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Comparative analysis of dislocation kink migration models in disordered solid solutions

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Models of dislocation kink migration in disordered solid solutions and alloys are analyzed. Mechanisms of interaction of kinks with impurity subsystem leading to the hardening of crystalline materials are studied. Various approaches to the description of statistical patterns of kinks overcoming random potential reliefs are critically discussed. Conceptual features of the movement of kinks in impurity materials, leading to disagreements in interpretations of the phenomenon are noted. Qualitative picture of the anomalous kinetics of kinks in disordered media is described.

Keywords: dislocation kinks, solid solution hardening, random processes, anomalous kinetics.

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1. Introduction

Dislocations move in crystalline materials with a high potential lattice relief (Peierls barriers), such as metals with a body-centered cubic (BCC) structure, semiconductors, intermetallic compounds, etc., via the emergence and propagation of kinks [1–4]. A kink is a bend coupling dislocation segments located in neighboring valleys of the crystal relief. It is not exotic or unique in any way. Dislocation kinks belong to the family of so-called topological solitons [5]. Phenomena similar to those discussed below may be observed in the propagation of steps on the surface of a growing crystal, in biological macromolecules and nanowires, in magnetization switching in spin chains and molecular magnets with a chain structure, and in various other similar one-dimensional and quasi-one-dimensional structures [6–9]. The kinetics of a reversible transition between crystalline and amorphous phases is also important for data recording and readout in optical and electrical low-dimensional data storage systems [10].

The dynamics of kinks governs dislocation mobility and the rate of plastic strain of crystals [1–4]. In addition to barriers for dislocations, which are associated with the intrinsic crystal lattice, most materials feature impurity relief. It may be induced by both uncontrolled impurities and impurities that are introduced deliberately to modify the material parameters. Foreign atoms diluted in crystals normally form a disordered subsystem. Its influence within different concentration and temperature intervals induces various dislocation dynamics modes [11,12].

In the general case, the migration (along with formation) of kinks is one of the key stages of propagation of dislocations; in certain modes in alloys, it is the governing stage [13]. A superposition of these stages is characterized in certain instances by a sum of the formation time of a pair of kinks (τ_{kp}) and the time of their propagation to the boundaries of a dislocation segment (τ_{km}). Dislocation velocity V_d is then written as $V_d = h/(\tau_{kp} + \tau_{km})$, where h is the lattice period. Arrhenius-type expressions $\tau_{kp,m} = \tau_{kp,m0} \exp(E_{kp,m}/k_B T)$ with certain activation energies E_{kp} and E_{km} and pre-exponential factors τ_{kp0} and τ_{km0} are used for τ_{kp} and τ_{km}. Here, k_B is the Boltzmann constant and T is temperature. According as which of the times τ_{kp} and τ_{km} is greater, one of the stages is assumed to be dominant and the corresponding simplified expression $V_d \approx h/\tau_{kp,km}$ is used. This approach is an example of a unfounded representation of a phenomenon and leads to its incorrect interpretation. This is evident even from the fact that multiple events of production of kink pairs and annihilation of kinks and antikinks occur in a segment within the time of kink propagation to its boundaries if $\tau_{kp} \ll \tau_{km}$; therefore, one needs to characterize a completely different process instead of simply using the $V_d \approx h/\tau_{km}$ expression. With this aspect taken into account, the following expression for dislocation velocity with halved activation energies was obtained for pure crystals: $V_d \sim h/\sqrt{\tau_{kp}\tau_{km}} = h/\sqrt{\tau_{kp0}\tau_{km0}} \exp[(E_{kp} + E_{km})/2k_B T]$ [1]. Naturally, this does not imply that the heights of barriers in elementary events of kink kinetics changed; instead, it is attributable to the self-consistent nature of competition between kink formation, migration, and annihilation processes.

Solution atoms, which are also called impurities for brevity, contribute occasionally to the formation of kinks. When this factor is dominant, the dislocation mobility and, consequently, the plasticity of a material increases („softening“ occurs [14–18]). The propagation of kinks in the presence of impurities is made harder than in pure materials, since the interaction of a dislocation with impurity

atoms produces energy relief variations that act as traps or barriers for kinks. Thermal activation may be required to overcome these barriers. An opposite trend is thus set: dislocations slow down and the plasticity decreases, contributing to „hardening“ [19–24]. The competition between these trends plays an important role not only in structural materials [25], but also in natural phenomena, such as the mechanical properties of ice in the context of climate warming [26].

