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Photon-photon scattering in the atomic ion field

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The analytical structure, absolute values and angular anisotropy of the triple differential cross section of elastic photon-photon scattering in the field of a multicharged neon-like atomic ion are theoretically predicted.

Keywords: elastic scattering, neon-like atomic ion, X-ray photon, scattering probability amplitude, triple differential cross section.

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1. Introduction

Elastic scattering of two real photons through vacuum of quantum electrodynamics (QED) is one of the fundamental processes in the microcosm [1]. The first theoretical studies of this process [2–6] have shown its low probability (the complete scattering sections $\sigma \sim 10^{-30}\,\text{cm}^2$). With the then-available power and radiance, the first attempts of experimental observation of this process failed [7–9]. However, with possibilities of the today experiment, for example, with X-ray free-electron laser (XFEL), we can hope for experimental detection of the photon-photon scattering through QED-vacuum in the nearest future [10,11]. At the same time, the scattering of two virtual photons (similar to scattering of two real photons) in collisions of heavy ions [12–14] was experimentally detected in Large Hadron Collider [15–17]. The authors' papers [18–20] include the first theoretical studies of elastic photon-photon scattering in the atom field (atomic ion) in the second order (in terms of a number of interaction apexes) of the quantum mechanics theory of perturbations. The main result of these papers is prediction of „mirror-symmetrical" (in relation of energy of Thomson scattering) ($\hbar\omega_1 \cong \hbar\omega$, $\hbar\omega$ — the energy of the incident photon, $\hbar\omega_1$ — the energy of the scattered photon) resonance structures of the scattering spectrum in the regions of birth of „cold" ($\hbar\omega_1 \leq \hbar\omega - I_{1s}$, I_{1s} — the energy of the ionization threshold of the $1s^2$-shell) and „hot" ($\hbar\omega_1 \geq \hbar\omega + I_{1s}$) scattered photons. In the present paper we generalized the theory and supplement the physical results of the papers [18–20] with a transition to the third order of the perturbation theory. Such studies are required, in particular, for interpretation of the short-wave regions (the radiation wavelength $\lambda \sim 1.5 - 2\,\text{Å}$) in the observed spectra of X-ray emission of hot astrophysical plasma [21,22]. The object of the study has included a neon-like ion of the ferrum atom (Fe^{16+}, the ion nucleus charge $Z = 26$, the configuration and ground state term $[0] = 1s^2 2s^2 2p^6 [^1S_0]$). It was selected due to the spherical symmetry of the ground state of the ion Fe^{16+}, a pronounced presence of the $K_{\alpha,\beta}$-lines of emission of the ferrum atom and its multi-charge ions, in particular, Fe^{16+} and Fe^{24+}, within $\lambda \sim 1.75 - 1.95\,\text{Å}$, and to availability of the ion Fe^{16+} in the gas phase for high-precision experiments, for example, for scattering of the XFEL-radiation by the ion captured in the „trap" [23,24].

2. Method theory

The subvalence ($2s^2$) and valence ($2p^6$) shells of the neon-like atomic ions are spatially and energetically strongly separated from the deep $1s^2$-shell. In case of the ion Fe^{16+} we have the following inequalities: for the shell radii $r_{1s} = 0.031\,\text{Å} \ll r_{2s}\,(r_{2p}) = 0.140\,(0.123)\,\text{Å}$ (the calculation in this paper), for the energy of the ionization threshold of the shells $I_{1s} = 7699.23\,\text{eV}$ (the calculation in this paper) $\gg I_{2s}\,(I_{2p}) = 1397.77\,(1270.60)\,\text{eV}$ [25]. Within the incident photon energies $\hbar\omega \geq I_{1s}$ this fact makes it possible to neglect the contribution by the $2s^2$- and$2p^6$-shells to the probability of the process of elastic photon-photon scattering in the field of the neon-like atomic ion. Then, the amplitude of the probability of the process turns out to be a sum of the partial amplitudes of the number 364. Let us accept the dipole approximation for operators of the radiation transition:

