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Transportic equations of Maxwell, their fundamental and generalized solutions at constant speed of moving emitters

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The article discusses transport solutions of the system of Maxwell's equations under the action of mobile sources of electromagnetic waves moving at a constant speed in a fixed direction. Fundamental and generalized solutions have been constructed for speeds of motion less than the speed of light in the medium, and their regular representation in analytical form.

To do this, in the space of Fourier transformftion over coordinates and time, the Green's tensor has been constructed. To restore the originals, the fundamental solutions of the wave equation and properties of Fourier transformation were used. Construction of solutions for arbitrary moving sourthes are based on the property of convolution of fundamental solutions differential equations with right-hand side.

Formulas are given for calculating the electric and magnetic intensities for moving emitters of various types, useful for radiodetechnical applications.

Keywords: light speed, speed of movement, Mach number, Green's tensor, generalized solutions, electromagnetic waves, radio waves.

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Introduction

Maxwell equations are fundamental in the modern electrodynamics and are determinant in the study of electromagnetic fields generated by various emitters of electromagnetic waves (EM). Many scientists since the second half of the XIXth century are engaged in building and study of solutions for such equations and for boundary value problems for them in the areas of different geometry. There is a vast bibliography in this area, beginning with varied educational materials in electromagnetism [1–6], etc.

First of all, we are interested here in generalized solutions of that system of equations, when action of emitters is described as singular generalized focused pulse functions, which are described by delta-functions and their derivatives, or as simple and double singular layers on the lines and surfaces of various shapes.

Green tensor and generalized solutions of non-steady Maxwell equations and their Hamiltonian form in isotropic and anisotropic media are built in the papers [7–9]. Based on it, by using the generalized functions method, a method of boundary integral equations has been developed to resolve non-steady and steady boundary value problems of electrodynamics in the areas with arbitrary geometry of boundaries in [10,11].

The most common sources of EM waves emission among existing ones are movable sources, located on platforms of different vehicles. It is evident that the travel speed significantly impacts the processes of EM waves propagation in the media with different electrical conductivity and permeability, as well as the shape of the source itself and the nature of its operation. The studies in this area are not that abundant, and these are related to certain type of the emission source [12–18].

In any medium the waves are propagated at certain speed. These are called acoustic waves according to the continuum mechanics — the name originating from acoustics. In a continuum, the waves propagation speed depends on the medium deformation type propagated by them. This is why there can be several sound speeds in a continuum. And in anisotropic media they also depend on the direction. The relation between the perturbation source movement speed in the medium and the sound speed is called the Mach number (M). At $M < 1$ the movement is subsonic, at $M > 1$ — is supersonic.

The specifics of acoustic waves during the aircrafts movement at subsonic and supersonic speeds are well known. During mathematic simulation of such transport problems the type of differential equations changes: elliptic in the subsonic mode and hyperbolic in the supersonic mode. It heavily impacts the problem resolution and drastically changes the wave field pattern in a medium.

In isotropic electromagnetic media, which are described by Maxwell equations (ME), the EM waves propagation speed is one, and it customary to call it as the speed of light. It is critical one, the same as the sound speed in the air is critical too. This is why we can consider subligh mode of movement, the light mode and the superlight mode. Herein the authors deal with the sublight range of movement of an emitter.

Here we consider transport solutions of the system of Maxwell equations for the case of movable sources of EM waves moving at a constant speed V in a fixed direction. It is assumed that the movement speed is lower than the light propagation speed c in that medium, which is called the sublight speed. The relation between the source movement and the speed of light $M = V/c$ in the medium we call the Mach number, in a similar way to its definition in the continuum mechanics.

Fundamental and generalized transport solutions of Maxwell equations at $M < 1$ were built. Their regular integral representations in analytical form are given. The formulas are given for calculation of electrical and magnetic intensity of EM fields for movable emitters of various type, which are useful for radiotechnical applications.

1. Maxwell equations. Transport sources of EM waves

Let us consider a classic system of Maxwell equations [1–3], being written as follows:

$$\begin{gathered} \operatorname{rot}\mathbf{E} + \mu\mu_0 \frac{\partial \mathbf{H}}{\partial t} = \mathbf{j}^m(x_1, x_2, x_3, t), \\ \operatorname{rot}\mathbf{H} - \varepsilon\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mathbf{j}^e(x_1, x_2, x_3, t), \\ \operatorname{div}\mathbf{B} = \rho^m, \quad \operatorname{div}\mathbf{D} = \rho^e, \end{gathered} \tag{1}$$

where $\mathbf{j}^m$ — is the vector of density of magnetic flux [V/m^2], $\mathbf{j}^e$ — is the vector of density of electric current [A/m^2], $\mathbf{E}$ — is the vector of intensity of electrical field [V/m], $\mathbf{H}$ — is the vector of intensity of magnetic field [A/m], ρ^e — is volumetric density of electric charge [C/m^3].

