{\mathrm{L}}}+\frac{1}{O S N R_{\mathrm{NL}}} \\
& =\sum_{n=1}^{N} \frac{C_{n}}{P_{n}}+\left[\sum_{n=1}^{N}\left(\eta_{n} P_{n}^{2}\right)^{\frac{1}{1+\varepsilon}}\right]^{1+\varepsilon} \tag{23}
\end{align*}
$$

The following power values satisfy the condition for minimum of (23):

$$
\begin{gather*}
P_{k}=2^{-\frac{1}{3}} \frac{1}{\sqrt{\eta_{k}}}\left(C_{k} \sqrt{\eta_{k}} \frac{1+\varepsilon}{3^{+\varepsilon}}\left[\sum_{n=1}^{N}\left(C_{n} \sqrt{\eta_{n}}\right)^{\frac{2}{3+\varepsilon}}\right]^{-\frac{\varepsilon}{3}}\right.  \tag{24}\\
k=1, \ldots, N
\end{gather*}
$$

If same-type fibers are used in different spans and the lengths of spans are greater than effective fiber length $L_{\text {eff }}$, nonlinearity coefficients of spans $\eta_{1}=\eta_{2}=\ldots \eta_{N}=\eta$ are the same, and (24) takes the form

$$
\begin{equation*}
P_{k}=\frac{1}{\sqrt[3]{2 \eta}} C_{k}^{\frac{1+\varepsilon}{3+\varepsilon}}\left[\sum_{n=1}^{N} C_{k}^{\frac{2}{3+\varepsilon}}\right]^{-\frac{\varepsilon}{3}}, k=1, \ldots, N \tag{25}
\end{equation*}
$$

which corresponds to the following expression in logarithmic units:

$$
\begin{equation*}
p_{k}=\frac{1+\varepsilon}{3+\varepsilon}\left(10 \lg C_{k}\right)+\text { const, } k=1, \ldots, N \tag{26}
\end{equation*}
$$

With the nonlinearity coefficients and the noise factors of amplifiers being the same, we find

$$
\begin{equation*}
p_{k}=\frac{1+\varepsilon}{3+\varepsilon} a_{k}+\text { const }, k=1, \ldots, N \tag{27}
\end{equation*}
$$

where $a_{k}$ is the power loss in a span in dB . The obtained expressions specify (up to a single common constant) the power levels at which the maximum transmission range is achieved.

If the power loss in span $k$ is $a_{k}$ and the amplifier gain factor is $g_{k}$, the signal powers at the span inlet $\left(p_{k}\right)$ and outlet $\left(p_{k+1}\right)$ are bound by the following relation (power balance):

$$
\begin{equation*}
p_{k}-a_{k}+g_{k}=p_{k+1} \tag{28}
\end{equation*}
$$

Figure 3 presents the power balance in a graphical form.
The gain factors for span $k$ were determined from relations (27) and (28):

$$
\begin{equation*}
g_{k}=\frac{2}{3+\varepsilon} a_{k}+\frac{1+\varepsilon}{3+\varepsilon} a_{k+1}, k=1, \ldots, N-1 \tag{29}
\end{equation*}
$$

A special case at $\varepsilon=0$ :

$$
\begin{equation*}
g_{k}=\frac{2}{3} a_{k}+\frac{1}{3} a_{k+1} . \tag{30}
\end{equation*}
$$

At $\varepsilon=1$ :

$$
\begin{equation*}
g_{k}=\frac{1}{2} a_{k}+\frac{1}{2} a_{k+1} \tag{31}
\end{equation*}
$$

which corresponds to the known relation obtained in [23]: the optimum gain factor of an amplifier in decibels is the arithmetic mean of losses (in decibels) in fiber segments on either side of it.

A multi-span FOCL 1800 km in length with 20 spans of alternating length ( 60 and 120 km ) was modeled in order to estimate the influence of $\varepsilon$ on the evolution of power and $o s n r_{\text {BER }}$ in a line (see Fig. 4).

It follows from the obtained data that $o s n r_{\text {BER }}$ at the end of a line at $\varepsilon=0$ is 17.2 dB , which is 4 dB higher than $o s n r_{\text {BER }}$ for $\varepsilon=1$. Thus, the $o s n r_{\text {BER }}$ value and, consequently, the maximum signal transmission range may be increased considerably in a line without chromatic dispersion compensators.

## Conclusion

It was demonstrated that the degree of correlation of nonlinear distortion in adjacent channels has a considerable influence on the degradation of optical signals due to nonlinear distortion in multi-span communication lines. Dependences of the operating range of a FOCL on power input into a span were examined at different values of coefficient $\varepsilon$ of correlation of nonlinear effects in spans. It was established that the maximum transmission range of a line without dispersion compensators is 2.8 times greater than the corresponding range of a line with dispersion compensators.

The optimum relation between EDFA gain factors and optical power losses in spans for multi-span FOCLs with arbitrary values of correlation coefficient $\varepsilon$ was determined. It was found that the optimum relation between EDFA gain factors and losses in spans, which provides the maximum operating range of a line, for coherent FOCLs with complete decorrelation of noise from adjacent spans $(\varepsilon=0$; this corresponds to FOCLs with digital dispersion compensation) is written as $g_{k}=\frac{2}{3} a_{k}+\frac{1}{3} a_{k+1}$. At the same time, the


Figure 4. Dependences of power and $o s n r_{\text {BER }}$ on the distance from the beginning of a line for $\varepsilon=0(a)$ and $1(b)$.
relation between EDFA gain factors and losses in spans for FOCLs with complete correlation of noise from adjacent spans takes a markedly different form: $g_{k}=\frac{1}{2} a_{k}+\frac{1}{2} a_{k+1}$. Such correlation properties are typical of systems with physical chromatic dispersion compensation.

## Conflict of interest

The authors declare that they have no conflict of interest.

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