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Features of the development of electrohydrodynamic instability of the molten metal boundary in a strong electric field

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The dynamics of the instability development of the free boundary of liquid metal (molten copper) in a strong electric field of about 10^8 V/cm has been studied. Under such local fields, natural submicron-scale protrusions on the cathode surface melt due to the flow of field emission current through them. Electrohydrodynamic instability of the melt boundary leads to a sharpening of the boundary, which provides a local increase in the electric field and, as a consequence, accelerates the processes of vacuum breakdown. It has been demonstrated that the feature of electrohydrodynamic instability under the considered conditions is the need to take into account viscous effects. A relatively simple nonlinear model is proposed to describe them.

Keywords: Electrohydrodynamic instability, molten metal, vacuum breakdown.

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Studies of the physical processes leading to vacuum breakdown have received a new impetus in development due to the development of accelerating equipment at terawatt power levels [1]. It is the vacuum breakdown of the accelerating structure when it is exposed to electromagnetic pulses of nanosecond duration that is the main problem on the way to achieving high accelerating gradients [2]. The study of nanosecond vacuum breakdown has shown [3] that its main mechanism is the formation of a conducting medium (plasma) on the cathode due to heating by the field emission high-density current of microprotrusions with the highest coefficients β of electric field amplification. The analysis of the heating of copper microprotrusions has shown [4,5] that the field strength at their vertices to realize this mechanism in accelerating structures [1] is on the order of 10^8 V/cm (for characteristic values $\beta = 50-100$ this corresponds to a macroscopic field of $1-2$ MV/cm).

The purpose of the present work is to analyze the features of the development of electrohydrodynamic (EHD) instability [6,7] of the molten metal surface in strong fields on the order of 10^8 V/cm. This instability determines the tendency for unrestricted surface sharpening. As a result, after melting of the microprotrusion, its geometrical parameters will change (increase β) and, as a consequence, pre breakdown processes will be accelerated.

Let us begin the analysis with estimates that will demonstrate the specifics of the development of EHD-instability of molten metal in fields three orders of magnitude higher than the instability threshold $E_c = (4\varepsilon_0^{-2}\rho g\alpha)^{1/4}$ [7], where ε_0 is electrical constant, ρ is density, g is free fall acceleration, α is surface tension coefficient (for liquid copper $E_c \approx 8.5 \cdot 10^4$ V/cm). For simplicity we will consider the boundary of the fluid in the unperturbed state as flat. The function η sets its perturbation — deviation from the plane. In the instability analysis, the boundary η is sought as a plane wave: $\eta(x,t) \propto \exp(ikx - i\omega t)$, where x is coordinate, t is time, $k > 0$ is wave number, ω is frequency. The linear (corresponding to the condition of smallness of slope angles) dynamics of the boundary is described by the dispersion relation (the relation between the values ω and k), which for an incompressible nonviscous perfectly conducting fluid in an external homogeneous field of strength E has the form [7]:

$$\rho\omega^2 = -\varepsilon_0 E^2 k^2 + \alpha k^3. \qquad (1)$$

It can be seen from (1) that ω is imaginary in the region of relatively small wave numbers $0 < k < k_c \equiv \varepsilon_0 E^2/\alpha$, which corresponds to the aperiodic instability of the boundary (at $k > k_c$ capillary forces suppress the destabilizing effect of electrostatic forces). The development of instability leads (at its nonlinear stages) to the formation of conical tips with opening angle 98.6° — Taylor cones [8,9]. The fastest developing instability will be for the wave number $k_d = 2k_c/3$ (the dominant mode of instability), which defines the scale of the conical formations as $\lambda_d = 2\pi/k_d \propto E^{-2}$. The characteristic time of instability development (the time for which the perturbation amplitude increases by a factor of $e \approx 2.718$) is $\tau_d = 1/\gamma_d \propto E^{-3}$, where $\gamma_d = \mathrm{Im}\,\omega(k_d)$ is instability increment. As can be seen, λ_d and τ_d decrease rapidly as E increases. Reducing the characteristic scales of the problem will inevitably lead to the fact that the ideal fluid approximation ceases to work and it will be necessary to take into account the viscosity of the medium (see, for example, [9,10]). According to the calculations [10], viscous effects cause the angle of the forming cone to decrease.

