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Scaling of variation intensity of flows under the influence of external noise

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The effect of noise on the change of intensity of axisymmetric flows of viscous incompressible fluid in spherical layers at rotation of boundaries with equal angular velocities is numerically investigated. Noise is introduced into the flows by adding random fluctuations with zero mean value to the time constant mean rotation velocity of the inner sphere. The response of the flows to the introduction of two types of noise with different spectra was investigated. Power laws between the increase in the time-averaged parameters of the flows and the increase in the noise amplitude were found, which qualitatively retain their form when the noise spectrum, the thickness of the spherical layer, and the Reynolds numbers are changed. It was found that noises with the same amplitude, but with different types of spectra, lead to relative changes in the time-averaged values of friction force moments, kinetic energy of the flows, and RMS deviations of kinetic energy that differ by more than an order of magnitude.

Keywords: noise, random fluctuations of rotational velocity, spherical Couette flow, flow control.

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Introduction

The presence of timing random in time coincidental fluctuations is a typical property of most natural processes and systems [1]. Such fluctuations differ both in the type of spectrum and in the wide range of time scales, varying from seasonal fluctuations in the intensity of solar radiation entering the Earth's atmosphere [2] to short-term oscillations in the Earth's rotation speed [3]. Random fluctuations in the Earth's rotation speed are being intensively studied at present, both with the aim of improving satellite navigation models [4], and in connection with the study of seismic activity [5]. Random fluctuations are extraneous noise that affects large-scale flows both in the atmosphere and in the liquid core of the Earth. Such flows are formed in spherical geometry with rotation; therefore, in this work we consider a model spherical Couette flow caused by the action of unidirectional rotation of spherical boundaries on a incompressible viscous fluid located between these boundaries. Noise added to fluid flows can lead to the generation of oscillations and turbulence [6,7], and to increasing the threshold for generating magnetic fields [8]. Under the influence of noise, the appearance of unstable states that differ from each other is possible [9,10], as well as a shift in the position of the stability limit of flows [11–13]. The last two of the above effects were observed in flows in rotating spherical layers [10,11,13] in the case where „white" noise was introduced by adding random fluctuations in the rotation speed with zero average value to the time-constant average rotation speed of the inner sphere. The same method of introducing various types of noise into flows is considered in this work. Such disturbances, regardless of the type of noise, are symmetrical relative to the equator and the axis of rotation. In this regard, when calculating flows with this method of introducing noise, an axisymmetric approximation was used [11,13], the admissibility of which is discussed below.

White noise with equal amplitudes of spectral components over the entire frequency range used, on the one hand, and periodic oscillations with a single frequency, on the other, are limiting cases of perturbations modeling unevenness in time of the Earth's rotation. It is shown numerically that in rotating spherical layers both periodic oscillations of rotation speed [14] and white noise which are added to the time-constant average rotation speed [11,13] are leading to the generation of an average flow. To study the effects arising from the uneven rotation of the Earth, not only white noise is used, which is characterized by the absence of time correlations, and its spectrum can be represented as $1/f^{\alpha}$, where $\alpha = 0$, but also other types of noise with nonzero amount of time correlation,as: $\alpha \neq 0$ [4]. At the same time, the question remains open about the nature of the dependence of flow parameters on the type of spectrum and the amplitude of noise supplied to the flow, which is the purpose of the study of this work. In this work, the purpose was not to study such well-studied phenomena in the case of periodic oscillations of the rotation speed of one of the spherical boundaries during unidirectional rotation as instability of flows [15] and/or inertial waves [16]. Therefore, the effect of noise on time-averaged flow parameters is considered here.

In this work, based on the solution of the complete system of Navier–Stokes equations for the isothermal flow of a viscous incompressible fluid, the dependence of the moments of friction forces, the average values of the kinetic energy of flows and fluctuations of the latter on the noise amplitude is studied numerically. The response of flows developing in rotating spherical layers to two types of noise, differing in the slope of the spectrum, is studied. It is shown that for the types of noise used in this work, the ratios of the sizes of spherical boundaries and Reynolds numbers, the dependences of the relative values of each of the above parameters on the noise amplitude are described by qualitatively similar exponential functions.

1. Calculation method and study area

Isothermal flows of a viscous incompressible fluid in a rotating spherical layer are described by the Navier–Stokes and continuity equations, which in this case have the form

$$\frac{\partial \mathbf{U}}{\partial t} = \mathbf{U} \times \operatorname{rot} \mathbf{U} - \operatorname{grad}\left(\frac{p}{\rho} + \frac{U^2}{2}\right) - \nu \operatorname{rot} \operatorname{rot} \mathbf{U},$$

$$\operatorname{div} \mathbf{U} = 0.$$

Here $\mathbf{U}$ — velocity field, p — pressure, both parameters are functions of time and coordinates, ρ — density, ν — kinematic viscosity of the liquid in the layer. The boundary conditions for no-slip and impermeability in a spherical coordinate system with radial (r) direction, polar (θ) one and (φ) one azimuthal have the form

