01

Cherenkov radiation of a fast vortex from the surface of a waveguide coupled to a Josephson junction

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Received June 27, 2023
Revised October 21, 2023
Accepted December 21, 2023

A system of equations for phase differences of the superconducting order parameter describing the coupled Josephson junction, waveguide, and semi-infinite dielectric medium is obtained. For such a layered structure, the Cherenkov radiation of a fast vortex emitted from the surface of the superconducting sandwich into the dielectric is described. In such a structure, the fast vortex moves with velocity close to the Swihart velocity of the waveguide, which can significantly exceed the velocity of an ordinary Josephson vortex. The characteristic emission frequencies and radiation losses of the fast vortex are determined.

Keywords: Cherenkov radiation, fast vortex, Josephson junction.

DOI: 10.21883/0000000000

Introduction

Over the past few decades, the search for terahertz radiation sources has been carried out in connection with the prospects for their use to solve a number of problems in fundamental and applied areas of research. One of the possibilities for creating compact sources of terahertz radiation involves the use of the Josephson effect. An important advantage of such sources is the continuity and tunability of terahertz radiation, which was previously demonstrated using high-temperature superconductors BSCCO [1–3]. Radiation generated by systems with Josephson interaction is usually detected near the end of an long Josephson junction or a stack of internal junctions, or is output to a strip line [2,4–7].

One of the sources of terahertz radiation is traveling Josephson vortices, emitting electromagnetic waves due to the Vavilov-Cherenkov effect [8–11]. It has been shown that Cherenkov radiation is possible from the entire area of the lateral surface of the layered structure [12,13]. Cherenkov radiation of a vortex into an external dielectric is possible if its velocity exceeds c_m , i.e. the speed of light in the dielectric. At the same time, the velocity of the vortex is limited by the value v_S of the Swihart velocity of the Josephson junction.

Experimentally realize conditions in which $v_S > c_m$ is wright is not easy. This limitation can be bypassed if the Josephson junction is connected to a waveguide. In such a layered structure, the propagation of a fast vortex [14] is possible, the velocity of which is limited by a velocity greater than the Swihart velocity in the waveguide V_{Sw}. At the same time, V_{Sw} can significantly exceed v_S. In connection with this, it is of interest to consider the radiation of a Josephson vortex traveling along a Josephson junction magnetically coupled to a plane waveguide into an external dielectric medium.

The text is organized as follows. In Section 1, the basic relations describing the fields in the layered structure under consideration are obtained, and the derivation of integro-differential equations for the phase differences of the superconducting order parameter on the Josephson junction and waveguide is given. Section 2 considers the situation when the field outside the sandwich is determined by a wave going deep into the outer dielectric. This made it possible to write a simpler system of differential equations for phase differences on the Josephson junction and waveguide, and to determine the velocity domains of the Josephson and fast vortices. In Section 3, the Poynting vector was found to describe the radiation of a fast vortex. An analysis is given of the dependence of the losses of such a vortex on its velocity. The Summary provides the main findings.

1. Basic electrodynamic relations

Let us consider a superconducting sandwich occupying the semi-infinite domain $x < x_s \equiv d + L_1 + 2d_w + L_2$ (Fig. 1). Inside the sandwich there is a Josephson junction occupying the $-d < x < d$ domain and a planar waveguide occupying the $d + L_1 < d + L_1 + 2d_w$ domain. To the right of the sandwich is a decelerating medium with dielectric permittivity ϵ_m.

Let us study the motion of a fast vortex in such a sandwich. The magnetic field of the vortex has one component $\mathbf{H} = (0, H, 0)$, and the electric field has two components $\mathbf{E} = (E_x, 0, E_z)$. To describe the fields in such a layered system, it is necessary to study the fields in each layer. When considering fields, we will use the