The present study is focused on one aspect of the process most prone to conflicting interpretations: the mechanism of solid-solution hardening induced by the influence of solution atoms on kink migration. This practically important effect has a long-standing history of being modeled, but the interest in such models has surged recently due to the emergence of a new class of promising multicomponent or high-entropy disordered alloys [27–30].

Studies relying on atomistic computer calculations of the interaction between dislocations and impurities are being published at present (see, e. g., [31] and other papers). However, they are focused mostly on individual impurities and, as was noted in [32], run into difficulties when characterizing alloys. Rare events and low-probability or extreme scenarios turn out to be significant in a disordered alloy. It is hard to model their influence on a computer, and conflicting interpretations often arise in such studies. Elementary mechanisms of dislocation dynamics are often left unrevealed in the results of modeling. If one needs to obtain a clear overall pattern of mechanisms of dislocation propagation in a disordered impurity subsystem, it is advisable to divide the problem: use, where possible, theoretical models with arbitrary values of microscopic parameters and perform atomistic calculations separately for the sole purpose of their parameterization. These atomistic calculations for specific materials are a challenge on their own and are not covered in the present study. Since the kinetics of kinks is non-trivial and invites conflicting interpretations, it is expedient, as a first step, to advance as much as possible toward a clear analytical description of this kinetics at the conceptual level, which is exactly the goal of the present study.

2. Energy relief for kink migration

Quite naturally, a barrier in the way of a kink was initially modeled by a local potential peak, as it is done for a common particle (see, e. g., [33,34]). The barrier height for identical impurities was characterized by equal energies of thermoactivation crossing. A presumptive dependence of activation energy on stress was used in analytical calculations and Monte Carlo modeling [32]. It has been understood eventually that the impurity barrier for kink migration is non-local in nature, since the interactions between an impurity and a dislocation on opposite sides of a kink differ [19,35–37]. Therefore, a barrier includes a step smoothed out over the kink width.

Step height U is the variation of the energy of interaction between an impurity and a dislocation upon displacement of this dislocation over a distance of lattice period h. This modification of an impurity barrier leads to significant qualitative changes, since steps from different impurities overlap and produce a cumulative effect, memory of past obstacles, and correlation. The cumulative effect induces strong barriers in the way of a kink, which also have random positioning and height. A statistical approach is needed to characterize kink migration within a chaotic relief of this kind. Such an approach was applied in [19,36,37]. It was assumed in these studies that the dislocation segment energy fluctuates proportionally to a random number of impurities in the dislocation core, producing a chaotic potential relief in the way of a kink with a term established by external driving force F superimposed on it:

$$U_k(x) = U[N_f(x) - N_i(x)] - Fx. \quad (1)$$

Here, x is the distance traveled by a kink along a dislocation; $F = \sigma bh$; σ is the stress that is assumed to be small relative to the Peierls stress at which barriers separating dislocation positions in lattice valleys vanish; b is the Burgers vector of a dislocation; h is the kink height that is equal to the crystal relief period; and $N_f(x)-N_i(x)$ is the difference between the numbers of impurities in the dislocation core at the new and initial positions of a dislocation, respectively. Only the maximum contact interaction of a dislocation with impurities from the corresponding lattice rows is featured in (1). The case of dilute solid solutions with a low impurity concentration, where the average distance between impurities is much greater than the crystal period and the lattice discreteness may be neglected, is considered.

The distribution of impurity atoms along a dislocation was assumed to be completely chaotic and Poissonian with the probability of finding n impurities within a segment containing N lattice sites being $P_{Pois} = (Nc)^n \exp(-Nc)/n!$ (c is the segment-average impurity concentration) [38]. When a kink moves in the field of chaotically positioned impurities, its potential shifts randomly along the energy scale, making differently directed „steps“ with a magnitude of U at random points of impurity positioning along a dislocation. This „random walk“ along the energy scale with drift under the influence of external driving force F [36,37,39–42] superimposed on it is illustrated in Figure 1.