$$\hat{R} = -\frac{e}{m_e c}\sum_{n-1}^{N}(\hat{p}_n \hat{A}_n), \tag{1}$$

and contact (non-linear for the electromagnetic field) interaction:

$$\hat{Q} = \frac{e}{2m_e c^2}\sum_{n-1}^{N}(\hat{A}_n \hat{A}_n). \tag{2}$$

(1) and (2) have determined: $\hat{A}_n$ — the operator of the electromagnetic field in representation of the secondary quantization, $\hat{p}_n$ — the operator of momentum of the n-electron of the ion, c — the speed of light in vacuum and N — the number of electrons in the ion. Let us also accept the Tamm-Dancoff approximation [26] with the maximum

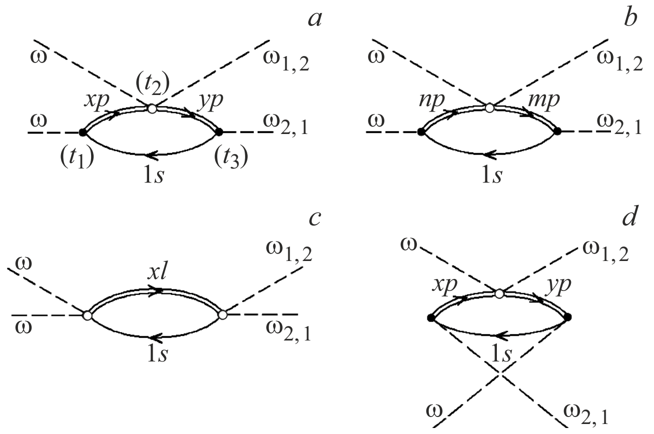

Figure 1. Amplitudes of probability of elastic photon-photon scattering in the field of the neon-like atomic ion (Fe^{16+}) in representation of Feynman diagrams: (*a*) Thomson scattering through the virtual states of the continuous spectrum, (*b*) resonance scattering through the virtual states of the discrete spectrum, (*c*) local double absorption of radiation, (*d*) „back–in–time" scattering. Right arrow — electron, left arrow — vacancy. Double line — the state was obtained in the Hartree-Fock field of the 1*s*-vacancy. Black (light) circle — top of radiative (contact) transition. $\omega(\omega_i, i = 1, 2)$ — incident (scattered) photon. Time direction — left to right ($t_1 < t_2 < t_3$).

number of the „particles" (electrons, vacancies and photons) in the sections of the Feynman diagrams $N_{\max} = 5$. Then, only the amplitudes of Fig. 1 *a*, *b* remain from the said number of the partial amplitudes. These amplitudes structurally take into account the completeness of the set of the intermediate (virtual) states of scattering in the discrete $(n, m) \in [3; \infty)$ and continuous $(x, y) \in [0; \infty)$ spectra. The remaining 362 partial amplitudes „disappear". For example, the amplitude of Fig. 1, *c* turns to zero in the dipole approximation for the $\hat{Q}$-operator: $\langle 1s|\hat{j}_l|xl\rangle \to \langle 1s|xl\rangle$ at $\hat{j}_0 \to 1$, $\hat{j}_{l\geq 1} \to 0$, where $\hat{j}_l$ — the spherical Bessel function of genus 1 of the order *l*. The amplitude of Fig. 1, *d* describes a process of spontaneous birth of the „particles" to a moment of absorption of the photons falling onto the ion, and it is omitted in the Tamm-Dancoff approximation. As a result, let us consider the following processes of elastic photon-photon scattering in the field of the neon-like atomic ion:

$$2\omega + [0] \to A \to B \to \omega_1 + \omega_2 + [0], \tag{3}$$

$$A = \omega + 1s \begin{Bmatrix} n \\ x \end{Bmatrix} p(^1P_1), \tag{4}$$

$$B = \omega_i + 1s \begin{Bmatrix} m \\ y \end{Bmatrix} p(^1P_1), \quad i = 1,\ 2. \tag{5}$$