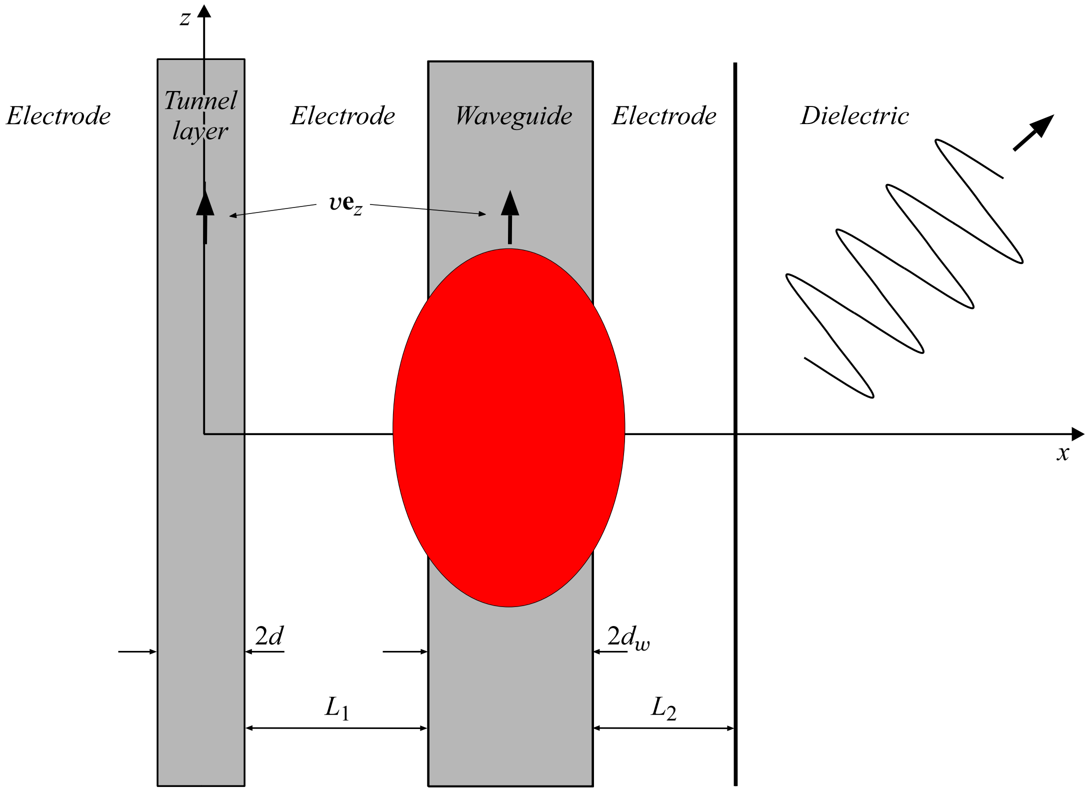

Figure 1. Schematic representation of a section of Josephson sandwich magnetically coupled to a waveguide in the xOz plane. The ellipse shows the region in which the magnetic field is mainly localized, considered below the fast vortex. The vortex moves with the velocity $v\mathbf{e}_z$. To the right of the sandwich, an electromagnetic wave is shown going deep into the outer dielectric, occupying the domain $x > x_s$

Fourier transform in time and coordinate z (it will use the designations ω and k, respectively). The Fourier transform of the magnetic field in the external dielectric is found from the wave equation $\Delta H(\epsilon_m/c^2)\partial^2 H/\partial t^2 = 0$:

$$H(x, k, \omega) = H_s(k, \omega)\exp[-\kappa(x - x_s)], \quad x > x_s, \quad (1)$$

where $H_s(k, \omega) \equiv H(x_s, k, \omega)$, $\kappa \equiv \sqrt{|k^2 - \omega^2/c_m^2|} \times \times[\eta(c_m^2k^2 - \omega^2) - i\,\mathrm{sgn}\,\omega\eta(\omega^2 - c_m^2k^2)]$, $c_m \equiv c/\sqrt{\epsilon_m}$, c is speed of light, $\eta(x)$ is the Heaviside function. The sign of the imaginary part κ corresponds to the wave moving away from the sandwich. Relation (1) and the $\mathrm{rot}\mathbf{H} = (\epsilon_m/c)\partial\mathbf{E}/\partial t$ equation allow us to write the Fourier transform of the z-component of the electric field at $x = x_s + 0$ in the following form:

$$E_z(x_s + 0, k, \omega) = -i\frac{c\kappa}{\epsilon_m\omega}H_s(k, \omega). \quad (2)$$

From the London's equation $\Delta H = \lambda^{-2}H$ we find the Fourier transforms of the magnetic field in superconducting electrodes occupying the domains $d + L_1 + 2d_w < x < x_s$, $d < x < d + L_1$ and $x < -d$, respectively:

$$H(x, k, \omega) = \frac{1}{\sinh[L_2/\lambda(k)]}\left\{-H_w(k, \omega)\sinh\left[\frac{x - x_s}{\lambda(k)}\right] + H_s(k, \omega)\sinh\left[\frac{x - d - L_1 - 2d_w}{\lambda(k)}\right]\right\},$$

$$d + L_1 + 2d_w < x < x_s, \quad (3)$$

$$H(x, k, \omega) = \frac{1}{\sinh[L_1/\lambda(k)]}\left\{-H_J(k, \omega)\sinh\left[\frac{x - d - L_1}{\lambda(k)}\right] + H_w(k, \omega)\sinh\left[\frac{x - d}{\lambda(k)}\right]\right\},$$

$$d < x < d + L_1, \quad (4)$$

$$H(x, k, \omega) = H_J(k, \omega)\exp[(x + d)/\lambda(k)], \quad x < -d, \quad (5)$$

where $H_J(k, \omega)$ and $H_w(k, \omega)$ is the Fourier transforms of magnetic fields inside the Josephson junction and waveguide, respectively, $\lambda(k) \equiv \lambda/\sqrt{1 + \lambda^2k^2}$, λ is the London penetration depth of magnetic field into the superconducting electrodes. We neglect the dependence of the magnetic fields inside the Josephson junction and the waveguide on the x coordinate, considering $2d$ and $2d_w$ small compared to the characteristic scales of variation of these fields along the Oz axis.