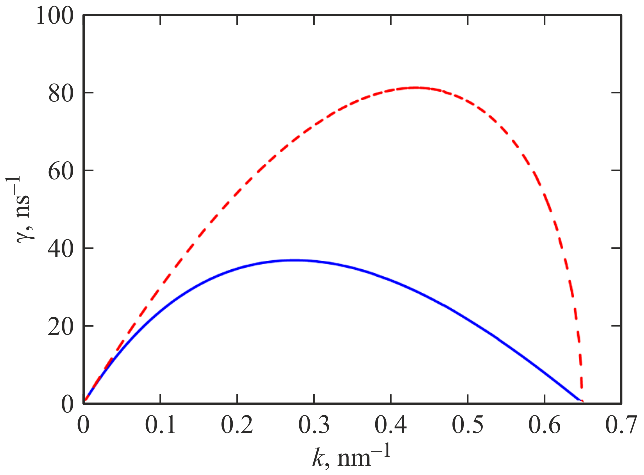

Figure 1. Dependence of the EHD-instability increment γ on the wave number k with (solid line) and without (dashed line) viscous effects. Field $E = E_\nu \approx 9.8 \cdot 10^7$ V/cm; liquid copper at melting point.

Note also the factor of additional heating of metal at its rapid deformations due to viscous friction.

We show that in fields of order 10^8 V/cm the role of viscous effects becomes comparable to that of capillary effects, and discuss how this affects the dynamics of the instability. The law of dispersion taking viscosity into account can be obtained by taking the well-known formula for gravitational waves [11] and making a formal substitution $\rho g k \to -\varepsilon_0 E^2 k^2 + \alpha k^3$ in it, corresponding to the fact that instead of gravity we will consider electrostatic and capillary forces. We obtain (here ν is kinematic viscosity)

$$\rho(2\nu k^2 - i\omega)^2 - \varepsilon_0 E^2 k^2 + \alpha k^3 = 4\rho\nu^{3/2} k^3 \sqrt{\nu k^2 - i\omega}. \tag{2}$$

It follows from the analysis (2) that it is possible to introduce a characteristic value of field strength $E_\nu = \alpha\nu^{-1}\varepsilon_0^{-1/2}\rho^{-1/2}$, depending only on the fluid parameters, such that at $E \ll E_\nu$ viscous effects can be neglected, and at $E \gg E_\nu$ they will determine the dynamics of instability. When the fields are comparable with E_ν, the most complicated case is realized when viscous forces will be comparable to electrostatic and capillary forces. For liquid copper at melting point, we can take $\rho = 8.0 \cdot 10^3$ kg/m^3, $\alpha = 1.3$ N/m, $\nu = 5.0 \cdot 10^{-7}$ m^2/s. We find $E_\nu \approx 9.8 \cdot 10^7$ V/cm, which falls exactly within the field range of interest.