The transitions from the initial state of the system „incident photons $\oplus$ ion" into the state *A* and from the state *B* into the final state of scattering occur in accordance with the operator $\hat{R}$. The transitions from the state *A* into the state *B* occur in accordance with the operator $\hat{Q}$. In (3)–(5) and below, the atomic system of units ($e = \hbar = m_e = 1$) is accepted, $\omega(\omega_i)$ — the energy of the incident (scattered) photon, $\omega_1 + \omega_2 = 2\omega$, the filled shells of the ion configurations are not specified. Following the Fermi's „golden rule" [27] and using the methods of the algebra of photon creation (annihilation) operators, the theory o irreducible tensor operators and the theory of non-orthogonal orbitals [28], for the triple differential cross-section of scattering we obtain:

$$\frac{d^3\sigma}{d\omega_1 d\Omega_1 d\Omega_1} \equiv \sigma^{(3)} = r_0^4 \mu\eta\omega_1(2\omega - \omega_1)\, G, \tag{6}$$

$$\mu = \frac{\pi}{18}\,\alpha\,\frac{(c\hbar)^2}{a_0 V}, \tag{7}$$

$$\eta = [(\mathbf{e}\cdot\mathbf{e}_1)(\mathbf{e}\cdot\mathbf{e}_2)]^2, \tag{8}$$

$$G = (M_1 - M_2)^2 + M_3^2, \tag{9}$$

$$M_1 = \frac{\omega\cdot\gamma_{1s}}{(\Delta_1^2 + \gamma_{1s}^2)}\langle 1s_0 \parallel \hat{r} \parallel x_0 p_+\rangle^2, \tag{10}$$

$$M_2 = \sum_{n=3}^{\infty}(x_n^2 - \Delta_1^2 - \gamma_{1s}^2)\cdot D_{np}, \tag{11}$$

$$M_3 = \Gamma_{1s}\sum_{n=3}^{\infty} x_n D_{np}, \tag{12}$$

$$D_{np} = \frac{1}{\pi\omega}\,\frac{(I_{1snp}J_{np})^2}{[(x_n + \Delta_1)^2 + \gamma_{1s}^2][(x_n - \Delta_1)^2 + \gamma_{1s}^2]}. \tag{13}$$

Here, it has determined: Ω_i — the spatial angle of escape of the scattered ω_l-photon, r_0 — the classical electron radius, α — the fine structure constant, a_0 — the Bohr radius, $V\ (\mathrm{cm}^3) = c$ — the quantization volume of the electromagnetic field [29], $\mathbf{e}(\mathbf{e}_i)$ — the polarization vector of the incident (scattered) photon, $2\gamma_{1s} = \Gamma_{1s}$ — the natural width of disintegration of the 1*s*-vacancy, $\Delta_1 = \omega - \omega_1$, $x_0 = \omega - I_{1s}$, $x_n = \omega - I_{1snp}$, I_{1snp} — the photoexcitation energy $1s \to np$. The radial part of the one-electron amplitude of probability of the radiative transition $J_{np} = \langle 1s_0 \parallel \hat{r} \parallel np_+\rangle$ is as follows:

$$J_{np} = N_{1s}\left(\langle 1s_0|\hat{r}|np_+\rangle - \frac{\langle 1s_0|\hat{r}|2p_+\rangle\langle 2p_0|np_+\rangle}{\langle 2p_0|2p_+\rangle}\right), \tag{14}$$

$$N_{1s} = \langle 1s_0|1s_+\rangle\langle 2s_0|2s_+\rangle^2\langle 2p_0|2p_+\rangle^6. \tag{15}$$