Here (1) magnetic currents and charges are introduced in the equations $\mathbf{j}^m$, ρ^m. Maxwell equations have no magnetic currents and charges: $\mathbf{j}^m = 0$, $\rho^m = 0$. Further, we will remove that constraint to build solutions of that system.

Material ratios

$$\mathbf{B} = \mu\mu_0\mathbf{H}, \quad \mathbf{D} = \varepsilon\varepsilon_0\mathbf{E}, \tag{2}$$

where μ — is the medium's permeability, $\mu_0 = 4\pi \cdot 10^{-7}$ H/m — is the magnetic constant, ε — is the medium's dielectric permittivity, $\varepsilon_0 = 8,85 \cdot 10^{-12}$ F/m — is the electric constant, $\mathbf{B}(x_1, x_2, x_3, t)$ — is the vector of magnetic field induction, $\mathbf{D}(x_1, x_2, x_3, t)$ — is the vector of electric field induction, C/m^2.

Note that two vector Maxwell equations for the currents refer to a closed system of equations, which is sufficient to determine EM field at given currents. After determination of it, scalar equations allow determining electrical and magnetic charge.

Let us consider movable transport sources of electromagnetic waves, which do not change their type and move at a constant speed V in the direction of axis X_3 ($\mathbf{e}_3 = (0, 0, 1)$). These can be described as currents of the type $\mathbf{J}(x_1, x_2, z)$, where $x_3 - Vt = z$. In a movable system of coordinates (x_1, x_2, z) the time derivative is:

$$\frac{\partial}{\partial t} = -V \cdot \frac{\partial}{\partial z} \tag{3}$$

and Maxwell equations for currents in that system of coordinates will be

$$\begin{gathered} \frac{\partial E_z}{\partial x_2} - \frac{\partial E_2}{\partial z} - V\mu\mu_0 \frac{\partial}{\partial z} H_1 = j_1^m(x_1, x_2, z), \\ \frac{\partial E_1}{\partial z} - \frac{\partial E_z}{\partial x_1} - V\mu\mu_0 \frac{\partial}{\partial z} H_2 = j_2^m(x_1, x_2, z), \\ \frac{\partial E_2}{\partial x_1} - \frac{\partial E_1}{\partial x_2} - V\mu\mu_0 \frac{\partial}{\partial z} H_z = j_z^m(x_1, x_2, z), \\ \frac{\partial H_z}{\partial x_2} - \frac{\partial H_2}{\partial z} + V\varepsilon\varepsilon_0 \frac{\partial}{\partial z} E_1 = j_1^e(x_1, x_2, z), \\ \frac{\partial H_1}{\partial z} - \frac{\partial H_z}{\partial x_1} + V\varepsilon\varepsilon_0 \frac{\partial}{\partial z} E_2 = j_2^e(x_1, x_2, z), \\ \frac{\partial H_2}{\partial x_1} - \frac{\partial H_1}{\partial x_2} + V\varepsilon\varepsilon_0 \frac{\partial}{\partial z} E_z = j_z^e(x_1, x_2, z). \end{gathered} \tag{4}$$

We will call that system of six equations as the transport Maxwell equations. It is represented as a matrix

$$\mathbf{M}(\partial_1, \partial_2, \partial_z)\mathbf{u} = \mathbf{J}, \tag{5}$$

$$M(\partial_1, \partial_2, \partial_z) =$$

$$= \begin{pmatrix} 0 & -\partial_z & \partial_2 & -V\mu\mu_0\partial_z & 0 & 0 \\ \partial_z & 0 & -\partial_1 & 0 & -V\mu\mu_0\partial_z & 0 \\ -\partial_2 & \partial_1 & 0 & 0 & 0 & -V\mu\mu_0\partial_z \\ V\varepsilon\varepsilon_0\partial_z & 0 & 0 & 0 & -\partial_z & \partial_2 \\ 0 & V\varepsilon\varepsilon_0\partial_z & 0 & \partial_z & 0 & -\partial_1 \\ 0 & 0 & V\varepsilon\varepsilon_0\partial_z & -\partial_2 & \partial_1 & 0 \end{pmatrix},$$

where $\mathbf{u}$, $\mathbf{J}$ — dimension vectors 6 composed of the components of serially specified values:

$$\mathbf{u} = \begin{pmatrix} \mathbf{E}(x_1, x_2, z) \\ \mathbf{H}(x_1, x_2, z) \end{pmatrix}, \quad \mathbf{J} = \begin{pmatrix} \mathbf{j}^m(x_1, x_2, z) \\ \mathbf{j}^e(x_1, x_2, z) \end{pmatrix}.$$

Next, let us use the following notations: $c = \sqrt{1/\mu\mu_0\varepsilon\varepsilon_0}$ — speed of light, $V/c = M$ — Mach number, $m^2 = 1 - M^2$.

