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On the generation of higher harmonics by a dipole electromagnetic pulse in vacuum

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The generation of nonlinear harmonics by a self-acting dipole pulse of electromagnetic field evolving in a vacuum is considered. Using Maxwell nonlinear equations obtained from the Heisenberg-Euler Lagrangian, it is shown that in the first order of perturbation theory the generation of the third harmonic does not occur, as follows from the law of conservation of energy-momentum. It is demonstrated that accounting for nonlinear effects leads to the generation of an octupole electromagnetic pulse at the frequency of the initial dipole pulse. Using numerical simulations the spectrum of the generated radiation for a Gaussian initial dipole pulse was obtained and it was shown that its maximum is shifted in direction of larger frequencies.

Keywords: nonlinear electrodynamics, conservation of energy-momentum, Heisenberg-Euler Lagrangian, radiation.

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Introduction

It is known that Maxwell's free-space equations in the presence of a strong electromagnetic field shall be changed to consider the effects associated with vacuum polarization. Field equations modified in this way will be nonlinear [1] and this will cause many new physical effects. For example, in a strong magnetic field, vacuum acts as a birefringent medium [2] and light propagates in a strong magnetic field (for example, magnetic dipole field) along curves that are geodesics in some pseudo-Riemannian space [3].

Generation of high-order harmonics by a strong electromagnetic pulse field was also a subject of extensive investigations. In particular, it has been shown that a plane monochromatic electromagnetic wave propagating in vacuum does not generate any harmonics even in the presence of a permanent homogeneous magnetic field. On the other hand, a spatially inhomogeneous wave in a permanent magnetic field or a plane monochromatic wave propagating in a spatially inhomogeneous magnetic field generates the second harmonic even in the one-loop approximation of the perturbation theory [4]. A review of the state of the art in this field is given in [5].

This work investigates (in one-loop approximation) another problem — higher-order harmonic generation by a strong dipole electromagnetic pulse [6] evolving in free space and interacting with itself.

Equations for strong electromagnetic field in vacuum

In case of a strong electromagnetic field (field strength $E \lesssim m_e^2 c^3/e\hbar$, where m_e is the electron mass, e is the electron charge and c is the speed of light), Lagrangian $\mathscr{L}$ of a free electromagnetic field is modified and the fourth-order and higher-order field terms appear in it [1,2]:

$$\mathscr{L} = \mathscr{L}_{(0)} + \mathscr{L}_{(1)} + \dots, \tag{1}$$

where $\mathscr{L}_{(0)}$ is the linear Lagrangian

$$\mathscr{L}_{(0)} = -\frac{1}{16\pi c} F^{ik} F_{ik}, \tag{2}$$

F^{ik} — is the electromagnetic field tensor expressed through the field potentials A^i in a conventional manner, $\mathscr{L}_{(1)}$ is the fourth-order nonlinear field correction that, at typical field quantum energy $\hbar\omega \ll m_e c^2$, may be expressed via the field invariants as

$$\mathscr{L}_{(1)} = \frac{a}{4}\left[(F^{ik}F_{ik})^2 + \frac{7}{4}(e_{ik\mu\nu}F^{ik}F^{\mu\nu})^2\right], \tag{3}$$

where $a = \hbar e^4/(90\pi^2 m_e^4 c^8)$. The Lagrangian (3) is called the Heisenberg-Euler Lagrangian.

The second couple of Maxwell's equations may now be derived from the Lagrangian (1) using the variation principle, while the first couple of Maxwell's equations remains unchanged compared with the linear case. As a result, we get for the second couple of equations

$$\frac{\partial F^{ik}}{\partial x^k} = -\frac{4\pi}{c} j^i, \tag{4}$$

where four-dimensional current on the right-hand side

$$j^i = -2c^2 \frac{\partial}{\partial x^k}\left(\frac{\partial L_{(1)}}{\partial F^{ik}}\right) \tag{5}$$