In (14) the length form for the one-electron radiative transition operator is implemented. A potential discrepancy between the forma of the length and the velocity in the single-configuration Hartree-Fock approximation can be removed by passing, for example, to the multi-configuration Hartree-Fock approximation when constructing the complete wave functions of the scattering states in (3). The indices „0" and „+" correspond to the radial parts of the electron wave functions obtained by solving the self-consistency Hartree-Fock field equations for the configurations of the initial ([0]) and the final ($[1s_+np_+]$) states of the ion. The axially-symmetrical (in relation to **k** — the wave vector of the incident photon) polarization multiplier η of (6) defines

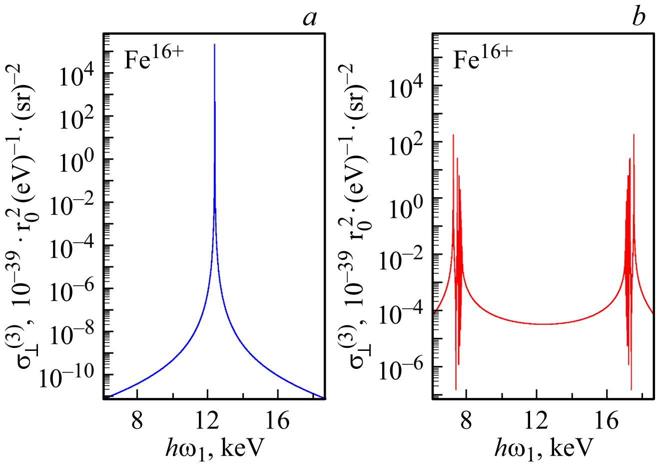

Figure 2. Partial triple differential sections of elastic photon-photon scattering in the field of the ion Fe^{16+} for the $\perp$-scheme of the experiment ($\eta^{\perp} = 1$): (a) the section of Thomson (only the amplitude M_1 is taken into account) scattering, (b) the section of the „mirror-symmetrical" resonance (only the amplitudes M_2 and M_3 are taken into account) scattering. The incident photon energy $\hbar\omega = 12.40$ keV, $\hbar\omega_1$ — the scattered photon energy.

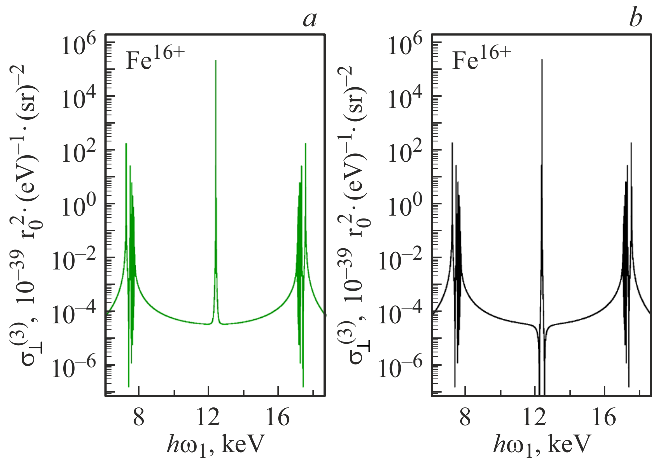

Figure 3. Complete triple differential sections of elastic photon-photon scattering in the field of the ion Fe^{16+} for the $\perp$-scheme of the experiment ($\eta^{\perp} = 1$): (a) without taking into account (the sum of the section is on Fig. 2, a, b), (b) taking into account quantum interference (9) of the amplitudes M_1 and M_2. The incident photon energy $\hbar\omega = 12.40$ keV, $\hbar\omega_1$ — the scattered photon energy.