Using equation $\Delta\mathbf{E} = (\lambda^2/c)\mathrm{rot}\,\partial\mathbf{H}/\partial t$ and relations (3)–(5), we find the Fourier transforms of the z-component of the electric field at the boundaries of superconductors:

$$E_z(x_s-0,k,\omega)=i\frac{\omega\lambda^2}{c\lambda(k)}\times\frac{H_w(k,\omega)-H_s(k,\omega)\cosh[L_2/\lambda(k)]}{\sinh[L_2/\lambda(k)]}, \quad (6)$$

$$E_z(d+L_1+2d_w+0,k,\omega)=i\frac{\lambda^2\omega}{c\lambda(k)}\times\frac{H_w(k,\omega)\cosh[L_2/\lambda(k)]-H_s(k,\omega)}{\sinh[L_2/\lambda(k)]}, \quad (7)$$

$$E_z(d+L_1-0,k,\omega)=i\frac{\lambda^2\omega}{c\lambda(k)}\times\frac{H_J(k,\omega)-H_w(k,\omega)\cosh[L_1/\lambda(k)]}{\sinh[L_1/\lambda(k)]}, \quad (8)$$

$$E_z(d+0,k,\omega)=i\frac{\lambda^2\omega}{c\lambda(k)}\times\frac{H_J(k,\omega)\cosh[L_1/\lambda(k)]-H_w(k,\omega)}{\sinh[L_1/\lambda(k)]}, \quad (9)$$

$$E_z(-d-0,k,\omega)=-i\frac{\lambda^2\omega}{c\lambda(k)}H_J(k,\omega). \quad (10)$$

From the condition of continuity of $E_z(x,k,\omega)$ at $x=x_s$, as well as from relations (2) and (6), we find the connection $H_s(k,\omega)$ and $H_w(k,\omega)$:

$$H_s(k,\omega)=\frac{H_w(k,\omega)}{\cosh[L_2/\lambda(k)]-R\sinh[L_2/\lambda(k)]}, \quad (11)$$

which allows to express $E_z(d+L_1+2d_w+0,k,\omega)$ (7) via $H_w(k,\omega)$:

$$E_z(d+L_1+2d_w+0,k,w)=i\frac{\lambda^2\omega}{c\lambda(k)}\times\frac{H_w(k,\omega)}{\tanh[L_2/\lambda(k)]}\frac{R-\tanh[L_2/\lambda(k)]}{R-\coth[L_2/\lambda(k)]}, \quad (12)$$

where $R\equiv R(k,\omega)\equiv c_m^2\kappa\lambda(k)/\omega^2\lambda^2$.

When considering fields in thin dielectric layers of the Josephson junction and waveguide, we will use Maxwell's equation $\mathrm{rot}\mathbf{E}=-(1/c)\partial\mathbf{H}/\partial t$. Neglecting the small change E_x and H along the axis Ox, from this equation we find the difference in the Fourier transforms z- of the electric field components on opposite sides of the layers:

$$E_z(d+0,k,\omega)-E_z(-d-0,k,\omega)\simeq 2idkE_{J,x}(k,\omega)=\frac{\phi_0}{2\pi c}k\omega\varphi(k,\omega), \quad (13)$$

$$E_z(d+L_1+2d_w+0,k,\omega)-E_z(d+L_1-0,k,\omega)\simeq 2id_wkE_{w,x}(k,\omega)=\frac{\phi_0}{2\pi c}k\omega\varphi_w(k,\omega), \quad (14)$$

where ϕ_0 the magnetic flux quantum. When writing the right-hand sides of these formulas, the Josephson relation for the $E_{J,x}(z,t)=(\phi_0/4\pi cd)\partial\varphi(z,t)/\partial t$ for the tunnel layer and a similar relation for the waveguide were used. These relations connect the x--component of the electric field with the phase differences φ and φ_w of the superconducting order parameter at the junction and waveguide.