$$u_\varphi(r = r_k) = \Omega_k(t) r_k \sin\theta, \quad u_r(r = r_k) = 0,$$

$$u_\theta(r = r_k) = 0, \quad k = 1, 2,$$

where u_φ, $u_r u_\theta$ — azimuthal, radial and polar components of velocity, respectively, Ω_1 and Ω_2 — angular velocities of rotation, and r_1 and r_2 — radii of the inner and outer spheres, respectively (index 1 refers to the internal sphere, 2 — to the external). The numerical solution method is based on a conservative finite-difference scheme for discretizing the Navie–Stokes equations in space and a semi-implicit Runge–Kutta scheme of 3rd order accuracy for integration over time [17]. The time integration scheme includes an estimate of the local error and automatic selection of the time step. When discretized over space, some important properties of the Navier–Stokes equations are preserved, including the exact invariance of law of conservation of energy by nonlinear and pressure gradient terms [17]. The algorithm and features of the finite-difference schemes used in the calculations were studied in detail in the work [17], and in the particular case of a three-dimensional problem in a spherical coordinate system using non-uniform θ and r mesh models — in work [18].

A system of equations was solved that described unsteady flows formed under the influence of rotation of boundaries with equal average angular velocities Ω_0 symmetrical with respect to the equatorial plane and the axis of rotation. Two configurations of a spherical layer with relative layer thicknesses $\delta = 1$ and 1.76 were considered,

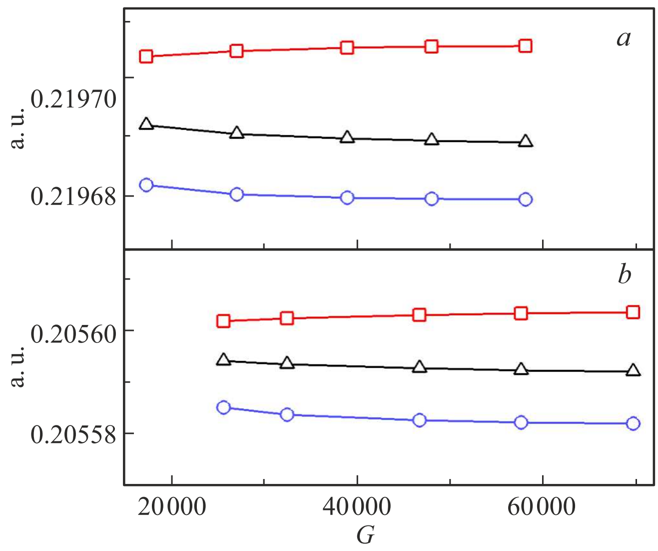

Figure 1. Dependence of the normalized quantities E_φ (red square symbols, vertical axis on the left),T (black triangular symbols) and E_ψ (blue round symbols) on the number of computational grid nodes G at $\mathrm{Re} = 4000$; a — $\delta = 1$, b — $\delta = 1.76$.

where $\delta = (r_2 - r_1)/r_1$. The $\delta = 1$ layer allows comparison with experimental results. For example, in [11,13] a good agreement was obtained between the calculated and experimental results regarding the increase in u_φ with increasing amplitude of white noise. Layer $\delta = 1.76$ is closer to currently known data on the internal structure of the Earth. In the calculations, uniform in θ and non-uniform in r meshes were used with the ratio of the smallest mesh size (near spherical boundaries) to the largest size equal to 0.5. Parametric study was carried out aimed at selecting the number of nodes of the computational mesh at different δ. Precision of results as the number of nodes increases is shown in Fig. 1 using the example of normalized values of the kinetic energy of the flow and the moment of friction forces transmitted to the outer sphere. The kinetic energy of currents is presented as the sum of the azimuthal E_φ and meridional E_ψ flows determined by integrating the corresponding components of the velocity component over the entire volume of the spherical layer:

$$E_\varphi = \int u_\varphi^2(r, \theta, t), \quad E_\psi = \int \left(u_r^2(r, \theta, t) + u_\theta^2(r, \theta, t)\right).$$

Further, the components of kinetic energy in the absence of additional noise will be denoted as $E_{\varphi 0}$ and $E_{\phi 0}$, but and with the noise dithered — $E_{\varphi n}$ and $E_{\psi n}$. For the type of flow under consideration, the condition$E_\psi \ll E_\varphi$ is satisfied. The moment of friction forces M is determined as [19]:

$$M = \nu r_2^3 \int_0^{2\pi} \int_0^{\pi} \left\{ \frac{\partial \nu_\varphi}{\partial r} - \frac{\nu_\varphi}{r} \right\} \sin^2\theta d\theta d\varphi.$$

