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Microwave scattering by blob in tokamak edge plasma

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Received June 21, 2023
Revised October 10, 2023
Accepted October 11, 2023

Microwave probing of tokamak edge plasma is a method of diagnostics of plasma blobs and filaments. The spectrum and amplitude of the scattering signal of the probing microwave radiation scattered by a blob stretched along the magnetic field line and having a finite radius in the transverse direction are investigated.

Keywords: microwave, scattering signal, turbulence, plasma blob.

DOI: 10.61011/TPL.2024.01.57828.19658

Recently, the structure and motion of plasma clots at the plasma column boundary in toroidal plasma confinement [1,2] devices have been actively studied. Plasma clots are relatively isolated structures that are stretched along the magnetic force line, have finite transverse dimensions, and move radially (in the literature they are called blobs or filaments). The study of these structures is important for understanding transverse transport at the plasma edge or near the magnetic separatrix of tokamaks, and for future comparisons of edge turbulence measurements with modelling results, e.g., for the ITER reactor tokamak. The concept of convective motion of an isolated blob near the magnetic separatrix of a tokamak was proposed in [3]. Since then, many refinements have been made to the analytical theory of blob structure and motion as described in the literature [4,5]. Given the complexity of blob theory and the difficulty of measuring blobs in tokamaks, it is not surprising that the relationship between experiments and blob theory is inaccurate and incomplete. For this reason, the development of experimental methods that could allow information on blob size and parameters to be obtained is an important application. A possible way to diagnose blobs at the plasma periphery is to probe with microwaves (e.g., classical reflectometry [6] and Doppler backscattering [7]) and then analyze the linear scattering signals. In the present work, the spectrum and amplitude of the signal of scattering of a probing microwave by a plasma blob at the boundary of a plasma column in a tokamak are studied.

Let us consider the near-wall region of the tokamak. We will assume that the plasma density profile has a constant component n_0 and a perturbation $n_b \gg n_0$, caused by the filament and having a Gaussian form in the transverse relative to the magnetic field:

$$n_e = n_0 + n_b \exp(-r^2/r_b^2), \tag{1}$$

where r_b — the transverse dimension of the blob. Consider a beam of microwaves of ordinary polarization, which propagates in the equatorial plane across the magnetic field from the outside of the plasma column inwards and is infinite along the magnetic field. Away from the blob at distance $|r| \gg c/\omega_0$ the electric field of the wave in the Wentzel–Kramers–Brillouin approximation we write as

$$\mathbf{E}|_{x/r_b\to\infty} = \frac{\mathbf{e}_z}{2}\int\limits_{-\infty}^{\infty} dk_y A_0(k_y) \times \exp\big(-ik_x(\omega_0, k_y)x - ik_y y - i\omega_0 t\big) + \text{c.c}, \tag{2}$$

where $A_0(k_y) = \exp(-k_y^2 w^2/2)$ — wave amplitude, ω_0 — wave frequency, which is much larger than the electron plasma frequency, w — beam width, $k_x(\omega_0, k_y) \approx \omega_0/c\sqrt{1 - k_y^2c^2/\omega_0^2}$, c.c. — member obtained from the former by complex conjugation. In cylindrical geometry, which is natural in describing the blob, given $\omega_0^2 w^2/c^2 \gg 1$, expression (2) has the form

$$\mathbf{E}|_{r/r_b\to\infty} \approx \frac{\mathbf{e}_z}{2}\sum_{m=-\infty}^{\infty} E_m^{(0)}(\omega_0 r/c)\exp(-im\theta - i\omega_0 t) + \text{c.c.}, \tag{3}$$

where $E_m^{(0)} = A_m^{(0)} J_m(\omega_0 r/c)$, J_m — the Bessel function, and the amplitude

$$A_m^{(0)} = \frac{1}{\sqrt{2\pi w^2}}\exp\left(-\frac{c^2m^2}{2\omega_0^2 w^2} - im\frac{\pi}{2}\right) \tag{4}$$

defines the spectrum of the probing beam given by the antenna. In the neighborhood of the blob partial amplitude of the ordinary wave $\mathbf{E} \sim \mathbf{e}_z \sum\limits_m E_m \exp(-im\theta - i\omega_0 t)$ is a solution to equation