Substituting expressions (8) – (10) and (12) into relations (13) and (14), we obtain a system of equations that gives a connection between $H_J(k,\omega)$ and $H_w(k,\omega)$ with $\varphi(k,\omega)$ and $\varphi_w(k,\omega)$:

$$\frac{e^{L_1/\lambda(k)}H_J(k,\omega)-H_w(k,\omega)}{\sinh[L_1/\lambda(k)]}=-\frac{i\phi_0\lambda(k)k\varphi(k,\omega)}{2\pi\lambda^2},$$

$$\frac{H_J(k,\omega)}{\sinh[L_1/\lambda(k)]}-\left\{\coth\left[\frac{L_1}{\lambda(k)}\right]+\frac{\sinh[L_2/\lambda(k)]-R\cosh[L_2/\lambda(k)]}{\cosh[L_2/\lambda(k)]-R\sinh[L_2/\lambda(k)]}\right\}\times H_w(k,\omega)=\frac{i\phi_0\lambda(k)k\varphi_w(k,\omega)}{2\pi\lambda^2}. \quad (15)$$

The determinant of this system is equal to $\lambda(k)D(k,\omega)/\lambda$, where

$$D(k,\omega)\equiv\frac{\lambda}{\lambda(k)}\frac{\exp[L_1/\lambda(k)]\exp[L_2/\lambda(k)]}{\sinh[L_1/\lambda(k)]}\times\frac{1-R}{\cosh[L_2/\lambda(k)]-R\sinh[L_2/\lambda(k)]}, \quad (16)$$

and its solution has the form

$$H_J(k,\omega)=-i\frac{\phi_0k}{2\pi\lambda}[A(k,\omega)\varphi(k,\omega)+B(k,\omega)\varphi_w(k,\omega)], \quad (17)$$

$$H_w(k,\omega)=-i\frac{\phi_0k}{2\pi\lambda}[B(k,\omega)\varphi(k,\omega)+C(k,\omega)\varphi_w(k,\omega)], \quad (18)$$

where

$$A(k,\omega)\equiv\frac{1}{D(k,\omega)}\left\{\coth[L_1/\lambda(k)]+\frac{\sinh[L_2/\lambda(k)]-R\cosh[L_2/\lambda(k)]}{\cosh[L_2/\lambda(k)]-R\sinh[L_2/\lambda(k)]}\right\}, \quad (19)$$

$$B(k,\omega)\equiv\frac{1}{D(k,\omega)}\frac{1}{\sinh[L_1/\lambda(k)]}, \quad (20)$$

$$C(k,\omega)\equiv\frac{1}{D(k,\omega)}\frac{\exp[L_1/\lambda(k)}{\sinh[L_1/\lambda(k)]}. \quad (21)$$

Relations (17) and (18) allow us to write the x-component of the generalized current $-(c/4\pi)\partial H/\partial z$ in the Josephson junction and waveguide in terms of $\varphi(z,t)$ and $\varphi_w(z,t)$, which, in turn, allows us to write the following system of coupled equations for phase differences:

$$\omega_J^2 \sin\varphi(z,t) + \frac{\partial^2\varphi(z,t)}{\partial t^2} = 2v_S^2 \times \frac{\partial}{\partial z}\iint dz'dt'\left[A(z-z',t-t')\frac{\partial\varphi(z',t')}{\partial z'} + B(z-z',t-t')\frac{\partial\varphi_w(z',t')}{\partial z'}\right], \quad (22)$$

$$\frac{\partial^2\varphi_w(z,t)}{\partial t^2} = 2v_{Sw}^2\frac{\partial}{\partial z}\iint dz'dt'\left[B(z-z',t-t')\frac{\partial\varphi(z',t')}{\partial z'} + C(z-z',t-t')\frac{\partial\varphi_w(z',t')}{\partial z'}\right], \quad (23)$$

where $\omega_J \equiv 4\pi\sqrt{c j_c d/\phi_0\epsilon}$ is Josephson plasma frequency, ϵ is dielectric permittivity of the Josephson junction, j_c is critical Josephson current density, $v_S \equiv c\sqrt{d/\epsilon\lambda}$ and $v_{Sw} \equiv c\sqrt{d_w/\epsilon_w\lambda}$ are the Swihart velocities of the Josephson junction and waveguide in the limit of bulk superconducting electrodes, respectively.

2. Fast Josephson vortex

The characteristic spatial size of the change in electromagnetic fields along the axis Oz is the value of $\sim 1/k$. In the future, we will assume that this size is greater than the London penetration depth, i.e. $\lambda k \ll 1$. Let us limit ourselves to considering the conditions when in the external dielectric there is only a wave outgoing from the sandwich. Under these conditions, the quantities κ and R are purely imaginary. We also assume that the outgoing wave is created by a Josephson vortex traveling at a constant velocity $v > c_m$. The last assumption allows us to limit ourselves to the analysis of only those Fourier components for which $\omega = vk$.

