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Energy losses minimization of the electromagnetic pulse propagating through the saprse media

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The analyses of the electromagnetic pulse with different shapes propagation through the media with astrophysical parameters was made. Energy losses of the pulse was calculated. The special shape of the pulse that provide pulse to propagate with the least energy losses was revealed.

Keywords: soliton of the self-induced transparency effect, two-level medium.

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Introduction

The effect of self-induced transparency is that an electromagnetic pulse of a certain shape propagates with minimal energy losses in a medium with significant concentrations of atoms. In the case of a two-level medium [1] (a medium consisting of atoms in which only one transition between two energy states — ground and excited, with a known transition frequency is resonant with radiation), this mechanism consists in the following. The leading edge of such an electromagnetic pulse transfers atoms to an excited state, transferring its energy to the medium, while further propagation of the pulse removes the excitation, and the energy returns from the medium back to the pulse, restoring its original shape. The shape of such a pulse was determined by S. McCall and E. Khan [2–4]. The question arises whether this effect manifests itself in rarefied media, such as astrophysical [5] ones, in which the concentration of atoms is significantly lower than in the case of [2–4]. We will discover that, for fixed energy and carrier frequency, the pulse has the least loss if its shape coincides with that indicated by McCall and Khan. We will show this by computer simulation with variations in the pulse shape.

System of equations

The propagation of a radiation pulse with a fixed (linear) polarization in a medium is described by the following wave equation from Maxwell's equations:

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(E + 4\pi P) = 0. \qquad (1)$$

Here E is the electric field strength with a fixed (linear) polarization, P — polarization of the medium, c — the speed of light in vacuum, t — time. We review a two-level medium, so the polarization is written in the following form $P = Nd_{21}(\rho_{21} + \rho_{21}^*)$, here N — concentration of atoms, d_{21} — dipole matrix element of the transition between the first and second levels, ρ_{21} — non-diagonal element of the density matrix describing an individual atom environment. The state of an atom at passing of a pulse is described by equations for the density matrix [1,6] which, together with the wave equation (1) make the system

$$\nabla^2 \tilde{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(\tilde{E} + 4\pi\tilde{P}\right) = 0,$$

$$\frac{\partial\tilde{\rho}_{21}}{\partial t} = -i\omega_{21}\tilde{\rho}_{21} - \gamma_{21}\tilde{\rho}_{21} - i\,\frac{d_{21}}{h}\,\tilde{E}(\rho_{22} - \rho_{11}),$$

$$\frac{\partial\rho_{22}}{\partial t} = -\gamma_2\rho_{22} - i\,\frac{d_{21}}{h}\,\tilde{E}(\tilde{\rho}_{21} - \tilde{\rho}_{21}^*),$$

$$\frac{\partial\rho_{11}}{\partial t} = \gamma_2\rho_{22} + i\,\frac{d_{21}}{h}\,\tilde{E}(\tilde{\rho}_{21} - \tilde{\rho}_{21}^*), \qquad (2)$$

where γ_{21} and γ_2 —relaxation rates, ρ_{11} and ρ_{22} — diagonal elements of the density matrix. The upper wavy line marks rapidly changing values. We will use the slowly varying amplitude approximation with a fixed carrier frequency. Let us introduce

$$\tilde{E} = \frac{1}{2}E\exp(i\phi) + \text{k.c.},$$

$$\tilde{\rho}_{21} = \rho_{21}\exp(i\phi),$$

where E and ρ_{21} — slowly changing amplitudes, and $\phi = kz - \omega t$ — rapidly changing phase. We will designate $\Delta\omega = \omega_{21} - \omega$. If diffraction effects are not taken into account, then the terms with $\partial^2/\partial x^2 + \partial^2/\partial y^2$ will disappear in the equation for the field. Let