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} E_m + \frac{\omega_0^2}{c^2}\left(1 - \frac{n_b}{n_c}\exp\left(-\frac{r^2}{r_b^2}\right) - \frac{m^2c^2}{r^2\omega_0^2}\right)E_m = 0, \tag{5}$$

where n_c — critical density for frequency ω_0. According to the available data [2], $n_b/n_c \ll 1$, which makes it possible to find a solution by perturbation theory

$$E_m \approx E_m^{(0)} + E_m^{(1)}, \tag{6}$$

where $E_m^{(0)}$ is determined by the boundary condition (3). The $E_m^{(1)} \sim \exp(-im\theta - i\omega_0 t)$ correction arises from scattering in the blob and satisfies the inhomogeneous equation

$$\frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi}E_m^{(1)} + \left(1 - \frac{m^2}{\xi^2}\right)E_m^{(1)} = \frac{n_b}{n_c}\exp\left(-\frac{c^2}{r_b^2\omega_0^2}\xi^2\right)E_m^{(0)}, \tag{7}$$

where $\xi = \omega_0 r/c$ — a new dimensionless variable. The solution to equation (7) can be found by the method of variation of constants

$$E_m^{(1)}(\xi) = \frac{\pi}{2}\frac{n_b}{n_c}A_m^{(0)}\left(H_m^{(1)}(\xi)\int_0^{\xi}\exp\left(-\frac{c^2}{r_b^2\omega_0^2}\xi^2\right) \times J_m(\xi)H_m^{(2)}(\xi)\xi d\xi + H_m^{(2)}(\xi)\int_{\xi}^{\infty}\exp\left(-\frac{c^2}{r_b^2\omega_0^2}\xi^2\right) \times J_m(\xi)H_m^{(1)}(\xi)\xi d\xi\right), \tag{8}$$

where the Hankel functions $H_m^{(1)} = J_m + iY_m$ and $H_m^{(2)} = J_m - iY_m$ describe waves propagating along $\mathbf{e}_r$ and in the opposite direction, respectively, Y_m — the Neumann function. At the plasma periphery far beyond the blob $\xi_a \gg r_b\omega_0/c$, we can use the asymptotics of the Hankel functions and, taking into account that the main contribution to the integrals in expression (8) comes from the region $r_b^2\omega_0^2/c^2$, replace the integrals with a variable upper limit by integrals with a constant upper limit $\xi_a \to \infty$ and represent the scattered wave field in the following form:

$$\mathbf{E}^{(1)}(\xi, \theta, t)|_{\xi_a c/(\omega_0 r_b)\to\infty} \approx \frac{\mathbf{e}_z}{2}\sqrt{\frac{\pi}{2\xi_a}}\frac{n_b}{n_c} \times \sum_{m=-\infty}^{\infty} A_m^{(0)}\exp\left(-i\left(\xi_a + \frac{\pi m}{2} - \frac{\pi}{4}\right) - im\theta - i\omega_0 t\right) \times \int_0^{\infty}\exp\left(-\frac{c^2}{r_b^2\omega_0^2}\xi^2\right)J_m(\xi)H_m^{(2)}(\xi)\xi d\xi + \text{c.c}, \tag{9}$$

Note that the main contribution to the single integral in expression (9) is made by the first term of the Hankel function $H_m^{(2)} = J_m - iY_m$, which allows us to reduce it to the second Weber exponential integral

$$\int_0^{\infty}\exp\left(-\frac{c^2}{r_b^2\omega_0^2}\xi^2\right)J_m(\xi)H_m^{(2)}(\xi)\xi d\xi \approx W_m = \frac{r_b^2\omega_0^2}{2c^2}\exp\left(-\frac{r_b^2\omega_0^2}{2c^2}\right)I_m\left(\frac{r_b^2\omega_0^2}{2c^2}\right)$$