Let us consider how the EHD instability develops at fields of order E_ν. We will be interested in the solutions of the dispersion law (2) b describing the development of aperiodic instability. This corresponds to the fact that $\omega = i\gamma$, where $\gamma > 0$ is instability increment. In Fig. 1 for $E = E_\nu$, the dashed and solid lines show the dependences of γ from k without and with viscosity (formulas (1) and (2), respectively). It can be seen that our conclusion about the essential influence of viscous effects on the development of instability at such a field is correct. The wave number of the dominant mode (k_d) decreases by a factor of 1.6, and the corresponding increment (γ_d) — by a factor of 2.2. The characteristic spatial and temporal scales are $\lambda_d \approx 23$ nm and $\tau_d \approx 27$ ps. The range of wave numbers for which the surface is unstable remains unchanged (scale over $2\pi/k_c \approx 10$ nm). In the case of the melt formed at the tip of the cathode protrusion, we can identify the radius of the tip apex R with a quarter of a wavelength for estimates, which gives $R > 2{-}6$ nm. These dimensions correlate with those considered in [12], where the deformation of the cathode tip with $R = 1{-}10$ nm was modeled by the molecular dynamics method. At $R > 3$ nm, the threshold value of the field at which the pulling of the copper nano-protrusion began was in the range $(10.5{-}11.5) \cdot 10^7$ V/cm, i.e., close to the value E_ν. According to calculations [12], the stretching occurred in times of tens to hundreds of picoseconds at high temperatures much higher than the melting temperature. Considered by us EHD-mechanism of sharpening the top of the protrusion does not require the achievement of high temperatures, and can be realized at lower fields (see below) in the case of increasing the scale of the melt region from units to tens of nanometers.

When developing a nonlinear model for the development of instability, it is important that the relationship of γ and k be described by the simplest possible expression. So, in the limit $k \ll k_c$ we have a linear dependence $\gamma \approx \varepsilon_0^{1/2}\rho^{-1/2}Ek$, which allowed us to construct a model describing the nonlinear evolution of the boundary up to the formation of singularities — points with infinite curvature. For a viscous fluid, we encounter the difficulty that the expression for the dispersion law (2) is very cumbersome and unsolvable with respect to frequency (increment). However, it turns out that the relation described by (2) between γ and k in the special case $E = E_\nu/2$ is perfectly approximated by the parabolic relation

$$\gamma \approx \alpha(2\rho\nu)^{-1}k - 2\nu k^2. \tag{3}$$

Fig. 2 shows the exact dependence $\gamma(k)$ (solid line) and the given by (3) approximation (dashed line). As can be seen, expression (3), despite its simplicity, reflects all the main features of the exact dispersion law in the whole range of wave numbers $0 < k < k_c$ determining the development of instability. This is extremely important when analyzing the nonlinear stages of instability, at which energy pumping from large-scale harmonics to small-scale harmonics takes place; it is this process that leads to the formation of tips.

We propose that when $E = E_\nu/2$ the following nonlinear model be used to describe the development of EHD-instability in the case of flat symmetry of the problem:

$$\eta_t = -\alpha(2\rho\nu)^{-1}\hat{H}\eta_x + 2\nu\eta_{xx} + \alpha(4\rho\nu)^{-1}\left[(\hat{H}\eta_x)^2 - (\eta_x)^2\right]. \tag{4}$$

Here η_t and η_x denote the partial derivatives on t and x, respectively; $\hat{H}$ is the Hilbert operator having the property $\hat{H}e^{ikx} = i\,\mathrm{sgn}(k)e^{ikx}$. The linear part of (4) corresponds to the approximate dispersion law (3) and allows us to adequately account for the influence of both electrostatic, capillary, and viscous forces. The nonlinear part of

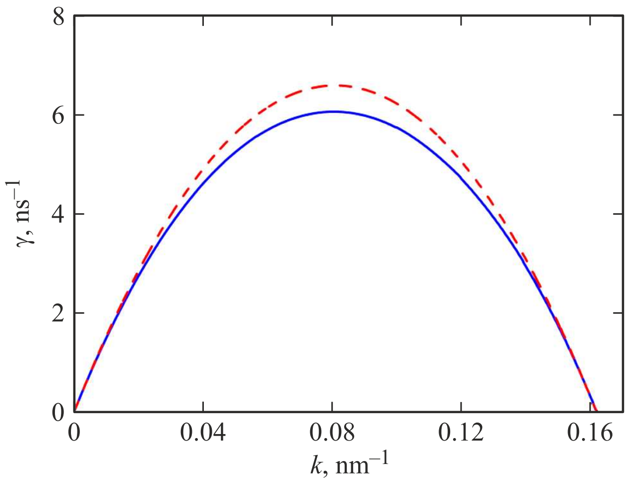

Figure 2. Dependence of the EHD-instability increment γ on the wave number k for the exact dispersion law (solid line) and the parabolic approximation (dashed line). Field $E = E_v/2 \approx 4.9 \cdot 10^7$ V/cm; liquid copper at melting point.