is expressed nonlinearly via the full filed

$$F^{ik} = F_{(0)}{}^{ik} + F_{(1)}{}^{ik} + \dots. \tag{6}$$

Now assume that nonlinear corrections are small compared with the initial field $F^{ik}_{(0)}$ that satisfies Maxwell's linear equations. Then, the first correction term to field $F^{ik}_{(1)}$ by a small parameter a will satisfy equations (4) where the current on the right-hand side depends only on the initial strong field $F^{ik}_{(0)}$. If this field is, in addition, a radiation field with finite total energy, for which the second invariant is always equal to zero [7], then only the first terms will remain in the Lagrangian (3). The resulting expression for current (5) can be written as:

$$j^i(x) = -2ac^2 \frac{\partial (F^{\mu\nu}_{(0)} F_{(0)\mu\nu})}{\partial x^k} F^{ik}_{(0)}, \tag{7}$$

where it was assumed that $\partial F^{ik}_{(0)}/\partial x^k = 0$.

Thus, the strong initial field $F^{ik}_{(0)}$ polarizes the vacuum and induces some vacuum four-dimensional current that will generate a new field $F^{ik}_{(1)}$ that will include both the radiation and the non-radiative part. This field may be found from equation (4) using a suitable electromagnetic propagator. When only the radiation needs to be found, then the Pauli-Jordan function $\Delta(x)$ should be selected [4].

Equations (4) can be rewritten in a somewhat different form, if the induction tensor H^{ik} is introduced for which we have

$$\frac{\partial H^{ik}}{\partial x^k} = 0, \tag{8}$$

where

$$H^{ik} = (1 + 8\pi a F^{st} F_{st}) F^{ik}, \tag{9}$$

whilst the expression in parentheses corresponds to the nonlinear permittivity and permeability of vacuum.

The approach used herein to build perturbation theory on the basis of the Heisenberg-Euler Lagrangian has been used before, for example, in [9,10].

Dipole pulse in vacuum

Now suppose the field $F^{ik}_{(0)}$ is the finite dipole electromagnetic pulse field [6]. Fields of this type ensure the maximum energy concentration in the center for a pulse with the set spectrum and energy [11]. The dipole pulse field can be generally written as

$$F^{ik}_{(0)} = \left(\frac{\partial}{\partial x^i} M^{k\mu} - \frac{\partial}{\partial x^k} M^{i\mu}\right) \frac{\partial g(x)}{\partial x^\mu}, \tag{10}$$

where $M^{\mu\nu}$ is the antisymmetric tensor of dipole moments written as

$$M^{\mu\nu} = \begin{pmatrix} 0 & d_1 & d_2 & d_3 \\ -d_1 & 0 & m_3 & -m_2 \\ -d_2 & -m_3 & 0 & m_1 \\ -d_3 & m_2 & -m_1 & 0 \end{pmatrix}, \tag{11}$$

$\mathbf{d} = (d_1, d_2, d_3)$ and$\mathbf{m} = (m_1, m_2, m_3)$ — are electric and magnetic dipole moment vectors, respectively, function $g(x)$ is the difference of converging and diverging spherical waves ($x = (ct, \mathbf{r})$ are four-dimensional coordinates) written as:

$$\begin{aligned} g(x) =& c \int\limits_{-\infty}^{\infty} f(s) \Delta(x - sn)\, ds \\ =& \frac{f(ct - |\mathbf{r}|) - f(ct + |\mathbf{r}|)}{|\mathbf{r}|}. \end{aligned} \tag{12}$$

Here, $f(s)$ is the arbitrary real one-variable quadratically integrable function and $n^2 = 1$ is the arbitrary single four-dimensional vector defining the frame of reference where the dipole rests. It can be shown that $g(x)$ satisfies the wave equation $\Box g(x) = 0$. Expressions for the electric and magnetic dipole pulse fields derived from (10) may be found in [6].