The values shown in Fig. 1 represent the kinetic energy components normalized to the square of the maximum flow velocity: $E_\varphi^{norm} = E_{\varphi 0}/(\Omega_0 r_2)^2$, $E_\psi^{norm} = E_{\psi 0}/(\Omega_0 r_2)^2$, as well as dimensionless frictional moment $M^{norm} = M/(\nu r_2^3 \Omega_0)$. In accordance with the data presented in Fig. 1, in the case $\delta = 1$ of the total number of nodes $4.8 \cdot 10^4$ selected, and in the case $\delta = 1.76$ of all number of nodes — $5.76 \cdot 10^4$ is selected. The time step Δt was chosen constant in all cases: $\Delta t = 6 \cdot 10^{-4}$ s, which provided 1178 and 2356 time steps per revolution of the spheres. Additional noise was introduced at each time step in the form of random fluctuations in the rotation speed of the inner sphere: $\Omega(t) = \Omega_0 + N\,rn(j)$. Here N noise amplitude and $rn(j)$ is pseudo-random number, the value of which changes at each time step. These numbers were selected one after another from a sequence calculated in advance for the selected type of spectrum with a standard normal distribution and a average zero-mean value. The number corresponded to the time step number. Two types of noise with spectral tilts $\alpha = 0.1$ and 1 were used in the calculations. The first type of noise is close to white, the second type is — well-studied flicker−noise. Methods for obtaining noise with different types of spectrum can be found, for example, in [20]. As in [11,13], the same sequence of the random numbers has been used in all the calculations. The calculations used dimensional parameters corresponding to the experimental conditions [10,11,13]: $\nu = 5 \cdot 10^{-5}\,\mathrm{m^2/s}$, $r_2 = 0.15\,\mathrm{m}$, $r_1 = 0.075\,\mathrm{m}$ for $\delta = 1$ and $r_1 = 0.0544\,\mathrm{m}$ for $\delta = 1.76$. In [13] it is shown that relative flow speed variation under the influence of white noise increase with decreasing Reynolds numbers $\mathrm{Re}_k = \Omega_k r_k^2/\nu$, determined for the inner $(k = 1)$ and outer $(k = 2)$ spheres. Therefore, the calculations were carried out at average angular velocities of rotation of the boundaries $\Omega_0 = 4.4444$ and 8.8888 1/s, which corresponds to relatively small numbers $\mathrm{Re}_2 = 2000$ and 4000, further designated as Re (number of Re_1 is not used in the presentation of the results). The sequence of stages in the numerical experiment is similar to their sequence in the laboratory experiment [13]. First, flows were calculated without introducing noise, the initial conditions for which were a stepwise change in the angular velocities of the spheres from a state of rest to selected values. The moment of completion for calculations without noise was determined by the achievement of stationaryby the achievement of stationary in time of all flow parameters In turn, the results of calculating flows without noise were chosen as the initial conditions for calculations with noise. The duration of one version of the calculation with noise was 1120 s, counted from the moment noise introduce begin into the signal of stationary rotation of the inner sphere, is amounted to 792 and 1584 revolutions of the spheres Time averaging was carried out based on the results of calculations in the last 420 s, which amounted to 297 and 594 revolutions of the spheres. The calculation results are presented below depending on the relative noise amplitude N:

$$\Delta = \sqrt{\frac{1}{I-1}\sum_{i=1}^{I}\left(\Omega(t_i) - \Omega_0\right)^2}, \quad N = \frac{\Delta}{\Omega_0}. \qquad (1)$$

Here I is time sampling length. The noise amplitude N varied from 0.01 to 0.06.

2. Results

The flows of a viscous incompressible fluid considered in this work are caused by the rotation of the boundaries of a spherical layer in the same direction with equal average angular velocities. At the above Re numbers, flows with stationary rotation are stable, and their structure is symmetrical with respect to the equatorial plane and the rotation axis. Both in the absence of additionally introduced noise and in its presence, such flows are not solid-state, since all three velocity components are non-zero ones. As shown first analytically in [21], and later numerically in [22], with stationary rotation of the layer boundaries at high velocities in the flow, a cylindrical Stewartson layer is formed, parallel to the rotation axis and touching at the equator internal sphere, in which the movement of fluid in the meridional plane is concentrated, i.e. meridional circulation. As noted above, in this work the calculations were carried out at low Re numbers, and the Stewartson layer at the boundary between circulations of the opposite direction has not yet been formed. However, in the meridional plane the lines of the stream function Ψ are clearly visible:

$$u_r = \frac{1}{r^2 \sin\theta}\frac{\partial\Psi}{\partial\theta}, \quad \Psi(r = r_1) = 0,$$

almost parallel to the axis of rotation and separating vortices of the opposite direction (Fig. 2). The flow structures are shown at $N = 0$ at different δ they are qualitatively similar in the position of the maxima and the direction of the stream function lines.

When random fluctuations in the rotation speed are introduced, all components of the flow velocity become time incidental. From the boundary conditions presented above it follows that the structure of disturbances transmitted during the flow is symmetrical relative to the equatorial plane,and at each moment of time it does not depend on the azimuthal angle. It is necessary to determine whether at such a structure of perturbations the axial symmetry of the flow is preserved For this purpose, test three-dimensional calculations of one of the flow options were carried out. The degree of asymmetry relative to the axis of rotation was determined by the following quantity [17]:

$$A_{3D}(t) = \frac{1}{V}\sum_{\beta}\int_{V}\left(u_\beta(r_i, \theta_j, \varphi_k, t) - u^{\varphi}_{\beta av}(r_i, \theta_j, t)\right)^2 d\nu_{ijk},$$