2. Green tensor of transport Maxwell equations

Definition. The Green tensor of Maxwell equations is a matrix of fundamental solutions of the equations (5) at

$$\mathbf{J} = \delta(x_1)\delta(x_2)\delta(z)\{\delta_{ij}\}_{6\times 6},$$

satisfying the emission conditions, which describe waves propagating from a movable wave source and attenuating in the infinity.

It satisfies the equation

$$\mathbf{M}(\partial_1, \partial_2, \partial_z)\mathbf{U}(x_1, x_2, z) = \delta(x_1, x_2, z)\{\delta_{ij}\}_{6\times 6}, \qquad (6)$$

where $\delta(x_1, x_2, z) = \delta(x_1)\delta(x_2)\delta(z)$ — is the singular delta-function, δ_{ij} — is the Kronecker delta.

To build it we use Fourier transformation in the space of slow-growth generalized functions [19]. In the space of Fourier transformation, the relation with initial coordinates $x_1, x_2, z \leftrightarrow k_1, k_2, k_3$ for regular functions is

$$F[f(x_1, x_2, z)] = \bar{f}(k_1, k_2, k_3)$$

$$= \int_{R^3} f(x_1, x_2, z)e^{i(x_1k_1+x_2k_2+zk_3)}dx_1dx_2dz,$$

$$F^{-1}[\bar{f}(k_1, k_2, k_3)] = \bar{f}(k_1, k_2, k_3)$$

$$= \frac{1}{(2\pi)^3}\int_{R^3} \bar{f}(k_1, k_2, k_3)e^{-i(x_1k_1+x_2k_2+zk_3)}dk_1dk_2dk_3. \qquad (7)$$

Using the property of Fourier transformation of the derivative

$$\partial_j \Leftrightarrow -ik_j,$$

from the equations (5) we get a system of linear algebraic equations

$$\mathbf{M}(-ik_1, -ik_2, -ik_z)\bar{\mathbf{U}}(k_1, k_2, k_3) = \{\delta_{ij}\}_{6\times 6}, \qquad (8)$$

where

$$\mathbf{M}(-ik_1, -ik_2, -ik_z) =$$

$$= \begin{pmatrix} 0 & ik_3 & -ik_2 & ik_3V\mu\mu_0 & 0 & 0 \\ -ik_3 & 0 & ik_1 & 0 & ik_3V\mu\mu_0 & 0 \\ ik_2 & -ik_1 & 0 & 0 & 0 & ik_3V\mu\mu_0 \\ -ik_3V\varepsilon\varepsilon_0 & 0 & 0 & 0 & ik_3 & -ik_2 \\ 0 & -ik_3V\varepsilon\varepsilon_0 & 0 & -ik_3 & 0 & ik_1 \\ 0 & 0 & -ik_3V\varepsilon\varepsilon_0 & ik_2 & -ik_1 & 0 \end{pmatrix}.$$

From (8) we have

$$\bar{\mathbf{U}}(k_1, k_2, k_3) = (\mathbf{M}(-ik_1, -ik_2, -ik_z))^{-1}. \qquad (9)$$

Inverse matrix components were obtained by resolving symbolic equations in MatCad-15. In matrix columns these are as follows:

$$\{\bar{U}_{m1}\} = \begin{bmatrix} 0 \\ \frac{ik_3}{k_1^2+k_2^2+k_3^2m^2} \\ \frac{-ik_2}{k_1^2+k_2^2+k_3^2m^2} \\ \frac{ik_1^2-ik_3^2M^2}{\alpha k_3(k_1^2+k_2^2+k_3^2m^2)} \\ \frac{ik_1k_2}{\alpha k_3(k_1^2+k_2^2+k_3^2m^2)} \\ \frac{ik_1}{\alpha(k_1^2+k_2^2+k_3^2m^2)} \end{bmatrix}, \quad \{\bar{U}_{m2}\} = \begin{bmatrix} \frac{-ik_3}{k_1^2+k_2^2+k_3^2m^2} \\ 0 \\ \frac{ik_1}{k_1^2+k_2^2+k_3^2m^2} \\ \frac{ik_1k_2}{\alpha k_3(k_1^2+k_2^2+k_3^2m^2)} \\ \frac{ik_2^2-ik_3^2M^2}{\alpha k_3(k_1^2+k_2^2+k_3^2m^2)} \\ \frac{ik_2}{\alpha(k_1^2+k_2^2+k_3^2m^2)} \end{bmatrix},$$