Under certain conditions formulated below, a Poissonian distribution of the number of impurities in the examined problem may be approximated by Gaussian distribution $P_{Gaus} = \exp[-(n-Nc)^2/2Nc]/(2\pi Nc)^{1/2}$ [38]. With a kink traveling over distance L, the impurity potential then undergoes „Brownian motion“ along the energy scale with its fluctuations increasing in magnitude in proportion to $\sqrt{L}$.

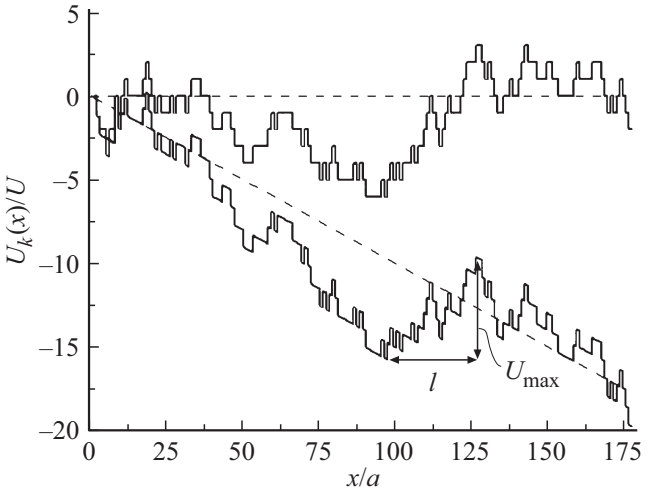

Figure 1. Particular realization of the energy landscape formed by superimposed contributions from the chaotically fluctuating density of solution atoms (upper curve) and driving force $-Fx$ (lower curve). Height $U_{\max}$ and size l of the most significant barrier to kink propagation are indicated.

3. Kink migration under Gaussian statistics of impurity clusters

Fluctuation nonuniformities of the potential relief shaped by chaotic clustering and diffusion of impurity atoms produce stochastic barriers to kink propagation. These barriers grow in scale as the magnitude of potential relief fluctuations increases with length of a traversed dislocation segment. Average kink propagation velocity v_{av} in the Gaussian limit was estimated in [19] by analyzing the fluctuations of a random relief ensemble. In a somewhat simplified form (without a small correction for the finite width of a kink), the result is written as

$$v_{av} \approx \nu L \exp\{-c_1 U^2 \ln[c_1 L/(2\pi)^{1/2} a]/\sigma a b h k_B T\}, \quad (2)$$

where L is the length of a dislocation segment traversed by a kink, $c_1 = zc$, c is the impurity concentration, z is the coordination number specified by the symmetry of lattice rows adjacent to the dislocation core ($z = 3$ was set for metals with a BCC lattice in [19,36]), a is the lattice period along a dislocation, $\nu \approx \nu_D b/d$, ν_D is the Debye frequency, and d is the kink width.

It was noted in [19] that estimation of the average velocity of a particle overcoming barriers in a random force field may be regarded as an interesting (and still unsolved) problem in mathematics of the probability theory. Although result (2) was obtained intuitively, it turned out to be largely correct. This was revealed in calculations [37] (by coincidence, contemporaneous) performed on a more regular basis with the use of an equation for the generating function of the distribution of kink delay times at fluctuation impurity clusters. Let us summarize the results reported in [37].

A spectrum of delay times associated with random barriers forms in a disordered medium. The time of delay at a potential barrier is commonly characterized by Arrhenius formula $\tau = \tau_0 \exp(U_{\max}/kT)$, where $U_{\max}$ is the height of the barrier potential maximum measured from the preceding minimum. If barrier potential $E(x)$ is substantially diffuse (as is the case when barriers from several impurities overlap), a more general expression should be used [43]:

$$\tau = C \int_0^\infty \exp[E(x)/k_B T] dx. \quad (3)$$

The constant in (3) may be estimated roughly as the ratio of characteristic time τ_1 and distance x_1 scales, $C = \tau_1/x_1$ (see below). In [37], we examined distribution function $P_d(z)$ for the integral in (3) with barrier potential (1)