$$\beta = r, \theta, \varphi,$$

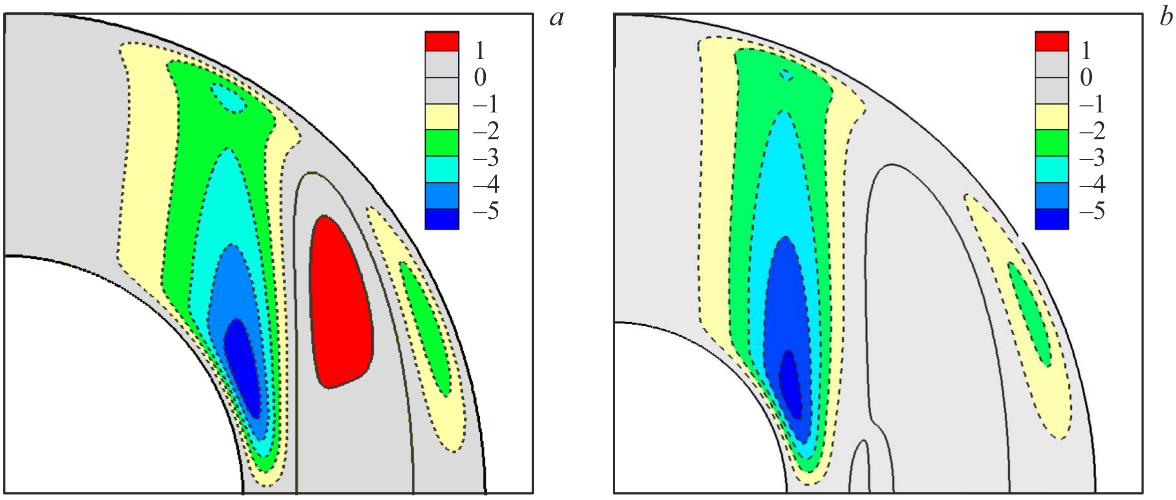

Figure 2. Levels of the stream function Ψ ($10^{-10}\,\mathrm{m}^3 \cdot \mathrm{s}^{-1}$) in meridional plane of an axisymmetric stationary flow (the dotted line indicates negative level values); *a* — $\delta = 1$, $\mathrm{Re}_1 = 1000$, $\mathrm{Re}_2 = 4000$; *b* — $\delta = 1.76$, $\mathrm{Re}_1 = 525$, $\mathrm{Re}_2 = 4000$.

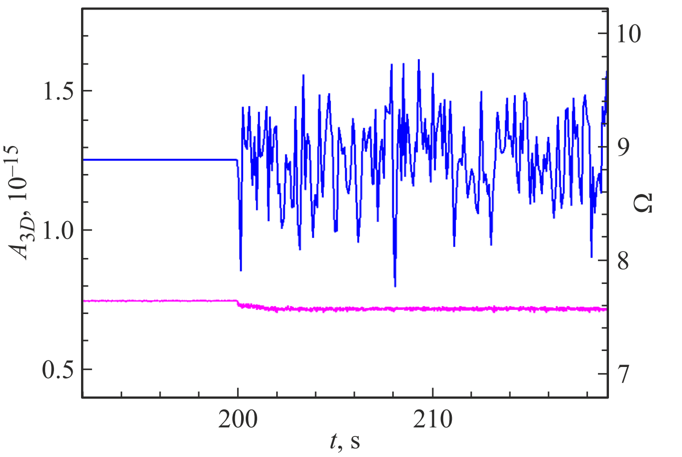

Figure 3. Results of three-dimensional calculations with $\mathrm{Re} = 4000$, $\delta = 1$, $N = 0.04$, number of computational mesh nodes $G = 1.152 \cdot 10^6$. Dependences of $\Omega_1(t)$ (top, vertical axis on the right) and $A_{3D}(t)$ (bottom, vertical axis on the left) are presented as a function of time t.

$$u^{\varphi}_{\beta av}(r_i, \theta_j, t) = \frac{1}{K}\sum_{1}^{K} u_\beta(r_i, \theta_j, \varphi_k, t).$$

Here $u_\beta(r_i, \theta_j, \varphi_k, t)$ and $u^{\varphi}_{\beta av}(r_i, \theta_j, t)$ — one of the velocity components and its value averaged over the angle φ, K — the number of nodes in the azimuthal direction, V — the volume of the spherical layer. If the flow is symmetrical relative to the axis of rotation, then the value of $A_{3D}(t)$ is close to zero.

If the flow is asymmetrical relative to the axis of rotation, for example, after loss of stability by the flow caused by the rotation of only the inner sphere, then the value of $A_{3D}(t)$ is in the range $10^{-4} < A_{3D}(t) < 10^{-3}$ [17]. Fig 3 shows the value of $A_{3D}(t)$ for the flow considered here in the absence and presence of random fluctuations in the rotation speed ($N = 0.04$). With the used dimensions of the computational grid and time step in the case of stationary rotation of the spheres $A_{3D}(t) < 10^{-15}$, which proves the axisymmetricity of such a flow. When additional noise is introduced, the average value of this quantity practically does not change, which indicates the preservation of the symmetry of the flow relative to the axis of rotation. Thus, the assumption adopted in the present and earlier works [11,13] that the types of flows under consideration are axisymmetric in the presence of low-amplitude noise is completely justified.