For further presentation, it is convenient to write down the quantities (19)−(21), clearly distinguishing the real and imaginary parts in them. Since under the conditions under discussion $R(k,vk) = -ic_m\sqrt{v^2-c_m^2}/v^2\lambda k$, then from (19)−(21) we find

$$A(k,vk) = \frac{\exp[-(L_1+L_2)/\lambda]}{1+|R|^2}\left[\cosh\left(\frac{L_1+L_2}{\lambda}\right) + |R|^2\sinh\left(\frac{L_1+L_2}{\lambda}\right) + \exp\left(-\frac{L_1+L_2}{\lambda}\right)R\right], \quad (24)$$

$$B(k,vk) = \exp\left(-\frac{L_1+L_2}{\lambda}\right) \times \frac{\cosh(L_2/\lambda) + |R|^2\sinh(L_2/\lambda) + \exp(-L_2/\lambda)R}{1+|R|^2}, \quad (25)$$

$$C(k,vk) = \exp\left(-\frac{L_2}{\lambda}\right) \times \frac{\cosh(L_2/\lambda) + |R|^2\sinh(L_2/\lambda) + \exp(-L_2/\lambda)R}{1+|R|^2}. \quad (26)$$

Let's also accept that the velocity of the vortex significantly exceeds the velocity of light in the external dielectric:

$$|R| \gg \coth^{1/2}(L_2/\lambda) > 1. \quad (27)$$

Here and in subsequent inequalities, the value R depends on $1/k$ — the characteristic size of the change in fields inside the sandwich along the axis Oz. Under conditions of small radiation losses (see below), this inequality allows us to consider the vortex under conditions where the influence of field penetration into the external dielectric does not lead to a significant change in the vortex field in the layered structure. Under these conditions, the expressions for $A(k,vk)$, $B(k,vk)$ and $C(k,vk)$can be significantly simplified:

$$A(k,vk) \simeq \exp^{-(L_1+L_2)/\lambda}\left[\sinh\left(\frac{L_1+L_2}{\lambda}\right) - \mathrm{i}\frac{\mathrm{sgn}k}{|R|}\exp^{-(L_1+L_2)/\lambda}\right], \quad (28)$$

$$B(k,vk) \simeq \exp^{-(L_1+L_2)/\lambda}\left[\sinh\left(\frac{L_2}{\lambda}\right) - \mathrm{i}\frac{\mathrm{sgn}k}{|R|}\exp^{-L_2/\lambda}\right], \quad (29)$$

$$C(k,vk) \simeq \exp^{-L_2/\lambda}\left[\sinh\left(\frac{L_2}{\lambda}\right) - \mathrm{i}\frac{\mathrm{sgn}k}{|R|}\exp^{-L_2/\lambda}\right]. \quad (30)$$

Using these simpler expressions, let's write equations (22) and (23) for the phase differences at the Josephson junction and at the waveguide as follows:

$$\omega_J^2\sin\psi + v^2\psi'' = V_S^2\psi'' + SV_S^2\psi_w'' - \frac{\lambda v^2 V_S^2}{c_m\sqrt{v^2-c_m^2}} \times \frac{\exp^{-L_2/\lambda}}{\sinh[(L_1+L_2)/\lambda]}(\exp^{-L_1/\lambda}\psi''' + \psi_w'''), \quad (31)$$

$$v^2\psi_w'' = V_{Sw}^2\psi_w'' + S_w V_{Sw}^2\psi'' - \frac{\lambda v^2 V_{Sw}^2}{c_m\sqrt{v^2-c_m^2}} \times \frac{\exp^{-L_2/\lambda}}{\sinh(L_2/\lambda)}(\exp^{-L_1/\lambda}\psi''' + \psi_w'''), \quad (32)$$

where

$$\psi(z-vt) \equiv \varphi(z,t), \quad \psi_w(z-vt) \equiv \varphi_w(z,t),$$

$$V_S^2 \equiv 2v_s^2\exp^{-(L_1+L_2)/\lambda}\sinh[(L_1+L_2)/\lambda],$$

$$V_{Sw}^2 \equiv 2v_{Sw}^2\exp^{-L_2/\lambda}\sinh(L_2/\lambda),$$

$$S \equiv \sinh(L_2/\lambda)/\sinh[(L_1+L_2)/\lambda] < 1,$$

$$S_w \equiv \exp^{-L_1/\lambda} < 1.$$

In equations (31) and (32), the terms containing the third derivatives of the phase differences clearly depend on c_m — the speed of light in the external dielectric. It's possible to say that these terms describe the inverse influence of the radiation field on the shape of the Josephson vortex. Further, this influence is considered weak and these terms are neglected. Omitting terms with third derivatives, from (32) we find that

$$\psi_w'' = \frac{S_w V_{Sw}^2}{v^2 - V_{Sw}^2}\psi''. \tag{33}$$

In this case, the condition for the applicability of such an approximation has the form

$$\frac{\exp(-L_2/\lambda)}{\sinh(L_2/\lambda)}\frac{v^2}{|v^2 - V_{Sw}^2|}. \tag{34}$$