(where I_m — modified Bessel function of the first kind) and present (9) in the form

$$\mathbf{E}^{(1)}(\xi, \theta, t)|_{\xi_a c/(\omega_0 r_b)\to\infty} \approx \frac{\mathbf{e}_z}{2}\sqrt{\frac{\pi}{2\xi_a}}\frac{n_b}{n_c}\sum_{m=-\infty}^{\infty}A_m^{(0)}W_m \times \exp\left(-i\left(\xi_a + \frac{\pi m}{2} - \frac{\pi}{4}\right) - im\theta - i\omega_0 t\right) + \text{c.c.}, \tag{10}$$

Consider the case when the receiving antenna is located next to the probing antenna and has the same radiation pattern (4). In this case, the amplitude of the received signal is equal to

$$A_s = \frac{\pi}{2}\frac{n_b}{n_c}\sum_{m=-\infty}^{\infty}\left|A_m^{(0)}\right|^2 W_m = \frac{n_b}{8n_c}\frac{r_b^2\omega_0^2}{w^2c^2}\sum_{m=-\infty}^{\infty} \times \exp\left(-\frac{c^2m^2}{2\omega_0^2w^2} - \frac{r_b^2\omega_0^2}{2c^2}\right)I_m\left(\frac{r_b^2\omega_0^2}{2c^2}\right). \tag{11}$$

From mathematical point of view, scattering on the blob results in a wave whose amplitude is determined by the relative amplitude of the blob $n_b/n_c \ll 1$, and whose spectrum is formed by spectral filtering of the initial distribution $A_m^{(0)}$ (4) with a weight function W_m.

Let us illustrate the obtained expression for the ordinary wave field (2). Fig. 1 shows its modulus in cross section in the absence of blob (3). The arrow shows the direction of propagation of ordinary waves with frequency $f_0 = 140$ GHz. The area marked in dark color corresponds to the area of beam localization with $w = 1$ cm. It can be seen that the beam propagates without distortion from the positive x region to the negative coordinate region. Fig. 2

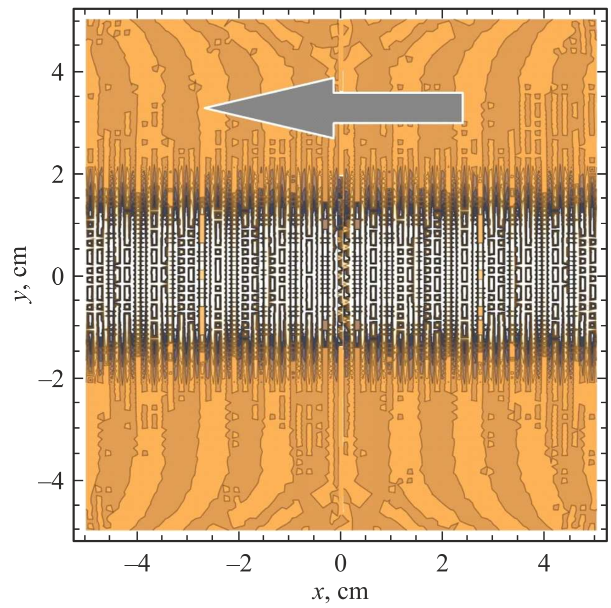

Figure 1. Modulus of the electric field in the cross section in the absence of blob (3). The dark color shows the localization of the beam with $w = 1$. The arrow indicates the direction of propagation of ordinary waves with frequency $f_0 = 140$ GHz.