Fig. 4 shows the spectra of the angular velocity of rotation and the components of the kinetic energy of the flow at the same numbers δ, Re and N, but at different α. The types of spectra $E_{\psi n}$ and $E_{\varphi n}$are significantly different: the former are practically independent of the magnitude of α and in the frequency range less than 1 Hz represent white noise, while the slope angle of the $E_{\varphi n}\alpha_\varphi$ spectrum depends on α: at I$\alpha = 0.1$ $\alpha_\varphi = 1.12$ and at $\alpha = 1$ $\alpha_\varphi = 1.97$. Thus, the slope of the spectra $E_{\varphi n}$ is greater than the slope of the spectrum of noise supplied to the flow from outside. At $\delta = 1$ $(\alpha_\varphi - \alpha) \sim 1$, and this relationship is preserved for all numbers Re and values N used in our work. It should be noted that the issues of transformation of the noise spectrum as it propagates in various media have been studied for a long time [23]. In our calculations, indicated attenuation of the spectrum amplitude $E_{\psi n}$ at high frequencies corresponds to the results [24], in which energy transfer from high frequencies to low frequencies was observed during rotational oscillations with two frequencies in a spherical layer, as well as a more rapid attenuation of the oscillation amplitude at high frequencies compared to low frequencies as you move away from the noise source.

It was discovered that under the influence of noise, not only the kinetic energy of flows can increase [10,11], but also the moment of friction forces transmitted to the boundaries of the layer can change. Let us consider

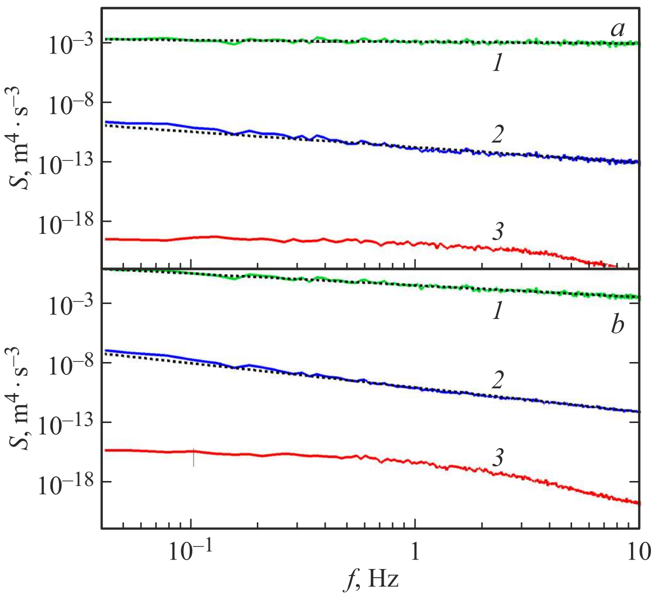

Figure 4. Spectra of the angular velocity of rotation (*1*), $E_{\varphi n}$ (*2*) and $E_{\psi n}$ (*3*); *a* — $\alpha = 0.1$, *b* — $\alpha = 1$, $N = 0.02$, $\delta = 1$, $\mathrm{Re}_1 = 1000$, $\mathrm{Re}_2 = 4000$. The dot-dash lines show the approximation of the slope of the spectra.

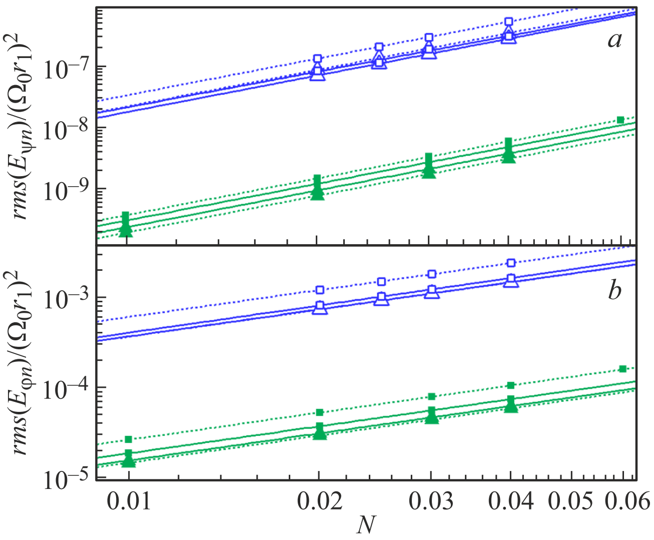

Figure 6. Normalized values of root-mean-square deviations $E_{\psi n}$ (*a*) and $E_{\varphi n}$ (*b*) depending up N. Light (blue) symbols are $\alpha = 1$, dark (green) symbols are $\alpha = 0.1$, square symbols are $\delta = 1$, triangular ones are $\delta = 1.76$, solid lines are dependency approximation changes $rms(E_{\psi n})$ and $rms(E_{\varphi n})$ up N at $\mathrm{Re}_2 = 4000$, broken lines are at $\mathrm{Re}_2 = 2000$.