$$\{\bar{U}_{m3}\} = \begin{bmatrix} \frac{-ik_2}{k_1^2+k_2^2+k_3^2m^2} \\ \frac{-ik_1}{k_1^2+k_2^2+k_3^2m^2} \\ 0 \\ \frac{ik_1}{\alpha(k_1^2+k_2^2+k_3^2m^2)} \\ \frac{ik_2}{\alpha(k_1^2+k_2^2+k_3^2m^2)} \\ \frac{ik_3-ik_3M^2}{\alpha(k_1^2+k_2^2+k_3^2m^2)} \end{bmatrix}, \; \{\bar{U}_{m4}\} = \begin{bmatrix} -\frac{ik_1^2-ik_3^2M^2}{\beta k_3(k_1^2+k_2^2+k_3^2m^2)} \\ -\frac{ik_2k_1}{\beta k_3(k_1^2+k_2^2+k_3^2m^2)} \\ -\frac{ik_1}{\beta(k_1^2+k_2^2+k_3^2m^2)} \\ 0 \\ \frac{ik_3}{k_1^2+k_2^2+k_3^2m^2} \\ -\frac{ik_2}{k_1^2+k_2^2+k_3^2m^2} \end{bmatrix},$$

$$\{\bar{U}_{m5}\} = \begin{bmatrix} -\frac{ik_2k_1}{\beta k_3(k_1^2+k_2^2+k_3^2m^2)} \\ -\frac{ik_2^2-ik_3^2M^2}{\beta k_3(k_1^2+k_2^2+k_3^2m^2)} \\ -\frac{ik_2}{\beta(k_1^2+k_2^2+k_3^2m^2)} \\ -\frac{ik_3}{k_1^2+k_2^2+k_3^2m^2} \\ 0 \\ \frac{ik_1}{(k_1^2+k_2^2+k_3^2m^2)} \end{bmatrix}, \; \{\bar{U}_{m6}\} = \begin{bmatrix} -\frac{ik_1}{\beta(k_1^2+k_2^2+k_3^2m^2)} \\ -\frac{ik_2}{\beta(k_1^2+k_2^2+k_3^2m^2)} \\ \frac{-ik_3+ik_3M^2}{\beta(k_1^2+k_2^2+k_3^2m^2)} \\ \frac{ik_2}{k_1^2+k_2^2+k_3^2m^2} \\ \frac{-ik_1}{k_1^2+k_2^2+k_3^2m^2} \\ 0 \end{bmatrix}, \qquad (10)$$

where $\alpha = \varepsilon\varepsilon_0 V$, $\beta = \mu\mu_0 V$.

Let us consider the following basis functions and their originals:

$$\bar{f}_0(k_1, k_2, k_3) = \frac{1}{k_1^2 + k_2^2 + m^2k_3^2} \Leftrightarrow f_0(x_1, x_2, z), \qquad (11)$$

$$\bar{f}_1(k_1, k_2, k_3) = -\frac{1}{ik_3}\bar{f}_0(k_1, k_2, k_3)$$

$$\Leftrightarrow f_0(x_1, x_2, z) = \partial_z f_1(x_1, x_2, z), \qquad (12)$$

$$\bar{f}_2(k_1, k_2, k_3) = -\frac{1}{ik_3}\bar{f}_1(k_1, k_2, k_3) = -\frac{1}{k_3^2}\bar{f}_0(k_1, k_2, k_3)$$

$$\Leftrightarrow f_1(x_1, x_2, z) = \partial_z f_2(x_1, x_2, z). \qquad (13)$$

By using them, the original Green tensor $\mathbf{U}(x_1, x_2, z)$ is represented as (by columns):

$$\{\bar{U}_{m1}\} = \begin{bmatrix} 0 \\ ik_3\bar{f}_0(k_1,k_2,k_3) \\ -ik_2\bar{f}_0(k_1,k_2,k_3) \\ \frac{k_1^2-k_3^2M^2}{\alpha}\bar{f}_1(k_1,k_2,k_3) \\ -\frac{ik_1ik_2}{\alpha}\bar{f}_1(k_1,k_2,k_3) \\ \frac{ik_1}{\alpha}\bar{f}_0(k_1,k_2,k_3) \end{bmatrix} \Rightarrow \{U_{m1}\} = \begin{bmatrix} 0 \\ -\partial_z f_0(x_1,x_2,z) \\ \partial_2 f_0(x_1,x_2,z) \\ \frac{1}{\alpha}(\partial_1^2 - M^2\partial_z^2) f_1(x_1,x_2,z) \\ -\frac{1}{\alpha}\partial_1\partial_2 f_1(x_1,x_2,z) \\ -\frac{1}{\alpha}\partial_1 f_0(x_1,x_2,z) \end{bmatrix},$$