Next, using relation (33) and considering the terms with third derivatives small, from (31) we have

$$\sin\psi = k_J^{-2}\psi'', \tag{35}$$

where

$$k_J \equiv \omega_J\sqrt{\frac{v^2 - V_{Sw}^2}{(v^2 - v_1^2)(v_2^2 - v^2)}}, \tag{36}$$

$$v_m \equiv \sqrt{\frac{V_{Sw}^2 + V_S^2}{2} + (-1)^m\sqrt{\frac{(V_{Sw}^2 - V_S^2)^2}{4} + SS_w V_S^2 V_{Sw}^2}},$$

$$m = 1, 2. \tag{37}$$

The third derivatives in equation (31) can be neglected, if the inequality is satisfied

$$|R| \gg \frac{\exp[-(L_1 + L_2)/\lambda]}{\sinh[(L_1 + L_2)/\lambda]}\frac{v^2 V_S^2}{|v^2 - v_1^2| \cdot |v_2^2 - v^2|}. \tag{38}$$

Inequalities (27), (34) and (38) ensure that the influence of radiation losses on the moving vortex is small

The solution to equation (35) is 2π, i.e. — kink soliton

$$\psi(\xi) = 4\arctan[\exp(-k_J\xi)], \tag{39}$$

which corresponds to a vortex traveling at a constant velocity v along the Josephson junction. The vortex velocity (39) can lie in two domains: $0 < v < v_1$ and $V_{Sw} < v < v_2$. The sizes of these domains and the width of the forbidden band between them depend on the ratio of the renormalized Swihart velocities V_S and V_{Sw}, as well as on the parameter SS_w, which determines the magnitude of the coupling between the Josephson junction and the waveguide. In the most interesting case, when V_{Sw} significantly exceeds V_S, and the value of SS_w is not too close to unity, we have

$$v_1 \simeq \left(1 - \frac{SS_w}{2}\right)V_S, \quad v_2 \simeq \left(1 + \frac{SS_w}{2}\frac{V_S^2}{V_{Sw}^2}\right)V_{Sw}. \tag{40}$$

It can be seen that the velocity interval $0 < v < v_1$ corresponds to the velocity of existence of an ordinary Josephson vortex. At the same time, the not very wide range of velocities $V_{Sw} < v < v_2$ corresponds to the region in which one can consider a fast vortex, the velocity of which significantly exceeds V_S. Such a vortex in a slightly different layered structure was described earlier [14], but its losses due to Cherenkov radiation have not been studied.

3. Radiation losses of a fast vortex

Derived above formulas for fields make it possible to obtain an expression describing the radiation losses of the layered superconducting structure under consideration. Indeed, with $v > c_m$ and the fulfillment of the condition $|R| \gg \coth(L_2/\lambda)$ somewhat more stringent than (27), it follows from (11) that

$$H_s(k, vk) \simeq -i\frac{H_w(k, vk)}{\sinh(L_2/\lambda)}\frac{\lambda k v^2}{c_m\sqrt{v^2 - c_m^2}}. \tag{41}$$

Under the same conditions, from (18), (29) and (30) we find

$$H_w(k, vk) \simeq -i\frac{\phi_0 k}{2\pi\lambda}\exp^{-L_2/\lambda}\sinh\left(\frac{L_2}{\lambda}\right)[S_w\psi(k) + \psi_w(k)]. \tag{42}$$

Next, taking into account relation (33), from (41) and (42) for the Fourier transform of the magnetic field at the sandwich boundary we have

$$H_s(k, vk) = -\frac{\phi_0\exp(-L_2/\lambda)}{2\pi}\frac{v^2k^2}{c_m\sqrt{v^2 - c_m^2}}\frac{S_w v^2}{v^2 - V_{Sw}^2}\psi(k). \tag{43}$$