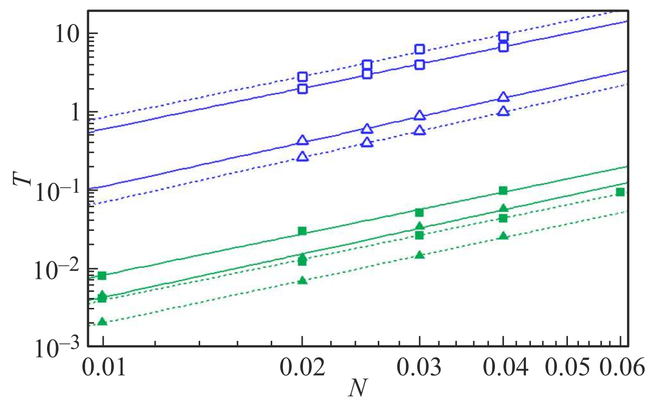

Figure 5. Dependence of time-averaged values T from N: light (blue) symbols — $\alpha = 1$, dark (green) symbols — $\alpha = 0.1$, square symbols — $\delta = 1$, triangular — $\delta = 1.76$, solid lines — approximation of the dependence of T from N at $\mathrm{Re}_2 = 4000$, dashed lines — at $\mathrm{Re}_2 = 2000$.

the magnitude of the relative change in the moment of friction forces T, transmitted to the outer sphere $T = (M_n - M_0)/M_0$, where M_0 and M_n are respectively, the moments of friction forces without noise and with noise (Fig. 5). Regardless of the values of spectrum slope α, relative layer thickness δ and numbers Re considered in this work, all the results obtained can be presented in the form $T \sim N^{\tau}$, where $\tau = 1.81 \pm 0.06$. At the same noise amplitudes N, the values T corresponding to $\alpha = 1$ (light symbols in Fig. 5) are two orders of magnitude higher than the values Tcorresponding to $\alpha = 0.1$(dark symbols in Fig. 5). At the same time, for the same noise amplitudes N, the values T strongly depend up the layer thickness δ and very weakly depend up on the Re number.

In the case when the spectrum of random fluctuations corresponds to the spectrum of white noise, an increase in its intensity leads to an increase in the average values of kinetic energy in flows caused by the rotation of both the internal sphere and the codirectional rotation of the spherical boundaries [11,13]. In this case, the increase in the average values $E_{\varphi n}$ could be accompanied by both an increase and a decrease in root-mean-square deviations $E_{\varphi n}$. The normalized root-mean-square deviations $E_{\varphi n}$ and $E_{\psi n}$ are determined as follows:

$$\frac{rms(E_{\varphi n})}{(\Omega_0 r_1)^2} = \frac{1}{(\Omega_0 r_1)^2}\sqrt{\frac{1}{I-1}\sum_{i=1}^{I}\left(E_{\varphi n}(t_i) - E_{\varphi n\,av}\right)^2},$$

$$\frac{rms(E_{\psi n})}{(\Omega_0 r_1)^2} = \frac{1}{(\Omega_0 r_1)^2}\sqrt{\frac{1}{I-1}\sum_{i=1}^{I}\left(E_{\psi n}(t_i) - E_{\psi n\,av}\right)^2}.$$

Here $E_{\varphi n}(t_i), E_{\psi n}(t_i)$ are instantaneous values, and $E_{\varphi n_{av}}, E_{\psi n_{av}}$ are time-averaged values of the corresponding components of kinetic energy. The calculation results show that the dependence of the quantities $rms(E_{\psi n})/(\Omega_0 r_1)^2$ (Fig. 6,*a*) and $rms(E_{\varphi n})/(\Omega_0 r_1)^2$ (Fig. 6,*b*) up the noise amplitude N for all considered values α, δ and Re can be represented in the form

$$rms(E_{\psi n})/(\Omega_0 r_1)^2 \sim N^{\beta}, \quad \beta = 2 \pm 0.002,$$

$$rms(E_{\varphi n})/(\Omega_0 r_1)^2 \sim N^{\gamma}, \quad \gamma = 1 \pm 0.006.$$

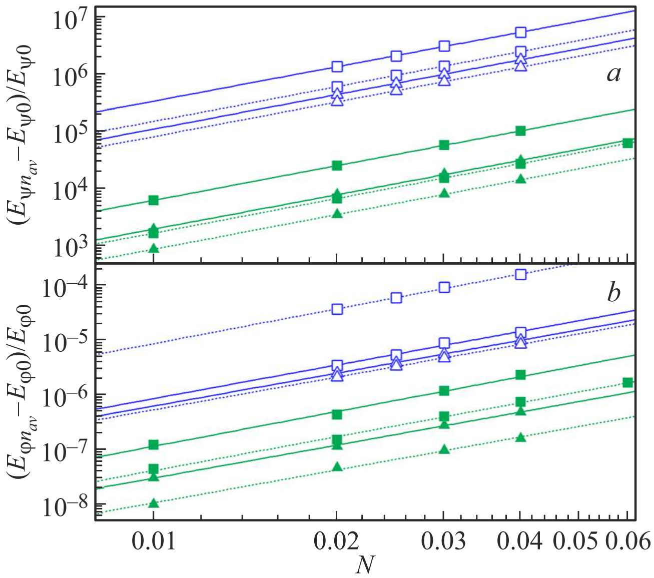

Figure 7. Relative change in the time-averaged values of the meridional (a) and azimuthal (b) components of the kinetic energy of flows with increasing N. Light (blue) symbols are $\alpha = 1$, dark (green) symbols are $\alpha = 0.1$, square symbols are $\delta = 1$, triangular ones are $\delta = 1.76$, solid lines are dependency approximation of the dependences of the relative change in the components of kinetic energy up N at $\mathrm{Re}_2 = 4000$, dashed lines — at $\mathrm{Re}_2 = 2000$.