$$z = \int_0^\infty \exp[U_k(x)/k_B T] dx = \tau/C.$$

The following equation was derived for generating function

$$\phi(s) = \left\langle \int_0^\infty \exp(-sz) P_d(z) dz \right\rangle$$

of distribution $P_d(z)$ (angular brackets denote averaging over random positions of impurity atoms at lattice sites):

$$\frac{c_1}{a}\left\{\phi[s \exp(U/k_B T)] + \phi[s \exp(-U/k_B T)]\right\} - s \frac{\sigma b h}{k_B T} \frac{d\phi}{ds} - s\phi(s) = 0. \quad (4)$$

In the Gaussian limit (after expansion in $U/k_B T \ll 1$), a simple equation is obtained. The inverse Laplace transform of its solution is

$$P_d(z) = \frac{D \exp(-1/dz)}{\Gamma(\delta)(zD)^{1+\delta}}, \quad (5)$$

where $D = (c_1/a)(U/k_B T)^2$ is the ratio of „coefficient of diffusion“ $(c_1/a)U^2$ of the impurity potential along the energy scale to thermal energy squared $(k_B T)^2$,

$$\delta = \sigma b h a k_B T/c_1 U^2, \quad \Gamma(\delta) = \int_0^\infty u^{\delta-1} \exp(-u) du$$

is the Euler gamma function. It follows from (5) that $1/D$ is the characteristic scale of variable z. This value may be used as the x_1 distance scale introduced above. This choice agrees with the estimates made in [39]. Inverse frequency $1/\nu$ may serve as the time scale. The distribution function for delay times $\tau = Cz$ is

$$P_d(\tau) = P_d(z) dz/d\tau \approx x_1 \nu P_d(z)_{z=\tau/C}.$$

It is rather inconvenient to characterize the propagation of a kink using the concept of motion velocity, since

the average velocity at $\delta \leq 1$ depends on the future path length and tends to zero in the limit of long path lengths. Kinetic dependences of typical (in the probabilistic sense) path lengths on time appear to be more relevant and intuitive. The path length statistics is governed by probability $P_{pr}(t) = \int_t^\infty P_d(t')dt'$ of encountering a barrier with a time delay of t (or longer). According to [19,37], $L \sim (a/c_1)/P_{pr}(t)$. It is seen clearly from (2) that

$$L = v_{av}t \approx (2\pi)^{1/2}(a/c_1)(\nu t)^{\delta}. \quad (6)$$

Long delay times are of the utmost interest. It follows in this case from (5) that

$$P_{pr}(t) = \int_t^\infty P_d(t')dt' \sim 1/[\Gamma(1+\delta)(\nu t)^{\delta}],$$

which yields (if insignificant prefactors are ignored) the same time dependence of the typical kink path length

$$L \sim (a/c_1)/P_{pr}(t) \sim (a/c_1)(\nu t)^{\delta}, \quad (7)$$

as (6).

As was noted in [37], anomalous dependence (7), which corresponds to nonlinear kink drift, holds true at $\delta \leq 1$, while the path length at $\delta > 1$ is characterized by a common linear dependence $L = v_k t$, where v_k is the average kink velocity that is finite in this case. Thus, although the magnitude of relief fluctuations and the scale of barriers increase with path length, the spatiotemporal nonuniformity of kink propagation in the statistical sense is retained within the parameter range corresponding to condition $\delta = \sigma abhk_B T/c_1 U^2 > 1$. At $\delta = 1$, a kinetic phase transition occurs with a qualitative change in the nature of kink propagation in the $\delta < 1$ region. This transition is associated with an expansion of influence of extremely strong fluctuations (i.e., heterogeneity) of a chaotic impurity relief due to the formation of slowly decreasing asymptotics of times of kink delay at them with an increase in the impurity atom concentration [37,44]. This is inconsistent with attempts at characterizing the influence of an impurity subsystem within „mean field“ approaches (e.g., uniform renormalization of the Peierls relief). Rare extreme fluctuations of the impurity relief produce fairly localized and widely spaced energy relief distortions and cannot yield uniform renormalizations.