$$\{\bar{U}_{m2}\} = \begin{bmatrix} -ik_3\bar{f}_0(k_1,k_2,k_3) \\ 0 \\ ik_1\bar{f}_0(k_1,k_2,k_3) \\ -\frac{ik_1ik_2}{\alpha}\bar{f}_1(k_1,k_2,k_3) \\ \frac{k_2^2-k_3^2M^2}{\alpha}\bar{f}_1(k_1,k_2,k_3) \\ \frac{ik_2}{\alpha}\bar{f}_0(k_1,k_2,k_3) \end{bmatrix} \Rightarrow \{U_{m2}\} = \begin{bmatrix} \partial_z f_0(x_1,x_2,z) \\ 0 \\ -\partial_1 f_0(x_1,x_2,z) \\ -\frac{1}{\alpha}\partial_1\partial_2 f_1(x_1,x_2,z) \\ \frac{1}{\alpha}(\partial_2^2 - M^2\partial_z^2) f_1(x_1,x_2,z) \\ -\frac{1}{\alpha}\partial_2 f_0(x_1,x_2,z) \end{bmatrix},$$

$$\{\bar{U}_{m3}\} = \begin{bmatrix} ik_2\bar{f}_0(k_1,k_2,k_3) \\ -ik_1\bar{f}_0(k_1,k_2,k_3) \\ 0 \\ \frac{ik_1}{\alpha}\bar{f}_0(k_1,k_2,k_3) \\ \frac{ik_2}{\alpha}\bar{f}_0(k_1,k_2,k_3) \\ \frac{ik_3-ik_3M^2}{\alpha}\bar{f}_0(k_1,k_2,k_3) \end{bmatrix} \Rightarrow \{U_{m3}\} = \begin{bmatrix} -\partial_2 f_0(x_1,x_2,z) \\ \partial_1 f_0(x_1,x_2,z) \\ 0 \\ -\frac{1}{\alpha}\partial_1 f_0(x_1,x_2,z) \\ -\frac{1}{\alpha}\partial_2 f_0(x_1,x_2,z) \\ -\frac{m^2}{\alpha}\partial_z f_0(x_1,x_2,z) \end{bmatrix},$$

$$\{\bar{U}_{m4}\} = \begin{bmatrix} \frac{k_3^2M^2-k_1^2}{\beta}\bar{f}_1(k_1,k_2,k_3) \\ \frac{ik_1ik_2}{\beta}\bar{f}_1(k_1,k_2,k_3) \\ -\frac{ik_1}{\beta}\bar{f}_0(k_1,k_2,k_3) \\ 0 \\ ik_3\bar{f}_0(k_1,k_2,k_3) \\ -ik_2\bar{f}_0(k_1,k_2,k_3) \end{bmatrix} \Rightarrow \{U_{m4}\} = \begin{bmatrix} \frac{1}{\beta}(M^2\partial_z^2 - \partial_1^2) f_1(x_1,x_2,z) \\ \frac{1}{\beta}\partial_1\partial_2 f_1(x_1,x_2,z) \\ \frac{1}{\beta}\partial_1 f_0(x_1,x_2,z) \\ 0 \\ -\partial_z f_0(x_1,x_2,z) \\ \partial_2 f_0(x_1,x_2,z) \end{bmatrix},$$

$$\{\bar{U}_{m5}\} = \begin{bmatrix} \frac{ik_1ik_2}{\beta}\bar{f}_1(k_1,k_2,k_3) \\ \frac{k_3^2M^2-k_2^2}{\beta}\bar{f}_1(k_1,k_2,k_3) \\ -\frac{ik_2}{\beta}\bar{f}_0(k_1,k_2,k_3) \\ -ik_3\bar{f}_0(k_1,k_2,k_3) \\ 0 \\ ik_1\bar{f}_0(k_1,k_2,k_3) \end{bmatrix} \Rightarrow \{U_{m5}\} = \begin{bmatrix} \frac{1}{\beta}\partial_1\partial_2 f_1(x_1,x_2,z) \\ \frac{1}{\beta}(M^2\partial_z^2 - \partial_2^2) f_1(x_1,x_2,z) \\ \frac{1}{\beta}\partial_2 f_0(x_1,x_2,z) \\ \partial_z f_0(x_1,x_2,z) \\ 0 \\ -\partial_1 f_0(x_1,x_2,z) \end{bmatrix},$$