It should be noted that for the same noise amplitudes and values α, δ and Re, $rms(E_{\varphi n})/(\Omega_0 r_1)^2 \gg rms(E_{\psi n})/(\Omega_0 r_1)^2$, the differences are at least three orders of magnitude. For the same values of α, the results weakly depend up δ (always more in the case of $\delta = 1$). At $\delta = 1.76$ the results independent up the numbers Re. At the same noise amplitudes, the normalized root-mean-square deviations of both components of the kinetic energy of the flow in the case of $\alpha = 1$ (light symbols in Fig. 6), other things being equal, are more than an order of magnitude higher than the similar values obtained in the case of $\alpha = 0.1$ (dark symbols in Fig. 6). Qualitatively similar results are observed for the dependences describing the relative change in the time-averaged values of the meridional $(E_{\psi n_{av}} - E_{\psi 0})/E_{\psi 0}$ (Fig. 7, a) and azimuthal $(E_{\varphi n_{av}} - E_{\varphi 0})/E_{\varphi 0}$ (Fig. 7, b) component of the kinetic energy of the flow from the noise amplitude N. As well as for the flow parameters considered above (Figs. 5, 6), so as the values of $(E_{\psi n_{av}} - E_{\psi 0})/E_{\psi 0}$ and $(E_{\varphi n_{av}} - E_{\varphi 0})/E_{\varphi 0}$ when increasing noise amplitude, they strongly depend up α: in the case of $\alpha = 1$ (light symbols in Fig. 7), the results are 2 orders of magnitude or more higher than the results obtained with $\alpha = 0.1$ (dark symbols in Fig. 7). As well as the normalized root-mean-square deviations (Fig. 6), both $(E_{\psi n_{av}} - E_{\psi 0})/E_{\psi 0}$ and $(E_{\varphi n_{av}} - E_{\varphi 0})/E_{\varphi 0}$ at the same noise level are slightly higher for $\delta = 1$ than for $\delta = 1.76$. The dependences of both $(E_{\psi n_{av}} - E_{\psi 0})/E_{\psi 0}$ and $(E_{\varphi n_{av}} - E_{\varphi 0})/E_{\varphi 0}$up the noise amplitude for the considered values of α, δ and Re are qualitatively similar (Fig. 7):

$$(E_{\psi n_{av}} - E_{\psi 0})/E_{\psi 0} \sim N^{\lambda}, \quad \lambda = 2 \pm 0.001,$$

$$(E_{\varphi n_{av}} - E_{\varphi 0})/E_{\varphi 0} \sim N^{\mu}, \quad \mu = 2.05 \pm 0.05,$$

but in absolute value, the relative change in the meridional component $(E_{\psi n_{av}} - E_{\psi 0})/E_{\psi 0}$ under the influence of noise significantly, by several orders of magnitude, exceeds the relative change in the azimuthal component $(E_{\varphi n_{av}} - E_{\varphi 0})/E_{\varphi 0}$. The same result was observed earlier in [14] in the case of using white noise with $\alpha = 0$.

The presented results, i.e. quadratic dependence on the noise level regardless of the type of its spectrum, higher growth of all components at $\delta = 1$, significant differences in the change in the meridional and azimuthal components of kinetic energy, can be obtained from the following approximate slightly expanded compared to the [13] model. For the flows parameters under consideration ($\mathrm{Re} = 4000$, $\delta = 1$) without adding additional noise, the ratio$E_{\psi 0}/E_{\varphi 0}$ is of the order 10^{-12}, which suggests that the ratio of the average values of the meridional and azimuthal velocities is equal to $u_\psi/u_\varphi = \varepsilon = 10^{-6}$. The azimuthal component of the kinetic energy can be represented in the form $E_{\varphi 0} \sim (\Omega_0 L)^2$, where L is linear layer size, $L = r_2 - r_1$. Then the meridional component can be defined as $E_{\psi 0} \sim (\varepsilon \Omega_0 L)^2$. Both components of the kinetic energy of flows increase under the influence of noise, and this increase can be approximately expressed using the value defined in section Δ (1) $E_{\varphi n\, av} \sim (\Omega_0 L + F\Delta L)^2$, $E_{\psi n\, av} \sim (\varepsilon \Omega_0 L + F\Delta L)^2$, where F is the coefficient that takes into account the type of spectrum of the input noise; for simplicity, we do not take into account the remaining coefficients. Then

$$(E_{\varphi n_{av}} - E_{\varphi 0})/E_{\varphi 0} = (E_{\varphi n_{av}}/E_{\varphi 0}) - 1$$

$$= 2F\Delta/\Omega_0 + F^2\Delta^2/\Omega_0^2 = 2FN + F^2N^2,$$

$$(E_{\psi n_{av}} - E_{\psi 0})/E_{\psi 0} = (E_{\psi n_{av}}/E_{\psi 0}) - 1 = \big((\varepsilon^2\Omega_0^2 + 2F\varepsilon\Omega_0\Delta$$

$$+ F^2\Delta^2)/(\varepsilon^2\Omega_0^2)\big) - 1 = 2FN/\varepsilon + F^2N^2/\varepsilon^2.$$

Since $\varepsilon \ll N$, from this model we get a faster growth of the meridional component under the influence of noise compared to the azimuthal one For the same noise amplitudes N and other equal parameters, the condition $(E_{\psi n_{av}} - E_{\psi 0})/E_{\psi 0} \gg (E_{\varphi n_{av}} - E_{\varphi 0})/E_{\varphi 0}$ is always satisfied.