A commonly accepted term characterizing the peculiar nature of kink propagation at $\delta < 1$ has not been found yet; it is called anomalous kinetics, „creep“ phase or nonlinear drift at $\delta < 1$, heterogeneous dynamics, quasi-localization mode (since average velocity $L/t \sim L^{1-1/\delta} \to 0$ at $L \to \infty$), etc. Slowly decreasing asymptotics of distribution functions for large values of quantities are sometimes referred to as fat-tailed or heavy-tailed distributions [45].

The specificity of energy relief within which a kink moves in a disordered medium was emphasized in [39,46,47]. This relief is not consistent with what is often called a random potential; instead, it is a „field of random forces“. When a kink travels a distance of x along a dislocation, this dislocation acquires energy (without regard to the external force)

$$U_k(x) = \int_0^x [U_d(x'+h) - U_d(x')]dx'. \quad (8)$$

Here, $U_d(x) = (c_1/a)\rho(x)$ is the actual energy relief of a dislocation with allowance for impurities, which are distributed with linear density $\rho(x)$, that has the statistical features of a random potential. The derivative of energy $U_k(x)$ with respect to displacement x (i.e., force) has similar statistical properties. According to the terminology proposed in [48], a random relief of the $U_k(x)$ form features strong correlation (i.e., the one extending over the entire system). The motion in a field of random forces differs significantly in its characteristics from the motion in a random potential. For example, it was demonstrated in [49] that diffusion displacement (without a driving force) follows the $x \sim \ln^2(t)$ law instead of the $x \sim t^{1/2}$ root dependence typical of Brownian motion.

The suggestion made by Suzuki [19] regarding several intriguing problems associated with particle motion in a field of random forces was verified in numerous studies (see, e.g., reviews [39,48,50,51,53]) in a broad context of statistical physics and in a more general context of mathematical and financial statistics [46,52]. Nonlinear drift law (7) was formulated rigorously based on Lévy distributions [53] in these and other studies.

Averaged characteristics (if, it should be pointed out, they exist and have a correct physical interpretation) are of prime importance in comparison of the results of theoretical calculations for disordered systems with experimental data. For example, it was demonstrated in [37] that average time $\langle \tau \rangle = \int_0^\infty P_d(t)dt$ of kink traversal over fluctuation obstacles is finite at $\delta > 1$, but diverges in an unbounded interval at $\delta \leq 1$. It is fairly evident if one examines the presented asymptotics of probability of long delay times $P_d(t) \sim 1/t^{\delta}$. The primary contribution to the time of kink propagation over distance L along a dislocation is the sum of times of delay at individual obstacles. According to mathematical statistics theorems [54], the sum of all terms at a large number of delays N_t has a fairly small deviation from average value $N_t\langle \tau \rangle$ if average $\langle \tau \rangle$ exists. Therefore, average time $\langle \tau \rangle$ within which a kink crosses an obstacle at $\delta > 1$ may be regarded as a quantity with a credible physical meaning, which is occasionally called a „self-averaging“ one.

The pattern changes at $\delta \leq 1$ and a diverging average obstacle crossing time, when the primary contribution to the sum is produced by the most significant obstacle with the greatest barrier height U_{max}, which is crossed via thermal activation in time $\tau_{max} \approx \tau_0 \exp(U_{max}/k_B T)$, within an interval. The authors of [41] characterized kink migration with the use of the same statistics of potential reliefs (called

a Wiener process with drift) as in [36,37]. A significant amount of effort went in [41,42] into computer modeling of an ensemble of random potential reliefs produced by an impurity subsystem, and the applicability of the Wiener model with drift to their statistics was demonstrated convincingly. However, no reasons for the application of average barrier height $\langle U_{\max}\rangle$ (i. e., logarithm of the maximum delay time $k_{\mathrm{B}}T\langle\ln(\tau_{\max}/\tau_0)\rangle$) instead of, e. g., $k_{\mathrm{B}}T\ln(\langle\tau_{\max}\rangle/\tau_0)$ in analytical characterization of the kink mobility at $\delta \leq 1$ were given. Applying the rigorous mathematical theory of sums of random variables [47,54], one arrives at the following: the distribution of kink displacements with time is qualitatively a diffuse packet that shifts at long times proportionally to $\sim t^{\delta}$ and has its width increasing as $\sim t^{\delta/(1+\delta)}$. Thus, as expected, the relations between characteristic scales of kink propagation time and path length in the case of motion through a series of random obstacles and estimation based on a single most formidable obstacle within an interval are the same.