$$\{\overline{U}_{m6}\} = \begin{bmatrix} -\frac{ik_1}{\beta}\overline{f}_0(k_1,k_2,k_3) \\ -\frac{ik_2}{\beta}\overline{f}_0(k_1,k_2,k_3) \\ \frac{ik_3M^2-ik_3}{\beta}\overline{f}_0(k_1,k_2,k_3) \\ ik_2\overline{f}_0(k_1,k_2,k_3) \\ -ik_1\overline{f}_0(k_1,k_2,k_3) \\ 0 \end{bmatrix} \Rightarrow \{U_{m6}\} = \begin{bmatrix} \frac{1}{\beta}\partial_1 f_0(x_1,x_2,z) \\ \frac{1}{\beta}\partial_2 f_0(x_1,x_2,z) \\ \frac{m^2}{\beta}\partial_3 f_0(x_1,x_2,z) \\ -\partial_2 f_0(x_1,x_2,z) \\ \partial_1 f_0(x_1,x_2,z) \\ 0 \end{bmatrix}. \quad (14)$$

Hence, the Green tensor components are defined through the original basic functions. Let us build them.

3. Building original basic functions at $M < 1$

3.1. Building original $\overline{f}_0(k_1, k_2, k_3)$

Let us consider the function,

$$\overline{f}_0(k_1,k_2,k_3) = \frac{1}{k_1^2+k_2^2+m^2k_3^2}, \quad (15)$$

which is Fourier transform of the fundamental solution of the equation

$$\frac{\partial^2 f_0}{\partial x_1^2} + \frac{\partial^2 f_0}{\partial x_2^2} + m^2\frac{\partial^2 f_0}{\partial x_3^2} + \delta(x) = 0. \quad (16)$$

In order to find its solution we need to separately consider three cases:

sublight case: $V < c \Rightarrow M < 1, m^2 = 1 - M^2 > 0$;

superlight case: $V > c \Leftrightarrow M > 1, m^2 < 0$;

light case: $V = c \Leftrightarrow M = 1, m^2 = 0$.

Here we consider the sublight one. At $m^2 > 0$ the Laplace transport equations are elliptic. In order to build solution of the equation (16) we use fundamental solution of Laplace equation:

$$\frac{\partial^2\Psi}{\partial x_1^2} + \frac{\partial^2\Psi}{\partial x_2^2} + \frac{\partial^2\Psi}{\partial x_3^2} + \delta(x) = 0, \quad (17)$$

$$\Psi(x_1,x_2,x_3) = \frac{1}{4\pi\|x\|}, \quad (18)$$

which satisfies the conditions of attenuation in the infinity [19]. Using the property of Fourier transformation

$$f(z) \leftrightarrow \overline{f}(k_3), \quad f(z/m) \leftrightarrow m\overline{f}(mk_3),$$

we get the original

$$f_0(x_1,x_2,z) = \frac{1}{4\pi\sqrt{(x_1^2+x_2^2)m^2+z^2}}$$

$$= \frac{1}{4\pi\sqrt{m^2r^2+z^2}} = \Phi_0(r,z), \quad (19)$$

where $r = \sqrt{x_1^2+x_2^2}$, $r_{,j} = \frac{x_j}{r}$. Its derivatives, which form part of the original Green tensor representation **U**, are equal to

$$\partial_j f_0 = -\frac{1}{4\pi}\frac{m^2x_j}{(m^2r^2+z^2)\sqrt{m^2r^2+z^2}} = -(4\pi m)^2 x_j\Phi_0^3(r,z),$$

$$\partial_j\partial_k f_0 = -\frac{m^2}{4\pi(m^2r^2+z^2)^{3/2}}\left\{\delta_{jk} - \frac{3m^2x_jx_k}{m^2r^2+z^2}\right\}$$

$$= -(4\pi m)^2\Phi_0^3(r,z)\left\{\delta_{jk} - \frac{3m^2x_jx_k}{m^2r^2+z^2}\right\}, \quad j,k = 1,2,$$

$$\partial_z f_0 = -\frac{1}{4\pi}\frac{z}{(m^2r^2+z^2)\sqrt{m^2r^2+z^2}} = -(4\pi)^2 z\Phi_0^3(r,z),$$

$$\partial_z\partial_z f_0 = \frac{2z^2-m^2r^2}{4\pi(m^2r^2+z^2)^2\sqrt{m^2r^2+z^2}}$$

$$= (4\pi)^4\Phi_0^5(r,z)(2z^2-m^2r^2).$$

3.2. Building original $\overline{f}_1(k_1, k_2, k_3)$

Let us consider the second basic function

$$\overline{f}_1(k_1,k_2,k_3) = \frac{1}{-ik_3^2(k_1^2+k_2^2+m^2k_3^2)}. \quad (20)$$