In both cases, there is a quadratic dependence on the noise amplitude, regardless of the type of its spectrum (regardless of the value F). Changing in the value of F at the same N leads to a changing in both components of kinetic energy, but the qualitative form of the dependence up Ndoes not change. On the other hand, an increase in F at the same values of the noise amplitude N leads to an increase in both components of kinetic energy. Thus, the simplified analytical model considered above corresponds to the numerically obtained patterns.

The results of the numerical study on the influence of the type of spectrum of additionally introduced noise on changes in the properties of viscous incompressible fluid flows are quite difficult to compare with experiment due to the difficulties in obtaining experimental date at $N = 0$. Indirect confirmation of the obtained numerical data can be the results of experiments [25]. According to these results, a reduction in the low-frequency part of the noise spectrum of the section with $\alpha = 0$ (i.e., removal of the spectrum type from white noise) at the same noise amplitudes leads to an increase in the time-averaged values of both flow velocity fluctuations and average values flow speed. Qualitatively, the experimental results completely correspond to the numerical results: with increasing spectrum slope, i.e., as the spectrum type moves away from white noise, at the same noise amplitudes, a higher relative increase in the averaged values of flow parameters is observed.

Conclusion

The results of a numerical study of the influence of noise in the form of time incidental broadband fluctuations of the rotation speed with zero average value, added to a constant average rotation speed, on changes in the properties of viscous incompressible fluid flows in a spherical layer are presented. Calculations were carried out for two relative layer thicknesses $\delta = 1$ and 1.76 at Reynolds numbers calculated from the parameters of the outer boundary of the layer $\mathrm{Re} = 2000$ and 4000. Two types of noise with different spectrum slopes $1/f^{\alpha}$ were considered: $\alpha = 0.1$ and 1. For all noise amplitudes N and Re numbers, the using of the slope of the spectrum for the azimuthal component of the kinetic energy of the flow is per unit less than the slope of the spectrum of input noise, while the low-frequency part of the spectrum of the meridional component of the kinetic energy is white noise. It is shown that both when using white noise ($\alpha = 0$) [13] and with $\alpha = 0.1$ and 1, an increase in the noise amplitude N leads to an increase in the time-averaged values of the azimuthal and meridional components of the kinetic energy of flows. It has been established that for all types of noise spectrum used in the work, ratios of the sizes of spherical boundaries and Reynolds numbers, for the same noise amplitudes, the relative increase in the meridional component of the kinetic energy of the flow is several orders of magnitude higher than the relative increase in the azimuthal component. Previously, the same relationship between the increase in various components of kinetic energy was discovered in [11,13] in the case of using white noise ($\alpha = 0$). The opposite relationship is observed for the root-mean-square deviations of the kinetic energy components. In the case of using in the work the types of noise spectrum, ratios of the sizes of spherical boundaries and Reynolds numbers, the increase in the root-mean-square deviations of the azimuthal component of kinetic energy is several orders of magnitude higher than the increase in the root-mean-square deviations of the meridional component at the same noise amplitudes.

It is shown that the relative values of such flow parameters as moments of friction forces on the outer sphere, fluctuations and average values of the azimuthal and meridional components of kinetic energy depend on the noise amplitude N as power functions N^n (n — this is one of theexponential factor: τ, γ, λ or μ). All values of n are in the range from 1 to 2, and the resulting scaling is independent of the δ, Reynolds numbers, and α numbers used in the work. It is shown that the response of flows to noise effect is determined by the type of spectrum of this noise: for the same noise amplitudes, the relative increase in flow parameters at $\alpha = 1$ is several orders higher of magnitude than at $\alpha = 0.1$. At the same time with the self-same N the influence of the type of noise spectrum on the relative change in all considered flow parameters is much stronger than the influence of the numbers δ and Re. Considered in the work, a simplified analytical model qualitatively corresponds to the obtained numerical results, and, according to this model, the type of spectrum of noise supplied to the flow does not affect the qualitative form of the dependence of the change in the components of the kinetic energy of flows. The dependences of changes in the kinetic energy of flows and their root-mean-square deviations from the noise level obtained in the work retain their qualitative form for any physically realizable values of the slope of the noise spectrum α. It can be assumed that the numerical results obtained are of a fairly universal nature. Firstly, the structure of the meridional circulation remains unchanged (Fig. 2), so the obtained dependencies will be valid for any values δ that are in the range of $1 < \delta < 1.76$. Secondly, it can be assumed that with physically realized slopes of the noise spectrum $0 \leq \alpha \leq 1$, the dependences of the considered flow parameters on the noise amplitude are preserved (Figs. 5–7): spectra E_{φ} retain a constant slope; the spectra of E_{ψ} are similar to those of white noise at low frequencies for $\alpha = 0$ [13], $\alpha = 0.1$ and $\alpha = 1$. The main result of the work is that for the same noise amplitudes, an increase in α is accompanied by an increase in the time-averaged relative values of the moment of friction forces, kinetic energy and its fluctuation

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Conflict of interest

The authors declare that they have no conflict of interest.

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