The four-dimensional Fourier transformation from $g(x)$ directly follows from (12),

$$g(k) = c\Delta(k) f(nk), \tag{13}$$

where $k = (k^0, \boldsymbol{\kappa})$, $f(\xi)$ — is the Fourier transformation from $f(s)$, and $\Delta(k)$ is the 4D Fourier transform from the Pauli-Jordan function [8]:

$$\Delta(k) = \frac{8\pi^2 i}{c} \operatorname{sign}(k^0) \delta(k^2). \tag{14}$$

From (10) and (13), we can also derive the expression for the Fourier harmonics of the electromagnetic field tensor of the dipole pulse.

Below we will use a so-called Gaussian dipole pulse for which

$$f(s) = \frac{1}{2\pi} \cos(\Omega s/c) \exp\left(-\frac{s^2}{2l^2}\right), \tag{15}$$

where l is the pulse half width and Ω is the carrier frequency. The Fourier transformation from (15) is equal to

$$\begin{aligned} f(\xi) =& \frac{l}{2\sqrt{2\pi}} \left[\exp\left(-\frac{l^2}{2}(\xi - \Omega/c)^2\right)\right. \\ &\left. + \exp\left(-\frac{l^2}{2}(\xi + \Omega/c)^2\right)\right] \end{aligned} \tag{16}$$

and at $l \to \infty$ it goes into an expression for the monochromatic dipole pulse spectrum:

$$f(\xi) = \frac{1}{2}(\delta(\xi - \Omega/c) + \delta(\xi + \Omega/c)). \tag{17}$$

In (16) and (17), two terms correspond to the positive-frequency and negative-frequency parts of the spectrum.

Higher-order harmonic generation by the dipole pulse

Now we obtain the radiation field that is generated by the strong dipole pulse written as (12) in accordance with equations (4) and (7). The calculations should be preferably conducted for the Fourier harmonics of the field $F_{(1)}^{ik}$ and current j^i. These Fourier harmonics are reduced to a double four-dimensional convolution of expressions for the Fourier harmonics of the tensor $F_{(0)}^{ik}$ taking into account an additional derivative and may be represented as

$$j^i(k) = \frac{2ac^2}{(2\pi)^4} \int F^{ik}(k-k')k'_k I(k')\, d^4k', \tag{18}$$

where I is the convolution written as

$$I(k) = \frac{1}{(2\pi)^4} \int F^{ik}(k'')F^*_{ik}(k-k'')\, d^4k''. \tag{19}$$

Now considering expression (13) as well as $k^2 = 0$, $(k-k')^2 = 0$, $(k'-k'')^2 = 0$ and $k''^2 = 0$ the expression for the current can be written as

$$\begin{aligned} j^i(k) =& \frac{aci}{\pi^2} \int \left[M^{k\mu}(k^i - k'^i)k'_k k_\mu + M^{i\mu}(kk')(k_\mu - k'_\mu))\right] \\ &\times I(k')\,\mathrm{sign}(k^0 - k'^0) f(k^0 - k'^0)\delta((k-k')^2)\, d^4k', \end{aligned} \tag{20}$$

where it is assumed that $k^0 = |\boldsymbol{\kappa}| > 0$. The negative-frequency spectrum part makes exactly the same contribution.

For I from (20), the following expression is derived:

$$\begin{aligned} I(k) =& 4 \int \Big[k'^2(M^{t\mu}M_t{}^{\nu}k''_t k''_\mu - M^{t\mu}M_t{}^{\nu}k''_t k'_\mu) \\ &-2(M^{t\mu}k''_t k'_\mu)^2\Big]\,\mathrm{sign}(k'^0 - k''^0) f(k'^0 - k''^0) \\ &\times \delta((k'-k'')^2)\,\mathrm{sign}(k''^0) f^*(k''^0)\delta(k''^2)\, d^4k'', \end{aligned} \tag{21}$$

that may be triply integrated and reduced to a one-dimensional integral.