This function corresponds to the regularization class

$$\frac{1-a}{-i(k_3+i0)(k_1^2+k_2^2+m^2k_3^2)} + \frac{a}{-i(k_3-i0)(k_1^2+k_2^2+m^2k_3^2)}, \quad (21)$$

where a — is an arbitrary constant. Here we use symmetrical regularization (a=0,5):

$$\overline{f}_1(k_1,k_2,k_3) = \frac{1}{2(k_1^2+k_2^2+m^2k_3^2)} \times\left(\frac{1}{-i(k_3+i0)} + \frac{1}{-i(k_3-i0)}\right). \quad (22)$$

It is clear that, when using the property of Fourier transformation of primitives, this function is the Fourier transformation of the function

$$f_1(x_1,x_2,z) = 0,5(f_0(x_1,x_2,z)H(z) \overset{*}{_z} H(z) - f_0(x_1,x_2,z)H(-z) \overset{*}{_z} H(-z)). \quad (23)$$

Here, convolution by z contains Heaviside function $H(z)$, Fourier transformation, which is

$$\bar{H}(k_3) = \frac{1}{-i(k_3 + i0)}.$$

By calculating the convolutions

$$f_0(x_1, x_2, z)H(z) \overset{*}{_z} H(z) = H(z)\int_0^z f_0(x_1, x_2, z - \tau)d\tau,$$

$$f_0(x_1, x_2, z)H(-z) \overset{*}{_z} H(-z) = H(-z)\int_z^0 f_0(x_1, x_2, z - \tau)d\tau. \tag{24}$$

we get the original function

$$f_1(x_1, x_2, z) = \frac{sgn(z)}{4\pi} ln\left(\frac{|z| + \sqrt{m^2r^2 + z^2}}{mr}\right) = \Phi(r, z). \tag{25}$$

Its derivatives

$$\partial_j\Phi_1 = \frac{x_j z}{r^2}\Phi_0(r, z), \quad \partial_z\Phi_1 = \Phi_0(r, z),$$

$$\partial_z\partial_z\Phi_1 = z(4\pi)^2\Phi_0^3(r, z),$$

$$\partial_k\partial_j\Phi_1 = \partial_k\left(\frac{x_j z}{r^2}\Phi_0(r, z)\right)$$

$$= \frac{z}{r^2}\{\Phi_0(\delta_{jk} - 2r_{,j}r_{,k}) + x_j\partial_k\Phi_0\},$$

$$\partial_j\partial_z\Phi_1 = (4\pi m)^2 x_j\Phi_0^3(r, z), \quad r_{,j} = \frac{x_j}{r}.$$

Therefore, all basic functions included into the tensor definition are found, the Green tensor is built.

4. Generalized solutions of transport Maxwell equations with different emitters

4.1. EM fields of movable volumetric emitters

Solution of transport Maxwell equations with arbitrary volumetric emitters is represented as a tensor-functional convolution of currents with Green tensor:

$$\mathbf{u}(x, z) = \mathbf{U}(x, z) * \mathbf{J}(x, z),$$

$$u_i(x, z) = \sum_{j=1}^{6} U_{ij}(x, z) * j_j(x, z), \quad j = 1, \ldots, 6,$$

which can be represented for regular currents as follows:

$$u_i(x, z) = \sum_{k=1}^{6} U_{ik}(x, z) * j_k(x, z)$$

$$= \sum_{k=1}^{6} \int\int_{R^3}\int U_{ik}(x - y, z - \xi) j_k(y, \xi)dy_1dy_2d\xi.$$

For singular currents we should use the convolution definition [19].

4.2. EM fields of movable superficial emitters

For an emitter with the intensity $\mathbf{I}^D(x, z)$ concentrated on the surface D, the solution is of a superficial convolution type

$$\mathbf{u}(x, z) = \mathbf{U}(x, z) * \mathbf{I}^D(x, z)\delta_D(x, z),$$

$$u_i(x, z) = \sum_{j=1}^{6} U_{ij}(x, z) * \mathbf{I}_j^D(x, z)\delta_D(x, z), \; j = 1, \ldots, 6.$$

Here $\delta_D(x, z)$ — is the simple layer on cylindrical surface $D = \{(x, z) \in L \times Z\} \subset R^3$, where the outline L — is the cross section of that surface. It is singular generalized function, whose convolution for regular currents on D can be represented as integral

$$u_i(x, z) = \sum_{j=1}^{6} \int_D U_{ij}(x - y, z - \xi) I_j^D(y, \xi)dD(y, \xi),$$

where $dD(y, \xi)$ — is the differential of the surface area D.