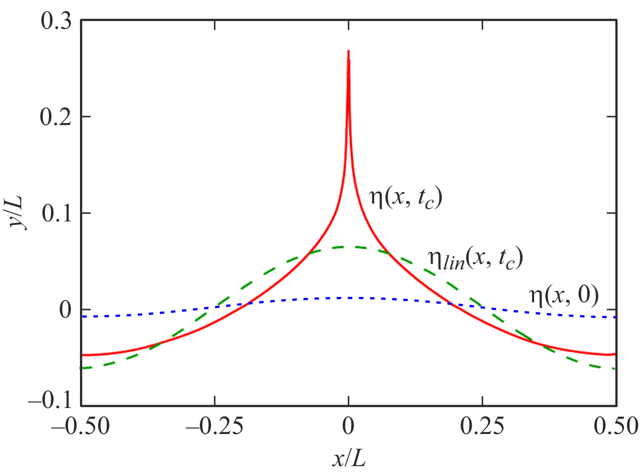

Figure 3. Evolution of the molten metal surface within the framework of the nonlinear model (4). The dotted line corresponds to the initial time point $t = 0$, the solid line — the moment of singularity formation $t = t_c$. For comparison, the dashed line shows the shape of the boundary at the moment $t = t_c$ for the situation when its evolution is described within the linear approximation (nonlinear terms are discarded in equation (4)). Field $E = E_v/2 \approx 4.9 \cdot 10^7$ V/cm; liquid copper at melting point.

(4) corresponds to the model from [13], in which only electrostatic forces were considered.

Let us consider in the framework of the model equation (4) the dynamics of singularity formation at the free boundary of the liquid. We consider that the fluid occupies a space bounded region $-L/2 \leqslant x \leqslant L/2$. We set the initial shape of the boundary as $\eta(x,0) = A\cos(2\pi x/L)$ with $L = 360$ nm (as applied to the melt at the tip apex, this corresponds to a radius of about 100 nm) and an amplitude of $A = 0.01\,L$. Equation (4) was solved numerically based on spectral methods with harmonic number $N = 8096$. Due to the specificity of the methodology used, the boundary conditions in space were taken as periodic. Time integration was performed by the explicit Runge–Kutt method of fourth-order accuracy with step $dt = 2.2 \cdot 10^{-15}$ s. The evolution of the boundary perturbation is demonstrated in Fig. 3. It can be seen that there is an unbounded sharpening of the surface in a finite time — by the moment $t = t_c \approx 0.73$ ns. This time is not directly determined by the dominant mode of instability, for which the characteristic time is much smaller: it is ~ 0.16 ns (Fig. 2). The time t_c is composed of the time of the main harmonic ($k = 2\pi/L$) amplitude rise (the corresponding characteristic time is ~ 0.41 ns) and the time of energy pumping from the main harmonic to the dominant harmonic due to their nonlinear interaction. The scale of the emerging tip can be estimated as the width of the region in which the tangent of the slope angle exceeds one. It amounted to ~ 30 nm, which is expectedly close to half the length of the dominant mode (~ 40 nm).

Also in Fig. 3, the solution of the linearized equation (4) to the moment $t = t_c$ is shown by the dashed line. From the comparison of linear and nonlinear solutions, it is clear that the nonlinear terms in (4) accelerate the development of instability, and it is they that cause the formation of singularity. Thus, model (4), despite its relative simplicity, shows a tendency to sharpen the boundary and, consequently, to increase the field enhancement factor β. In connection with the fact that this growth occurs at fields at the apex of a submicron-sized microprotrusion smaller than the breakdown fields, it can be concluded that melting of the apex will inevitably lead to an acceleration of vacuum breakdown.

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Conflict of interest

The authors declare that they have no conflict of interest.

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