Now assume that the magnetic moment $\mathbf{m}_\alpha = \varepsilon_{\alpha\beta\gamma}M^{\beta\gamma}/2$ is equal to zero and the electromagnetic pulse is fully electrodipole with the dipole moment $\mathbf{d} = M^{0\alpha}$. Finally, the following expression for $I(k'^0, \kappa')$ is derived:

$$\begin{aligned} I(k) =& -4 \int_{-\infty}^{+\infty} \left[P_1|\mathbf{d}|^2 + P_2(\mathbf{n}'\mathbf{d})^2\right] \\ &\times \Big(\theta((k'^0)^2 - \kappa'^2 - 2\kappa''(k'^0 - \kappa')) - \theta((k'^0)^2 \\ &-\kappa'^2 - 2\kappa''(k'^0 + \kappa'))\Big) f^*(\kappa'') f(k'^0 - \kappa'') \\ &\times \mathrm{sign}(k'^0 - \kappa'')\, d\kappa'', \end{aligned} \tag{22}$$

where $\kappa'' = |\boldsymbol{\kappa}''|$, $\boldsymbol{\kappa}''$ — is the spatial part of the 4D vector k'', $\kappa' = |\boldsymbol{\kappa}'|$, $\boldsymbol{\kappa}'$ — is the spatial part of the 4D vector k',

$$\begin{aligned} P_1 =& \frac{1}{2}\Big[((k'^0)^2 - 3\kappa'^2)S_{00} + ((k'^0)^2 + \kappa'^2)S_{11} \\ &-2((k'^0)^2 - \kappa'^2)k'^0 v^0\Big], \end{aligned} \tag{23}$$

$$\begin{aligned} P_2 =& \frac{1}{2}\Big[-((k'^0)^2 - 3\kappa'^2)S_{00} + 3((k'^0)^2 + \kappa'^2)S_{11} \\ &+8\kappa' k'^0 S_{01} 2v_1\kappa'((k'^0)^2 - \kappa'^2)\Big] \end{aligned} \tag{24}$$

and

$$S_{00} = \frac{\pi}{2\kappa'}\kappa''^2, \tag{25}$$

$$S_{01} = \frac{\pi((k'^0)^2 - \kappa'^2)}{4k'^2}|\kappa''| - \frac{\pi\kappa'^0}{2\kappa'^2}|\kappa''|\kappa'', \tag{26}$$

$$\begin{aligned} S_{11} =& \frac{\pi((k'^0)^2 - \kappa'^2)^2}{8\kappa'^3} - \frac{\pi((k'^0)^2 - \kappa'^2)k'^0\kappa''}{2\kappa'^3} \\ &+ \frac{\pi(k'^0)^2\kappa''^2}{2\kappa'^3}, \end{aligned} \tag{27}$$

$$v_0 = \frac{\pi}{2\kappa'}|\kappa''|, \tag{28}$$

$$v_1 = -\frac{\pi((k'^0)^2 - \kappa'^2)}{4\kappa'^2} + \frac{\pi k'^0}{2\kappa'^2}\kappa''. \tag{29}$$

After substitution (22) into expression for current (20), integration may be conducted and an expression for current components may be derived in the form of a triple integral.

It should be noted that only transverse components of current (20) orthogonal to the radial vector $\mathbf{n} = \boldsymbol{\kappa}/|\boldsymbol{\kappa}|$ will contribute to radiation. Now by isolating explicitly in (20) these transverse current components and by dividing the expression into the dipole and octupole parts, we can write, taking into account expression (14) for the Pauli-Jordan function, for the three-dimensional Fourier transformation from (positive-frequency) transverse parts of the electric field of radiation:

$$\begin{aligned} E^\alpha(\kappa) =& \frac{8aci|\mathbf{d}|^3}{\pi}\Big[(G'_1 + \frac{G'_2}{5})(\delta^{\alpha\beta} - n^\alpha n^\beta)\frac{d^\beta}{|\mathbf{d}|} \\ &+G'_2(\delta^{\alpha\beta}n^\gamma n^\sigma - n^\alpha n^\beta n^\gamma n^\sigma)O^{\beta\gamma\sigma}\Big], \end{aligned} \tag{30}$$