4. Kink migration under Poissonian statistics of impurity clusters

It is believed that Gaussian statistics of impurities is applicable if their number is small relative to the average number within the considered interval. However, this may not be the case when extreme fluctuations in the distribution of impurity atoms happen to be important. The relations obtained in [37] may be used to generalize the key results presented in the previous section to the case with an arbitrary number of impurities and clarify the conditions of applicability of Gaussian statistics in the examined problem.

Since long kink delay times are of primary interest, one should search for a solution of Eq. (4) at small values of time-conjugate Laplace variable s. It is easy to show that this solution is $\phi(s) \sim s^{\delta}$. Equation (4) provides a more general expression for the index of power $\delta = (k_{\mathrm{B}}T/U)\varphi$, where φ is determined from relation [55]

$$\frac{\sigma bha}{c_1 U} = [\exp(\varphi) + \exp(-\varphi) - 2]/\varphi. \qquad (9)$$

Inverting $\phi(s)$, we find that probability $P_{pr}(\tau)$ of encountering a kink delay in excess of τ behaves at large τ as

$$P_{pr}(\tau) = B/\tau^{\delta}, \qquad (10)$$

where B is a certain constant. At $\varphi \ll 1$, $\varphi \approx \sigma bha/c_1 U$ follows from (9), and the expression for $\delta = (k_{\mathrm{B}}T/U)\varphi$ transforms into $\delta = (\sigma bha k_{\mathrm{B}}T/c_1 U^2)$ that was discussed in the previous section. Thus, the condition of applicability of the Gaussian approximation is

$$\varphi \approx \sigma bha/c_1 U \ll 1, \qquad (11)$$

that is, the energy acquired within an average distance between atoms due to the external force should be insufficient to cross a barrier produced by a single impurity. However, even in the case when condition (11) is not satisfied and this energy is sufficient to cross a single barrier, rarer obstacles produced by impurity clusters may come into play. The expression for key parameter δ then assumes a more general form of $\delta = (k_{\mathrm{B}}T/U)\varphi$ with φ given by formula (9) and illustrated in Figure 2.

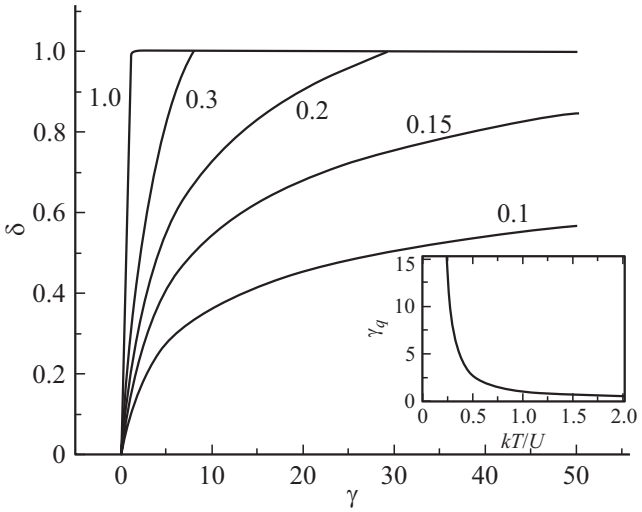

Figure 2. Dependence of index δ on stress $\gamma = \sigma bha/c_1 U$ at different values of temperature $k_{\mathrm{B}}T/U$, which are indicated next to the curves. The inset shows the dependence of stress of quasi-localization boundary $\delta = 1$ on temperature (stress and temperature are characterized by dimensionless parameters $\gamma = \sigma bha/c_1 U$ and $k_{\mathrm{B}}T/U$, respectively).