$$\tilde{z} = \frac{\omega_R z}{c},$$

$$\bar{t} = \omega_R t,$$

$$\bar{E} = \frac{E}{E_0},$$

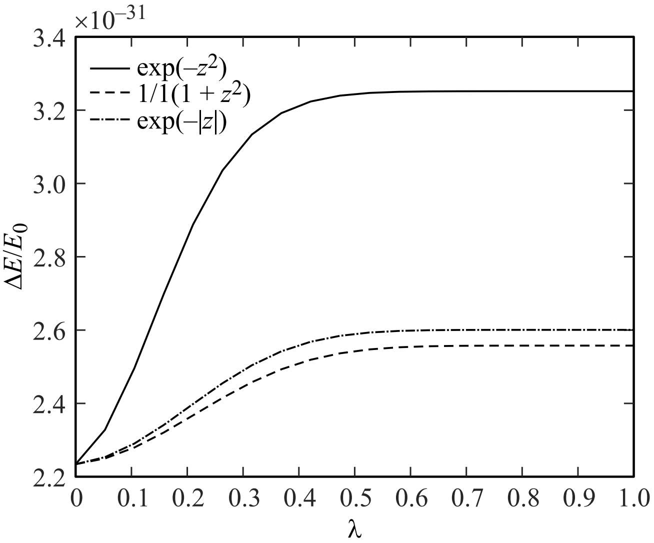

Figure 1. Dependences of energy losses of various pulses with maximum energy amplitude $E_0 = \frac{\hbar\omega_R}{d_{21}}$ and $\omega_R = 10^{15}\,\text{s}^{-1}$ on the parameter λ when propagating in a medium over a length $z = 30c/\omega_R$. The solid line shows the energy loss for a pulse with $f(z) = \exp(-z^2)$, the dashed line — $\frac{1}{1+z^2}$, the dash-dot — $\exp(-|z|)$.

where $E_0 = \frac{\hbar\omega_R}{d_{21}}$ and ω_R — Rabi frequencies. Let us introduce a designation

$$\alpha = \frac{2\pi N d_{21}^2 \omega}{hc},\ \bar{\alpha} = \frac{c\alpha}{\omega_R^2},\ \Delta\bar{\omega} = \frac{\Delta\omega}{\omega_R},\ \bar{\gamma}_{21} = \frac{\gamma_{21}}{\omega_R},\ \bar{\gamma}_2 = \frac{\gamma_2}{\omega_R}.$$

Further, the dashes above dimensionless variables will be omitted, and all variables in the equations will be assumed to be dimensionless. Let us write the elements of the density matrix in terms of the components of the Bloch vector $\mathbf{R} = (R_1, R_2, R_3)$:

$$\rho_{21} = \frac{R_1 + iR_2}{2},\ \rho_{11} = \frac{1 + R_3}{2},\ \rho_{22} = \frac{1 - R_3}{2}.$$

Then $\rho_{22} - \rho_{11} = -R_3$, $\rho_{22} + \rho_{11} = 1$. We also write the electric field strength in terms of the imaginary and real parts: $E = E_1 + iE_2$. Thus, we obtain the following system of equations:

$$\frac{\partial E_1}{\partial t} = -\frac{\partial E_1}{\partial z} - \alpha R_2,$$

$$\frac{\partial E_2}{\partial t} = -\frac{\partial E_2}{\partial z} + \alpha R_1,$$

$$\frac{\partial R_1}{\partial t} = \Delta\omega R_2 - \gamma_{21} R_1 - E_2 R_3,$$

$$\frac{\partial R_2}{\partial t} = -\Delta\omega R_1 - \gamma_{21} R_2 + E_1 R_3,$$

$$\frac{\partial R_3}{\partial t} = \gamma_2(1 - R_3) - (E_1 R_2 - E_2 R_1). \quad (3)$$

The self-consistent system of equations (3) represents the process of propagation of an electromagnetic pulse in a two-level medium. The soliton solution is possible in the absence of relaxation and has the form

$$E(z,t) = \frac{2}{t_p}\,\text{sech}\left(\frac{t - z/V}{t_p}\right),$$

where t_p — a free parameter that specifies the pulse width, and $V^{-1} = c^{-1} + \alpha t_p^2(1 + t_p^2\Delta\omega^2)^{-1}$ is the inverse velocity of the soliton of self-induced transparency [7]. When the area under the graph of such a pulse is equal to 2π, it is called 2π-pulse [8,9].

Energy loss minimization

Passing through the medium, the pulse excites atoms, i.e. its energy goes into the medium, but this energy does not return back into the pulse in low-concentration media, as happens in the effect of self-induced transparency. The excited atoms will return to the ground state after some time and the energy they emit will be the energy lost by the impulse. The radiation power of an atom in all directions is $S = \frac{2|\ddot{d}|^2}{3c^3}$, where $d = d_{21}(\rho_{21} + \rho_{21}^*)$. In our approximation $S \approx \frac{4d_{21}^2|\rho_{21}^2|\omega^4}{3c^3}$. By integrating this expression over time, we obtain the energy emitted by the atom. A soliton of self-induced transparency propagates in a medium with a significant concentration of atoms with minimal energy losses. For low concentrations, $N \approx 10^3$–$10^4\,\text{cm}^{-3}$, corresponding to dense astronomical media, the effect of self-induced transparency is transformed into a certain effect in which a pulse of this shape also undergoes the least energy loss. This work examines the evolution of various forms of initial profiles of an electromagnetic pulse with different areas. Meanwhile, various environmental parameters were considered. The work contains [5] information on the interstellar medium between the Sun and various sources. We can estimate the average hydrogen concentration as $N = (4.7$–$6.2) \cdot 10^2\,\text{cm}^{-3}$. When a pulse propagates through such a medium, it can be absorbed in the line Ly-α with the transition frequency $\omega_{21} = 15.5 \cdot 10^{15}\,\text{s}^{-1}$. The dipole moment of such a transition is $d = 1.5$ Debye. Let us imagine the pulse strength at the initial moment of time in the following form:

$$E(z) = (1 - \lambda)\frac{2}{t_p}\,\text{sech}\,(z/Vt_p) + \lambda f(z),$$

where $f(z)$ — arbitrary function, and λ — parameter for variation. Using computer simulation, we will launch pulses with a certain energy, varying the shape by changing the value of the parameter λ from zero to one, and we will look at the energy loss of the pulse. For the function $f(z)$, let us take, for example, $\exp(-z^2)$, $\frac{1}{1+z^2}$ and $\exp(-|z|)$. For a pulse with intensity $E_0 = 2.1 \cdot 10^{10}$ V/m, energy losses practically do not increase as the parameter λ changes (Fig. 1). However, for pulses with lower

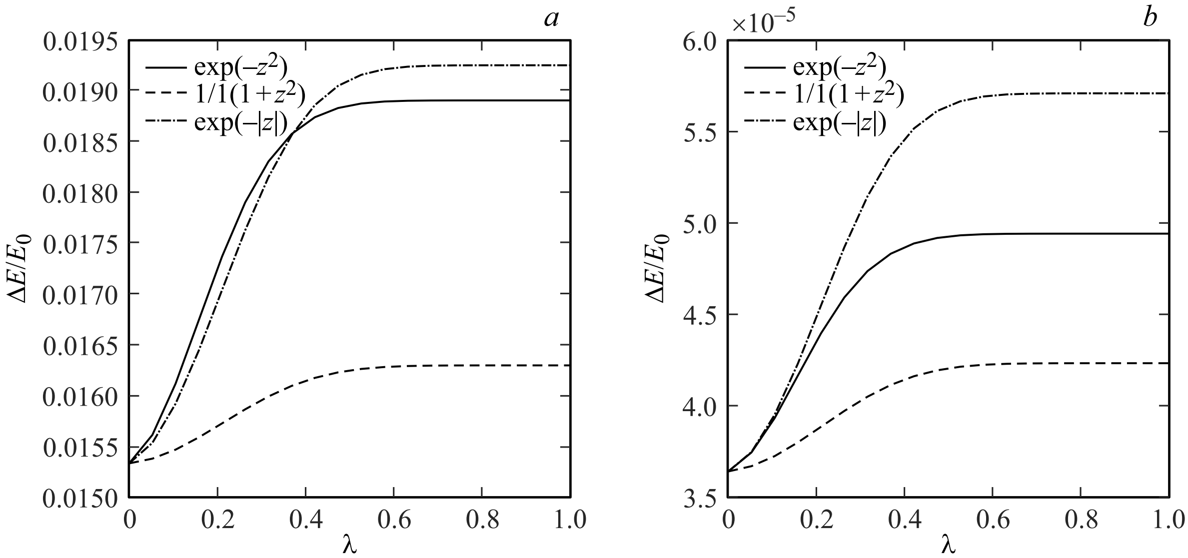

Figure 2. Dependences of energy losses of various pulses with maximum energy amplitude $E_0 = \frac{\bar{h}\omega_R}{d_{21}}$ and $\omega_R = 10^4\,\mathrm{s}^{-1}$ (a), $\omega_R = 10^5\,\mathrm{s}^{-1}$ (b) on the parameter λ when propagating in a medium over a length $z = 30c/\omega_R$. Medium concentration $N = 10^2\,\mathrm{cm}^{-3}$ (a) and $N = 10^3\,\mathrm{cm}^{-3}$ (b). The solid line shows the energy loss for a pulse with $f(z) = \exp(-z^2)$, the dashed line — $\frac{1}{1+z^2}$, the dash-dot — $\exp(-|z|)$.