where the octupole moment tensor is equal to

$$O^{\beta\gamma\sigma} = \frac{d^\beta d^\gamma d^\sigma}{|\mathbf{d}|^3} - \frac{(\delta^{\beta\gamma}d^\sigma + \delta^{\beta\sigma}d^\gamma + \delta^{\gamma\sigma}d^\beta)}{5|\mathbf{d}|}, \tag{31}$$

and G_1' and G_2' are equal to:

$$G_1' = \int_{-\infty}^{+\infty}\int_{0}^{+\infty}\int_{-\infty}^{+\infty} \kappa'^2 f(\kappa - k'^0) f^*(\kappa'') f(k'^0 - \kappa'')$$
$$\times(\theta((k'^0 - \kappa')(k'^0 + \kappa' - 2\kappa)) - \theta((\kappa' + k'^0)$$
$$\times(k'^0 - \kappa' - 2\kappa)))(\theta((k'^0 - \kappa')(k'^0 + \kappa' - 2\kappa''))$$
$$-\theta((k'^0 + \kappa')(k'^0 - \kappa' - 2\kappa''))) \operatorname{sign}(\kappa - k'^0)$$
$$\times \operatorname{sign}(k'^0 - \kappa'') G_1(k'^0, k', k'')\, d\kappa_0'\, d\kappa'\, d\kappa'', \quad (32)$$

$$G_2' = \int_{-\infty}^{+\infty}\int_{0}^{+\infty}\int_{-\infty}^{+\infty} \kappa'^2 f(\kappa - k'^0) f^*(\kappa'') f(k'^0 - \kappa'')$$
$$\times(\theta((k'^0 - \kappa')(k'^0 + \kappa' - 2\kappa)) - \theta((\kappa' + k'^0)$$
$$\times(k'^0 - \kappa' - 2\kappa)))(\theta((k'^0 - \kappa')(k'^0 + \kappa' - 2\kappa''))$$
$$-\theta((k'^0 + \kappa')(k'^0 - \kappa' - 2\kappa''))) \operatorname{sign}(\kappa - k'^0)$$
$$\times \operatorname{sign}(k'^0 - \kappa'') G_2(k'^0, k', k'')\, d\kappa_0'\, d\kappa'\, d\kappa''. \quad (33)$$

In (32), (33) , the coefficients are calculated as follows:

$$G_1 = \kappa k'^0 (k'^0 - \kappa) P_2 m_2^1 + \kappa\kappa'(k'^0 - \kappa)(P_1 m_0 - P_1 m_1^1 - P_2 m_3^1) - \kappa\kappa'^2 (P_1 m_2^1 + 2P_2 m_4^1), \quad (34)$$

$$G_2 = P_2 \left[\kappa k'^0 (k'^0 - \kappa) m_2^2 - \kappa\kappa' k'^0 m_3^2 + 2\kappa^2\kappa'(m_3^1 + m_3^2) - \kappa\kappa'^2 m_4^2\right], \quad (35)$$

where

$$m_0 = \frac{\pi}{\kappa\kappa'}, \quad (36)$$

$$m_1^1 = -\pi \frac{(\kappa - k'^0)^2 - \kappa'^2 - \kappa^2}{2\kappa^2\kappa'^2}, \quad (37)$$

$$m_2^1 = \frac{\pi}{2\kappa\kappa'}\left[1 - \frac{((\kappa - k'^0)^2 - \kappa'^2 - \kappa^2)^2}{4\kappa^2\kappa'^2}\right], \quad (38)$$

$$m_2^2 = \frac{\pi}{2\kappa\kappa'}\left[\frac{3((\kappa - k'^0)^2 - \kappa'^2 - \kappa^2)^2}{4\kappa^2\kappa'^2} - 1\right], \quad (39)$$