From here, taking into account expression (1), after the inverse Fourier transform we obtain the distribution of the magnetic field in the external dielectric

$$H(x, \xi) \simeq \frac{\phi_0\exp(-L_2/\lambda)}{2\pi}\frac{v^2}{c_m\sqrt{v^2 - c_m^2}}$$

$$\times\frac{S_w v^2}{v^2 - V_{Sw}^2}\psi''\left[\xi + \sqrt{\frac{v^2}{c_m^2} - 1}\,(x - x_s)\right], \quad x > x_s, \tag{44}$$

where $\xi = z - vt$. Here, from Maxwell's equation $\mathrm{curl}\mathbf{H} = (\epsilon_m/c)\partial\mathbf{E}/\partial t$ for the electric field components we have

$$E_x(x, \xi) = \frac{c}{\epsilon_m v}H(x, \xi), \quad E_z(x, \xi) = -\frac{\sqrt{v^2 - c_m^2}}{\sqrt{\epsilon_m}v}H(x, \xi),$$

$$x > x_s. \tag{45}$$

These relations allow us to find the Poynting vector $\mathbf{S} \equiv (c/4\pi)[\mathbf{EH}]$ in the external dielectric

$$\mathbf{S} = \frac{\phi_0^2\exp(-2L_2/\lambda)}{16\pi^3}\frac{v^3}{v^2 - c_m^2}\left(\frac{S_w v^2}{v^2 - V_{Sw}^2}\right)^2$$

$$\times\left\{\psi''\left[\xi + \sqrt{\frac{v^2}{c_m^2} - 1}(x - x_s)\right]\right\}^2\left(\sqrt{\frac{v^2}{c_m^2} - 1}\,\mathbf{e_x} + \mathbf{e_z}\right),$$

$$x > x_s. \tag{46}$$

By integrating the x -component (46) over ξ on any plane $x = \mathrm{const} > x_s$, we find the loss of vortex energy due to radiation per unit time per unit length along the axis Oy.

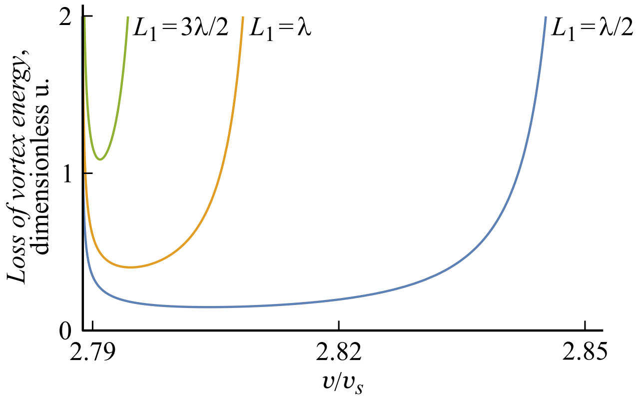

Figure 2. Dimensionless energy losses of a fast vortex due to the electromagnetic radiation. The curves are plotted for $L_2 = \lambda$, $\lambda/\lambda_J = 0.01$, $c_m = 2v_S$, $v_{Sw} = 3v_S$ and for three different values of L_1 — $\lambda/2$, λ and $3\lambda/2$

For a vortex(39) traveling along a Josephson junction, the radiation loss power is given by the expression

$$\frac{\phi_0^2 \exp(-2L_2/\lambda)}{6\pi^3} \frac{v^3 k_J^3}{c_m\sqrt{v^2 - c_m^2}} \left(\frac{S_w v^2}{v^2 - V_{Sw}^2}\right)^2. \quad (47)$$

The dependences of the radiation loss power (47) on the vortex velocity are shown in Fig. 2. In this case, the loss power is dimensionless by the amount $\phi_0^2 v_S/4\pi^3\lambda\lambda_J^2$. This pattern corresponds to fast vortices moving at velocities greater than V_{Sw}. For selected system parameters $V_{Sw} \simeq 2.79v_S$.

From Fig. 2 it can be seen that, with the remaining parameters of the system fixed (see caption to Fig. 2), with increasing thickness L_1 of the „middle" electrode, the losses of the fast vortex to radiation increase. In order to understand the reasons for this behavior of these U-types curves, let's find their minimum. To do this, let us approximately write (47) in a narrow range of velocities of a fast vortex

$$\frac{\phi_0^2\omega_J^3 V_{Sw}^3}{6\pi^3 c_m} \frac{\exp\left[-2(L_1 + L_2)/\lambda\right]}{\sqrt{v^2 - V_{Sw}^2}(v_2^2 - v^2)^{3/2}}. \quad (48)$$

Note that this value has a minimum at $v_{\min} = \sqrt{3V_{Sw}^2 + v_2^2}/2$, equal to

$$\frac{2}{3^{5/2}\pi^3} \frac{\phi_0^2\omega_J^3 V_{Sw}^3}{c_m v_S^4 \sinh^2(L_2/\lambda)} \exp^{2L_1/\lambda} \propto \exp^{2L_1/\lambda}. \quad (49)$$

This exponential dependence of the loss minimum on L_1 is clearly visible in Fig. 2. When moving away from $v_{\min}$ to left and to right „hierarchy" for loss-velocity curves will be preserved.