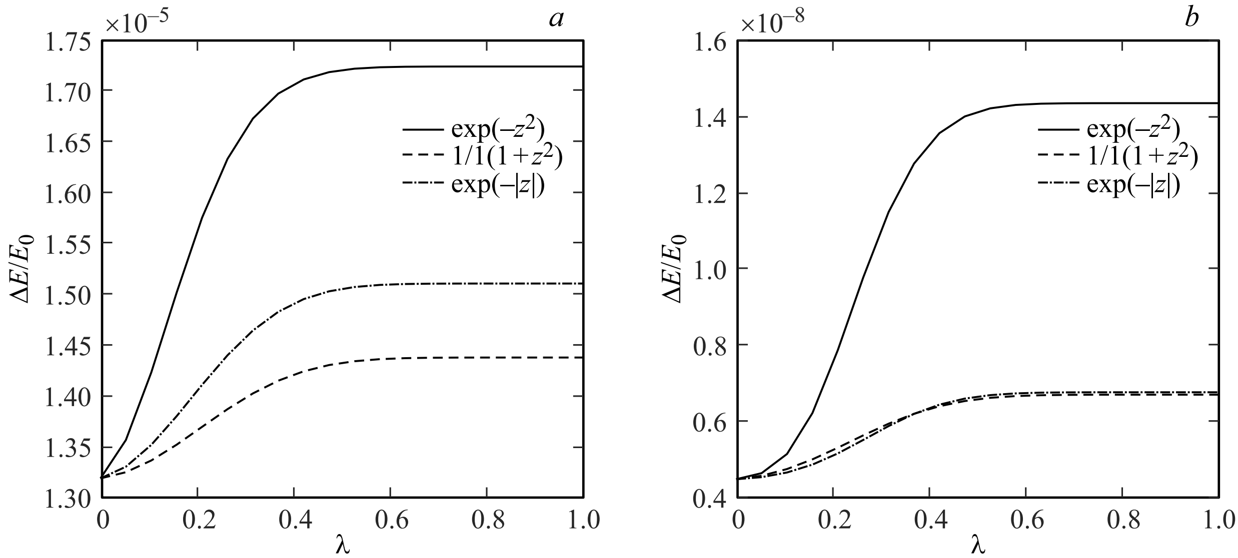

Figure 3. Dependences of energy losses of various pulses with maximum energy amplitude $E_0 = \frac{\bar{h}\omega_R}{d_{21}}$ and$\omega_R = 10^6\,\mathrm{s}^{-1}$ (a), $\omega_R = 10^7\,\mathrm{s}^{-1}$ (b) on the parameter λ when propagating in a medium over a length $z = 30c/\omega_R$. Medium concentration $N = 10^3\,\mathrm{cm}^{-3}$ (a) and $N = 10^4\,\mathrm{cm}^{-3}$ (b). The solid line shows the energy loss for a pulse with $f(z) = \exp(-z^2)$, the dashed line — $\frac{1}{1+z^2}$, the dash-dot — $\exp(-|z|)$.

intensity, energy losses become significant when deviating from the required pulse shape. From Fig. 2 *a, b* it is clear that a pulse with an unperturbed shape undergoes the least loss. The field strength here is $E_0 = 21\,\mathrm{V/m}$. Meanwhile , the higher the pulse energy, the lower its relative energy losses, but even in this case, with an unperturbed pulse shape, energy losses are minimal (Fig. 3).

Conclusion

For pulses with low field strength when passing through a medium with low concentrations, its shape is very important. Calculations were carried out for the case when the transition frequency in a two-level medium ω_{21} was close to the pulse frequency ω, namely $\Delta\omega \sim \omega_R$. It is shown that the effect of self-induced transparency in rarefied media is transformed into the following effect: when an electromagnetic pulse propagates through an interstellar medium with a profile having the shape of a hyperbolic secant, the least energy loss occurs.

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Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] L. Allen, J. H. Eberly, *Opticheskiy rezonans i dvukhurovnevyye atomy* (Mir, Moskva, 1978) (in Russian).
[2] S.L. McCall, E.L. Hahn. Phys. Rev., **183**, 457 (1969). DOI: 10.1103/PhysRev.183.457
[3] I.A. Poluektov, Yu.M. Popov, V.S. Roitberg. UFN, **114** (1), 97 (1974). (in Russian). DOI: 10.3367/UFNr.0114.197409e.0097
[4] A.I. Maimistov. Kvant. elektron., **40**(9), 801 (2010) (in Russian). DOI: 10.1070/QE2010v040n09ABEH014396
[5] C. Pinto, J.S. Kaastra, E. Costantini1, C. de Vries. Astronomy & Astrophysics, **551**, A25 (2013). DOI: 10.1051/0004-6361/201220481
[6] N.N. Rozanov. *Dissipativnye opticheskie solitony. Ot mikro- k nano- i atto-* (FIZMATLIT, Moskva, 2011). (in Russian).
[7] A.I. Maimistov, A.M. Basharov. *Nonlinear Optical Waves* (Springer Science+Business Media B.V., Dordrecht, 1999). DOI: 10.1007/978-94-017-2448-7
[8] R.M. Arkhipov, M.V. Arkhipov, N.N. Rozanov. Kvant. elektron., **50** (9), 801 (2020) (in Russian). DOI: 10.1070/QEL17348
[9] N.N. Rozanov, R.M. Arkhipov, M.V. Arkhipov. UFN, **188** (12), 1347 (2018). (in Russian). DOI: 10.3367/UFNr.2018.07.038386

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