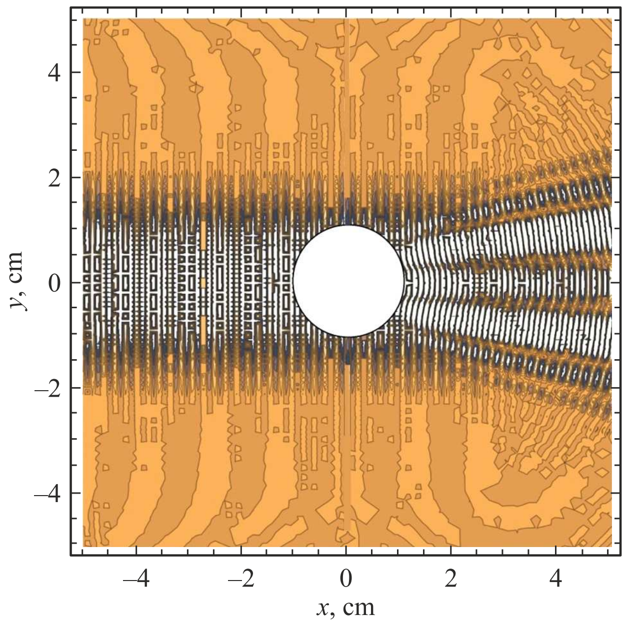

Figure 2. Modulus of the electric field in the cross section in the presence of blob (1). The parameters of the initial beam correspond to those of Fig. 1. Blob parameters: $n_b/n_c = 0.2$, $r_b = 0.75$ cm. White spot — the region where the asymptotic expression (10) is incorrect.

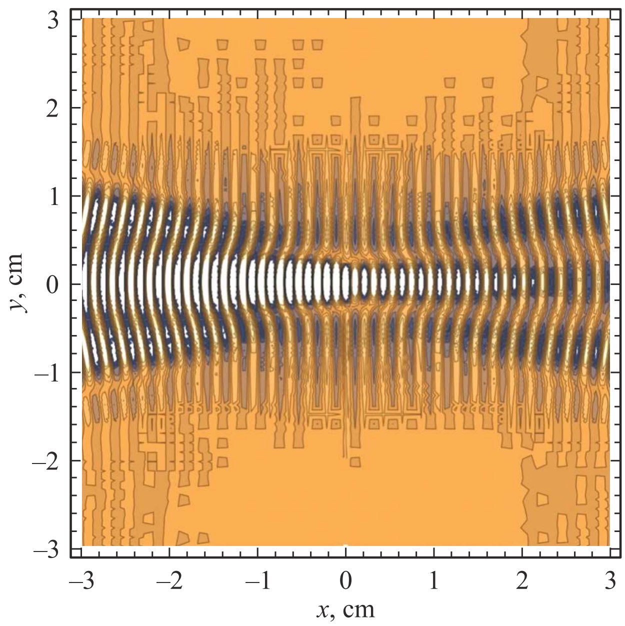

Figure 3. The electric field modulus obtained from the numerical solution of equation (5) with boundary condition (3). Blob parameters: $n_b/n_c = 0.2$, $r_b = 0.75$ cm.

shows the modulus of electric field (3) c amplitude (6), (10) in the presence of blob (1). The parameters of the initial beam correspond to those of Fig. 1. Blob parameters: $n_b/n_c = 0.2$, $r_b = 0.75$ cm. White spot — the region where the asymptotic expression (10) becomes incorrect and it is necessary to use the full expression (8) to correct for the wave amplitude in the presence of the blob.

Then we solve equation (5) with boundary condition (3) numerically. Fig. 3 shows the result of the solution — the modulus of the electric field. The blob parameters used were the same as in Fig. 2. Comparison of Figs. 2 and 3 indicates the adequacy of the solution (10) obtained analytically.

The results obtained for the spectrum and amplitude of the scattering signal of a probing microwave by a blob stretched along the magnetic force line and having finite dimensions in the transverse direction without sharp boundaries can be useful in studying the structure and dynamic of plasma clots at the plasma edge in toroidal plasma confinement facilities.

Funding

The analytical study was supported by RNF Grant No. 23-72-00024, the numerical modelling and code development — supported by the state contract of Ioffe Institute FFUG-2024-0028.

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by J.Deineka