In the Gaussian limit, the value of δ given below formula (5) is $\delta = \sigma bha k_{\mathrm{B}}T/c_1 U^2 = \gamma k_{\mathrm{B}}T/U$, which agrees with the behavior of curves in Figure 2 in the region of a weak driving force $\gamma = \sigma bha/c_1 U \ll 1$, but deviates noticeably from the pattern specified by Poissonian statistics in a wider parameter range. The inset in Figure 2 presents the relation that follows from Eq. (9) for parameters corresponding to quasi-localization boundary $\delta = 1$ ($\varphi = U/k_{\mathrm{B}}T$):

$$\frac{\sigma bha}{c_1 U} = \frac{k_{\mathrm{B}}T}{U}\left[\exp\left(\frac{U}{k_{\mathrm{B}}T}\right) + \exp\left(-\frac{U}{k_{\mathrm{B}}T}\right) - 2\right]. \qquad (12)$$

Let us use it to estimate the stress of transition to quasi-localization mode σ_q at typical (for BCC metals) energy $U \approx 0.05$ eV of interaction between an impurity atom and a dislocation, low impurity concentration $c_1 \sim 0.1$, and temperature $T = 580^{\circ}$ K. It follows from (12) that $\sigma_q \approx 120$ MPa. As the temperature decreases, σ_q grows rapidly and covers the region of plastic stress in most mechanical tests [3].

Let us also discuss briefly certain more advanced and formalized approaches using probability theory. If a kink travels over a long distance under the influence of an external force, it needs to overcome a series of obstacles, and the times of delay at each obstacle are summed. Approximately $N \approx (c_1/a)x$ obstacles are encountered within a sufficiently

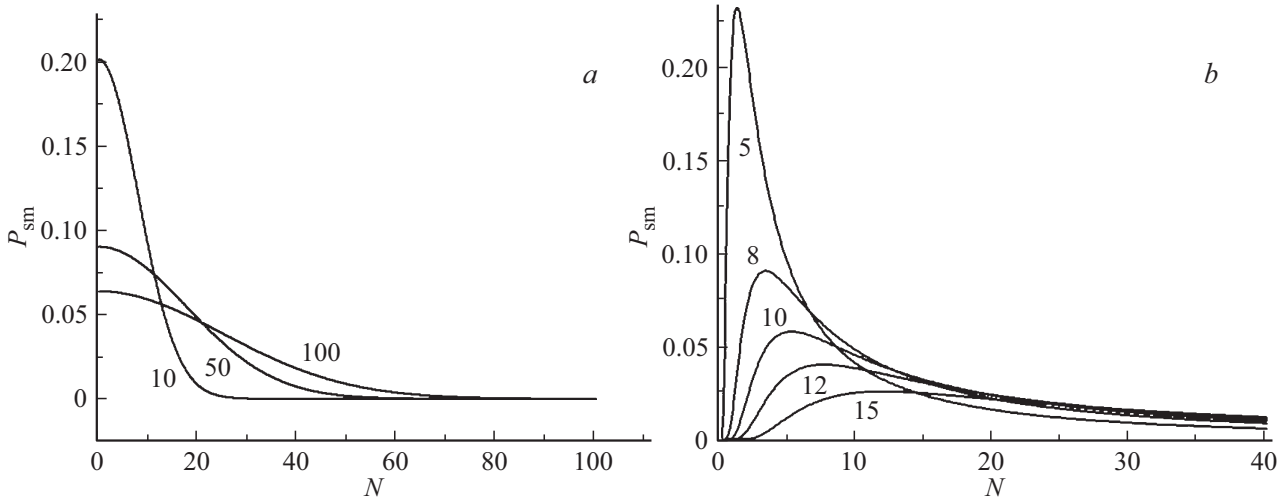

Figure 3. *a* — Distributions of path lengths for different time intervals, which are indicated next to the curves; *b* — distributions of times of propagation over different distances, which are indicated next to the curves.