$$m_3^1 = \pi \frac{(\kappa - k'^0)^2 - \kappa'^2 - \kappa^2}{4\kappa^2\kappa'^2} \times \left[\frac{((\kappa - k'^0)^2 - \kappa'^2 - \kappa^2)^2}{4\kappa^2\kappa'^2} - 1\right], \quad (40)$$

$$m_3^2 = \pi \frac{(\kappa - k'^0)^2 - \kappa'^2 - \kappa^2}{4\kappa^2\kappa'^2} \times \left[3 - \frac{5((\kappa - k'^0)^2 - \kappa'^2 - \kappa^2)^2}{4\kappa^2\kappa'^2}\right], \quad (41)$$

$$m_4^1 = \frac{\pi}{8\kappa\kappa'}\left[1 - \frac{((\kappa - k'^0)^2 - \kappa'^2 - \kappa^2)^2}{4\kappa^2\kappa'^2}\right], \quad (42)$$

$$m_4^2 = -\frac{\pi}{8\kappa\kappa'}\left[1 - \frac{5((\kappa - k'^0)^2 - \kappa'^2 - \kappa^2)^2}{4\kappa^2\kappa'^2} + \frac{((\kappa - k'^0)^2 - \kappa'^2 - \kappa^2)^4}{4\kappa^4\kappa'^4}\right]. \quad (43)$$

As can be seen from (30), the angular radiation spectrum changes compared with the initial dipole pulse and will include the octupole component corresponding to a photon with the angular moment $j = 3$ derived as a result of addition of three dipole photons with the angular moment $j = 1$ or the elastic scattering of two dipole photons on each other. The absence of the quadrupole term may be explained by the conservation of parity: three odd-parity dipole photons of the initial pulse $P = -1$ are combined into a single photon that also will have odd parity; or two odd-parity dipole photons are scattered on each other giving two photons, one of which will be a dipole photon and the second one shall be, respectively, odd-parity. But only odd-multipole electrical photons have odd parity because $P = (-1)^j$ for electrical type photons [1].

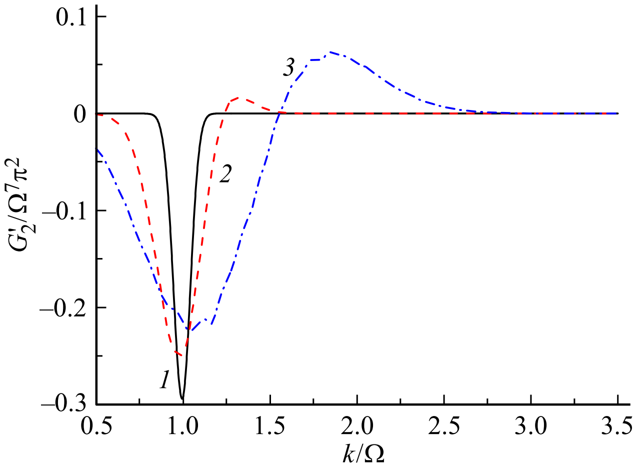
In case when the initial dipole pulse is described by f as defined by equation (17), integration over k'^0 and κ'' in expressions (32), (33) is conducted by substituting $\kappa'' = \pm\Omega$ and $k'^0 = \kappa \mp \Omega$. For the third harmonic, this gives (taking into account that $\kappa > 0$ and $\kappa' > 0$) $\kappa = 3\Omega$ and $k'^0 = 2\Omega$. These expressions with substitution into (32) and (33) set the integrals over κ' from 0 to $+\infty$ to zero because product of theta functions in this interval is equal to zero. This means that the third harmonic is not generated in the monochromatic electrodipole pulse field in vacuum. This is also a direct consequence of the laws of conservation of pulse energy applicable to the system of interacting photons.