the effect of angular anisotropy of the differential section of scattering. It is specified for the three schemes of the proposed experiment. The first scheme — the photon polarization vectors are perpendicular ($\perp$) to the scattering plane. The second scheme — the photon polarization vectors are parallel ($\|$) to the scattering plane. The third scheme — the scheme with unpolarized (NP) photons. The scattering plane passes through the wave vectors of the incident and scattered ($\mathbf{k}_i$) photons. Then we get:

$$\eta^{\perp} = 1, \tag{16}$$

$$\eta^{\|} = \cos^4\theta, \tag{17}$$

$$\eta_{NP} = \frac{1}{2}(\eta^{\perp} + \eta^{\|}), \tag{18}$$

where θ — the scattering angle (the angle between the vectors $\mathbf{k}$ and $\mathbf{k}_i$). As expected, the section (6) meets the asymptotic condition $\sigma^{(3)} \to 0$ at $\omega \to \infty$ and the QED-theorems of Furry [30] and Landau-Yang [31,32] on zero probability of the process with an odd number of external photon lines on the „loop" in representation of the Feynman diagrams: $\sigma^{(3)} \to 0$ at $\omega_1 \to 0$ (the elastic merge of the two ω-photons into one, $\omega_2 \to 2\omega$ is forbidden).

3. Results and discussion

The calculation results are presented in Fig. 2–4. The following values are accepted for the parameters of the section (6): $I_{1s} = 7699.23$ eV, $\Gamma_{1s} = 1.046$ eV [33], $\omega = 12400$ eV [34]. In this case, the criterion of applicability of the dipole approximation for the $\hat{R}$- and $\hat{Q}$-operators of the transition: $\lambda_\omega / r_{1s} \gg 1$, where the average radius of the $1s^2$-shell of the ion Fe^{16+} $r_{1s} = 0.031$ Å and the wavelength of radiation falling onto the ion $\lambda_\omega \cong 1$ Å. The values I_{1snp} and J_{np} for $n \in [3; 10]$ are obtained by calculation in the single-configuration Hartree-Fock approximation. For $n \in [11; \infty)$, the transition energies are obtained by an approximation of the form

$$I_{1snp} = I_{1s} - \frac{1}{n^2}\left(\alpha + \frac{1}{n}\beta\right), \tag{19}$$

where the numbers α and β are defined by the values I_{1smp} for $m = 9,\ 10$. For $n \in [11; \infty)$, the amplitudes of probabilities of the radiative transition are obtained by an approximation of the form

$$J_{np} = \frac{1}{n^2}\left(\xi + \frac{1}{n}\rho + \frac{1}{n^2}\chi\right), \tag{20}$$

where the numbers ξ, ρ and χ are defined by the values J_{mp} for $m = 8,\ 9,\ 10$. In the formulas (11) and (12) the summation is limited by the value of the principal quantum number $n_{\max} = 500$.

The result of Fig. 2, a demonstrates the leading role of the Thomson resonance in the section of scattering.In the region of the „cold" ($\omega_1 \sim I_{1snp}$) and „hot" ($\omega_1 \sim 2\omega - I_{1snp}$) photons, the „mirror-symmetrical" resonance structures of the section of scattering of Fig. 2, b demonstrate the effect of destructive (quenching) quantum interference of the partial probability amplitudes $1s \to np$ of photoexcitation in the amplitude (11), having the negative part. The found new „mirror-symmetrical"structures of the section of scattering are similar to those described in the papers [18–20]. However, they are different in „attributability" to the principal quantum numbers of the resonances of scattering and strong (~ 2 keV) energy separation inside the regions