long path length x. According to the statistical theory of distributions of sums of random variables (see, e. g., [50,54]), the density of distribution of a normalized sum of N random positive terms $t = \tau_1 + \tau_2 + \ldots \tau_N$, each of them having a distribution with asymptotics B/τ^{δ} $(\tau \to \infty)$, is given by Lévy function

$$L_{\delta,1}(t/N^{1/\delta}) = \frac{1}{2\pi i}\int\limits_{d-i\infty}^{d+i\infty} \exp\left[st/N^{1/\delta} - \frac{\pi B}{\sin(\pi\delta)\Gamma(\delta)}\, s^{\delta}\right]$$

$$= -\frac{N^{1/\delta}}{\pi t}\sum_{k=1}^{\infty}\left(\frac{-\pi BN}{\Gamma(\delta)\sin(\pi\delta)t^{\delta}}\right)^{k}\frac{\Gamma(1+k\delta)\sin(\pi\delta k)}{k!}. \quad (13)$$

Function $L_{\delta,1}(t/N^{1/\delta})$ (13) provides a fairly complete description of propagation of the front of kink motion in a field of randomly distributed impurities.

The key qualitative features of kink paths in the region of anomalous mobility are illustrated in Figure 3 that presents a particular case with $\delta = 1/2$, where the Lévy distribution is expressed in terms of elementary functions $L_{1/2,1}(u) = [B/(\pi^{1/2}u^{3/2})]\exp(-B^2/4u)$. Here $u = \nu t/N^2$; $N = Lc_1/a$.

Extended asymptotics in Figure 3, *b* illustrate the slow decrease of probability of long delays of kinks at obstacles produced by random impurity clusters.

5. Conclusion

The peculiar nature of kinks, which is manifested, e. g., in the nonlocality of their interaction with impurity atoms in solid solutions and alloys, and consequent effects are accepted less than enthusiastically by many researchers. It is no coincidence that the laws of motion of such objects are commonly referred to as anomalous or strange kinetics [51]. The treatment of kinks as common particles successively overcoming uncorrelated barriers of individual impurities is entrenched too deeply. This approach is consistent with the familiar rendering of an elementary event as a thermally activated jump that is characterized by the Arrhenius law with a certain activation energy. However, this leads to several conceptual inaccuracies and inconsistency of models of the discussed phenomenon. Let us list several inaccuracies of this kind that are of crucial significance.

1. Treatment of the superposition of formation and propagation of kinks as successive processes with additive durations.

2. Characterization of an obstacle to kink propagation produced by an impurity atom as a local barrier.

3. Treatment of the impurity relief affecting the motion of a kink as a random potential instead of a field of random forces.

4. Disregard of the cumulative effect of a superposition of individual impurity barriers due to their nonlocality.

5. Disregard of the infinite correlation length of a potential impurity relief.

6. Incorrect choice of statistical parameters for characterization of kink kinetics in a random impurity relief.

7. Disregard of the conditions and boundaries of applicability of various types of potential relief statistics to kink migration.

8. Disregard of the pivotal role of extreme fluctuations in the anomalous mode of dislocation motion in an attempt at characterizing the influence of an impurity subsystem by uniform renormalizations of Peierls barriers.

Although a well-developed modern theory of random walks in a random environment is available, studies into dislocation kink dynamics correcting certain inaccuracies from the above list to various degrees are still being published today. The difficulty of isolating kink migration from a multitude of overlapping processes in macroscopic plastic flow of materials (or even in the propagation

of individual dislocations) hampers direct verification of theoretical models. The large number of influencing factors often makes it difficult to compare experimental data even for same-type materials. However, certain experimental data for semiconductor materials agree with the above simplified theory of anomalous kink migration [56,57]. Let us outline, without going into details, several current trends of development of models of kink kinetics in disordered materials. These are, e.g., the transition from dilute solid solutions to more concentrated ones, which is especially relevant to multicomponent or high-entropy alloys [22]; the inclusion of dynamic dislocation aging, which leads to an asymmetric distribution of impurity atoms in the vicinity of a dislocation core [58]; the competition between solid-solution hardening and softening [14,59]; and the inclusion of correlation of solution atoms [60].

The current trends toward tighter control over the experimental conditions and toward the transition to a deeper nanolevel hold out the promise of advances in such research. Computer modeling may facilitate the verification of theoretical concepts. Several items from the above list have already been checked in [41,42], and the list itself may serve as a useful guide for a more complete inspection leading to a deeper understanding of the processes of solid-solution hardening of materials.

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Conflict of interest

The author declares that he has no conflict of interest.

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