For the first harmonic we have $\kappa = \Omega$ and $k'^0 = 0$ or $k'^0 = 2\Omega$ for the monochromatic pulse, which after integration over κ' from 0 to $+\infty$ in expressions (32), (33), results to the following expression for the three-dimensional Fourier transformation of transverse components of the electric field:

$$E^\alpha(\kappa) = -\frac{\sqrt{3}i}{15\pi}\frac{\alpha P}{\lambda I_s}\sqrt{\frac{P}{c}}\left[(G_1'' + \frac{G_2''}{5})(\delta^{\alpha\beta} - n^\alpha n^\beta)\frac{d^\beta}{|\mathbf{d}|} + G_2''(\delta^{\alpha\beta} n^\gamma n^\sigma - n^\alpha n^\beta n^\gamma n^\sigma) O^{\beta\gamma\sigma}\right]\delta(\kappa - \Omega/c), \quad (44)$$

where $P = |\mathbf{d}|^2\Omega^4/(3c^3)$ is the mean power of the initial dipole pulse, $I_s = m_e^4 c^7/(8\pi e^2\hbar^2) \approx \approx 1.82 \cdot 10^{29}\,\mathrm{W/cm^2}$ is the Schwinger intensity, $\alpha = e^2/(\hbar c)$ is the fine structure constant, $\lambda = 2\pi c/\Omega$ is the wavelength, $G_1'' = -73/630$ and $G_2'' = -1271/3465$.

Expression (44) means that, in addition to the initial dipole, the octupole with the same frequency appears as a result of scattering of two photons on each other. The mean intensity of the octupole component will be proportional to $\sim (\alpha P/\lambda I_s)^2$, i.e. is very low.



The normalized coefficient $G_2'/(\pi^2\Omega^7)$ calculated by means o numerical integration of the Gaussian dipole pulse field (16) by the Monte Carlo method for three different values of $(\Omega l)^{-1}$: *1* — $(l\Omega)^{-1} \sim 0.03$, *2* — $(l\Omega)^{-1} \sim 0.1$ and *3* — $(l\Omega)^{-1} \sim 0.3$.

It should be noted that, for the Gaussian pulse with the finite spectrum width (16), some radiation field does appear at the third harmonic frequency. This can be seen from the curves in the Figure for G_2' derived by the numerical calculation of triple integral in (33) by the Monte Carlo method in Matlab programming platform. G_2' was chosen because it defines the intensity of the generated octupole pulse in (44). It can be also seen from the Figure that with the increase in the initial pulse spectrum width $((l\Omega)^{-1})$, general shift of the octupole component spectrum to the high frequency region is observed, and this may be used, for example, to isolate this component against the background of the strong initial dipole pulse.

It should be noted that the conclusions made for the electrodipole pulses will be applicable also to the magnetic dipole pulses due to the symmetry between the electric and magnetic field in vacuum.

Conclusion

Generation of nonlinear harmonics due to self-interaction of the dipole electromagnetic pulse field evolving in free space has been considered. For this, the study uses the known nonlinear electromagnetic field equations considering the quantum electrodynamic effects in the first non-vanishing order by the fine structure constant in so-called one-loop approximation. It is shown that the periodic dipole pulse does not generate the third harmonic in vacuum. Impossibility of addition of several photons also follows from the law of conservation of pulse energy-momentum for the system of interacting photons. Nevertheless, self-interaction of the dipole pulse results in the occurrence of the octupole filed component at the initial frequency that corresponds to elastic scattering of two dipole photons on each other, as a result a photon with a angular moment equal to three and with the initial frequency appears.

Further numerical simulation has shown that for the real dipole pulses with the finite spectrum width — for example, Gaussian — a nonzero intensity does occur at the third harmonic frequency. However, this phenomenon is not associated with pulse generation at the third harmonic frequency and is explained by the finite width of the initial dipole pulse spectrum. It should be noted that for the intense dipole pulses, for example, in presence of a strong permanent magnetic field, the third harmonic will be generated, but the strength of this effect has yet to be assessed. The third harmonic will be also generated when the Heisenberg-Euler Lagrangian includes the sixth-order and higher-order terms [9].

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Conflict of interest

The author declares that he has no conflict of interest.

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