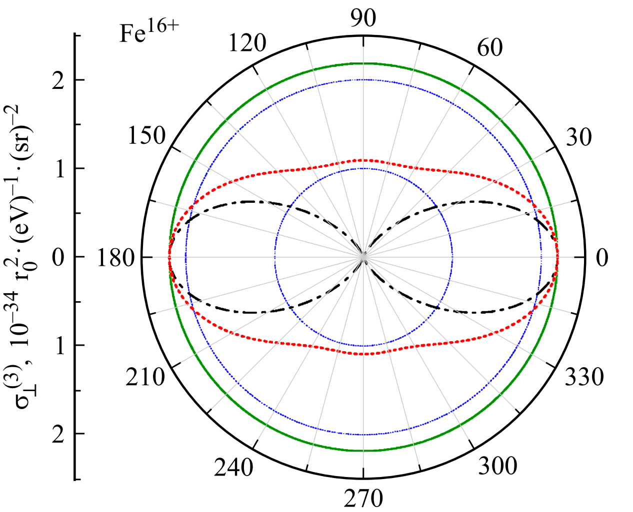

Figure 4. Indicatrices of elastic photon-photon scattering in the field of the ion Fe^{16+} with the polar radius $\sigma^{(3)}$ and the polar angle θ at the fixed values of the energies of the incident ($\hbar\omega = 12.40\,\text{keV}$) and scattered ($\hbar\omega_1 = \hbar\omega_2 = \hbar\omega$; Thomson resonance) photons. The schemes of the experiment: $\perp$ (the solid curve), $\|$ (the dash-dotted curve), the unpolarized curve (the dashed curve).

of the „cold" ($\omega_1 \sim \omega - I_{1snp}[20] \to \omega_1 \sim I_{1snp}$) and „hot" ($\omega_1 \sim \omega + I_{1snp}[20] \to \omega_1 \sim 2\omega - I_{1snp}$) photons. It has been evaluated that the respective sections of the resonance photon-photon scattering of the third order of the perturbation theory exceed those of the second order by the ~ 12 orders. The results of Fig. 3, *a* and Fig. 3, *b* have been compared to demonstrate the effect of destructive quantum interference of the amplitudes of probability of Thomson (M_1) and „mirror-symmetrical" (M_2) resonances, which enter into the full probability of scattering with different signs (G). At the same time, the wide ($\Delta\omega_1 \sim 5.2\,\text{keV}$) windows of „transmission" (abrupt drop in the probability of elastic photon-photon scattering) between the Thomson and „mirror-symmetrical" resonances. The result of Fig. 4 for the scheme of the experiment with non-polarized photons (18) demonstrates pronounced angular anisotropy of scattering — predominant scattering in the direction of the angles $\theta = 0^\circ, 180^\circ$. This result qualitatively reproduces the same for elastic photon-photon scattering through QED vacuum [35].

4. Conclusion

The nonrelativistic version of the quantum theory of the elastic photon-photon scattering process in the field of the multi-charge neon-like atomic ion has been formulated. The following is established: the leading role of Thomson resonance in the section of scattering; the presence of pronounced resonance structures of the section of scattering in the regions of „cold" ($\omega_1 \sim I_{1snp}$) and „hot" ($\omega_1 \sim 2\omega - I_{1snp}$) scattered photons; the presence of the destructive quantum interference both between the components of the „mirror-symmetrical" structures of the section and between the amplitudes of probability of the transitions into the virtual electron states of the continuous and discrete spectra; the presence of angular anisotropy (in the corresponding schemes of the proposed experiment) of scattering. Going beyond the dipole approximation for the $\hat{R}$- and $\hat{Q}$-operators of the transition and taking into account the following orders of the Tamm-Dancoff approximation belongs to future development of the theory. At last, we evaluate the section of the „observed" Thomson resonance (Fig. 3, 4) in the XFEL-experiment. At the average radiance of the laser (the number of photons in the laser pulse $N = 10^{17}$ [34], European XFEL), by virtue of the theorem on the sum of mutual exclusivity probabilities (selection of two of N photons falling onto the ion) we have

$$\frac{N!}{2!(N-2)!} \cdot \sigma_{\perp}^{(3)} \cong 80 \left(\frac{\text{mbarn}}{\text{eV} \cdot \text{sr}^2} \right).$$

For the differential section integrated by the angles of photon scattering in the axially-symmetrical scheme of the experiment, we have

$$\frac{N!}{2!(N-2)!} \left(\frac{d\sigma_{\perp}}{d\omega} \right) \cong 2.6 \left(\frac{\text{barn}}{\text{eV}} \right).$$

The values obtained are quite measurable.

Conflict of interest

The authors declare that they have no conflict of interest.

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