4.3. EM fields of movable linear emitters

For emitters concentrated on the curves $L \subset R^3$, with the intensity $\mathbf{I}^L(x, z)$ the solution is

$$\mathbf{u}(x, z) = \mathbf{U}(x, z) * \mathbf{I}^L(x, z)\delta_L(x, z),$$

$$u_i(x, z) = \sum_{j=1}^{6} U_{ij}(x, z) * I_j^L(x, z)\delta_L(x, z), \quad j = 1, \ldots, 6.$$

Here $\delta_L(x, z)$ — is the simple layer on L. It is singular generalized function, whose convolution can also be represented as integral

$$u_i(x, z) = \sum_{k=1}^{6} \int_L U_{ik}(x - y, z - \xi) I_k(y, \xi)dL(y, \xi),$$

where $dL(y, \xi)$ — is the differential of the arc length on L.

In particular, for the emitters with the intensity $\mathbf{f}(z)$ concentrated on the axis X_3:

$$u_i = \sum_{j=1}^{6} U_{ij}(x, z)\delta(x) f_j(z)$$

$$= \sum_{j=1}^{6} U_{ij}(x, z) \overset{*}{_z} f_j(z) = \sum_{j=1}^{6} \int_{-\infty}^{\infty} U_{ij}(x, z - y) f_j(y)dy,$$

or on X_2:

$$u_i = \sum_{j=1}^{6} U_{ij}(x, z) * \delta(x_1)\delta(z) f_j(x_2)$$

$$= \sum_{j=1}^{6} U_{ij}(x, z) \overset{*}{_{x_2}} f(x_2) = \sum_{j=1}^{6} \int_{-\infty}^{\infty} U_{ij}(x_1, x_2 - \xi, z) f_j(\xi)d\xi.$$

Here, the variable under the convolution sign refers to the coordinate, by which the convolution is performed.

EXAMPLE 1. Movable linear emitter on the segment of Z axis with the length $2L$:

$$\mathbf{j}^e(x_1, x_2, z) = \delta(x_1, x_2)H(L - |z|)(0, 0, 1), \quad \mathbf{j}^m = 0.$$

In the matrix form

$$\mathbf{u}(x_1, x_2, z) = \mathbf{U}(x_1, x_2, z) * \mathbf{j}(x_1, x_2, z),$$

$$j(x_1, x_2, z) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \delta(x_1, x_2)H(L - |z|) \end{bmatrix}.$$

Component by component

$$u_i = \sum_{j=1}^{6} U_{ij} * j_j = U_{i6} * \delta(x_1, x_2)H(L - |z|)$$

$$= U_{i6} \overset{*}{_z} H(L - |z|) = \int_{-\infty}^{\infty} U_{i6}(x_1, x_2, y)H(L - |z - y|)dy$$

$$= \int_{-L+z}^{L+z} U_{i6}(x_1, x_2, y)dy = \int_{-L+z}^{L+z} U_{i6}(x_1, x_2, y)dy.$$

Subject to the introduced notations and values of the Green tensor components that formula is as follows:

$$\begin{pmatrix} E(x, z) \\ H(x, z) \end{pmatrix} = \int_{-L+z}^{L+z} \begin{bmatrix} \beta^{-1}\partial_1\Phi_0 \\ \beta^{-1}\partial_2\Phi_0 \\ m^2\beta^{-1}\partial_3\Phi_0 \\ -\partial_2\Phi_0 \\ \partial_1\Phi_0 \\ 0 \end{bmatrix} dy.$$

By calculating these integrals, we get the intensities of the EM field

$$E_1 = -\frac{x_1}{4\pi r\beta}g(r, z), \quad E_2 = -\frac{x_2}{4\pi r\beta}g(r, z),$$

$$E_3 = -\frac{m^2 z}{4\pi r\beta}g(r, z),$$

$$H_1 = \frac{x_2}{4\pi r}g(r, z), \quad H_2 = -\frac{x_1}{4\pi r}g(r, z), \quad H_3 = 0.$$

Here

$$g(r, z) = \sqrt{m^2 + \frac{(L + z)^2}{r^2}} - \sqrt{m^2 + \frac{(z - L)^2}{r^2}}.$$

Conclusion

The results obtained can be used for studying EM fields of various light emitters and radio waves emitters located on movable objects (trains, cars, ships, etc.).

Note also that the tensor of fundamental solutions built herein is required for resolution of transport boundary value problems of electrodynamics in constrained areas based on the method of boundary integral equations, which is planned by the authors to do in the nearest future.

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Conflict of interest

The authors declare that they have no conflict of interest.

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