The reason for the counterintuitive behavior of quantities (48) and (49) with increasing thickness L_1 is as follows. Let the magnitude of the phase difference at the Josephson junction be fixed. Then in the region of fast vortex velocities, but without approaching too close to the velocities V_{Sw} and v_2, in accordance with (33) we have: $\psi_w \propto \exp(L_1/\lambda)\psi$. Therefore, the magnetic field inside the waveguide is $\propto \psi_w'$, and the field on the surface of the sandwich is $\propto \psi_w''$. Accordingly, the radiation losses of the vortex $\propto (\psi_w')^2 \propto \exp(2L_1/\lambda)$. Note that this behavior of losses is due to the fact that in our formulation of the problem the fields in the Josephson junction and in the waveguide were taken into account nonperturbatively.

Note that the appearance of the factor k_J^3 on the right side of (47) is due to the fact that in the case of a vortex (39) the contribution to the integral from S_x is given by a relatively extended interval of the axis Oz of the order of several k_J^{-1}. This, in turn, leads to the fact that the magnitude of radiation losses (49) depends on the width of the tunnel layer as $\propto 1/\sqrt{2d}$. This compares favorably with our situation from considering radiation from the end of the junction, when the radiation power behaves as $\propto \sqrt{2d}$, which is associated with a mismatch between the wave impedance of the Josephson junction and the wave impedance of standard external waveguides [15,16], which, in turn, leads to the reflection of an electromagnetic wave from the end of the transition.

Recall that when deriving relation (47), in addition to the condition $|R| \gg \coth(L_2/\lambda)$, the inequalities $\lambda k \ll 1$, (34) and (38) were used. Under fixed system parameters, these conditions indicate how closely the vortex velocity can approach the velocities of c_m, v_1, V_{Sw} and v_2. The degree of this approximation is determined by the small parameter λ/λ_J, the magnitudes L_1/λ and L_2/λ, as well as the velocity relationships c_m, v_S and v_{Sw}.

Note that the fast vortex emits electromagnetic waves in the terahertz range. Indeed, the Cherenkov radiation spectrum of the sandwich under consideration is $\omega = kv$ Since for the vortex (39) $k \sim k_J$, then the characteristic frequencies of the radiation are $\sim vk_J$. In the case of a fast vortex, due to (36) and (40), the characteristic frequencies are $\gtrsim \omega_J$, i.e. for typical superconductors fall within the terahertz region.

Cherenkov radiation of electromagnetic waves from the surface of an external superconducting electrode into a dielectric medium is possible if the velocity of the vortex exceeds c_m. On the other hand, the characteristic velocities of the fast vortex is $\sim V_{Sw}$. The condition $V_{Sw} > c_m$ leads to the following inequality for the dielectric permittivity of the external medium: $\epsilon_m > [\exp(L_2/\lambda)/2\sinh(L_2/\lambda)](\lambda/d_w)\epsilon_w$. Note that due to the fact that the width of the waveguide $2d_w$ significantly exceeds the width of the tunnel interlayer $2d$, this inequality can be satisfied for a wider range of media than a similar inequality for a superconducting sandwich: $\epsilon_m > (\lambda/d)\epsilon$. In this sense, we can say that a fast vortex is easier to emit than an ordinary Josephson vortex.

Conclusion

The basic principles of the electrodynamics of a multilayer system consisting of a coupled Josephson junction, a plane waveguide and a semi-infinite dielectric medium are formulated. For such a system, integro-differential equations are obtained for the phase differences of the superconducting order parameter at the Josephson junction and at the waveguide. In the case when the field from the superconducting sandwich propagates deep into the external dielectric in the form of an electromagnetic wave, the system of equations for phase differences is reduced to differential equations. This made it possible to determine the permissible range of velocities for the existence of a traveling Josephson vortex, and also to indicate in them the range of velocities of a fast vortex, which moves faster than an ordinary Josephson vortex.

It is shown that vortices traveling along a superconducting sandwich can create electromagnetic radiation in the external dielectric medium, emitted from the entire lateral surface of the sandwich. This radiation is associated with the Vavilov-Cherenkov effect and occurs at vortex velocity greater than the speed of light in the medium. For a fast vortex, the characteristics of Cherenkov radiation emitted from the surface of a superconducting sandwich into the medium are analyzed. Namely, the characteristic frequencies of radiation, the Poynting vector and the dependence of radiation losses on the vortex velocity are determined.

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by V.Prokhorov