The influence of space curvature on the moment of inertia tensor of axisymmetric magnetic field of radiopulsar

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Received May 12, 2023
Revised July 24, 2023
Accepted October, 30, 2023

The influence of space curvature on input of magnetic field outside neutron star to moment of inertia tensor of radiopulsar in case of Shwartzshild metric is considered. It is considered both the case of pure dipolar magnetic field and the case of „small scale" magnetic field but only in the case of a single axisymmetric harmonic.

Keywords: general relativity, neutron star, precession.

DOI: 10.61011/TP.2023.12.57743.f240-23

Introduction

Isolated radio pulsars are considered to be one of the most stable sources of periodic signals. Nevertheless, due to the constant loss of rotational energy to accelerate the particles and form their radiation, the period of pulsars P is constantly increasing. At the same time, slowly rotating pulsars with periods $P > 100$ ms, in addition to regular deceleration, show some cyclic changes in their parameters with a characteristic time scale $T \sim 10^3 - 10^4$ year [1]. One possible explanation for such variations — is the precession of the neutron star due to the difference in its shape from the spherical or anomalous braking moment (see, for example, [2]), which is actually the moment of inertia of the star's magnetic field [3]. Apparently, the precession of the pulsar B1828-11 is due to the difference in the shape of the neutron star from the spherical one [4]. It is possible that the recurrence of FRB flares is also related to the free precession of the neutron star (see, e.g., [5]). It should be noted, however, that the flattening of a neutron star due to its rotation does not contribute to precession, and the flattening caused by the deformation of the star by its magnetic field is comparable to the „oblateness" due to the anomalous magnetic moment (see, for example, estimates in [6]). It is worth noting that both of these effects provide a precession period of the order of $T \sim 10^3 - 10^4$ year for slowly rotating pulsars with periods of $P > 100$ ms. In this paper, we consider the effect of the curvature of space-time around a neutron star on the moment of inertia of its magnetic field outside the star. At the same time, we assume that the metric outside the star coincides with the Schwarzschild metric, and neglect the corrections to the metric due to the rotation of the star. In this paper, we consider the case of both the dipole magnetic field and the non-dipole „of the small-scale" field, but we limit ourselves to the case of only one axisymmetric harmonic.

1. Model

The solution for the magnetic field of a neutron star in the Schwarzschild metric was considered in the paper [7], in the axisymmetric case $m = 0$ it has the form [7]:

$$B^\alpha = B_{\hat r} \cdot \sqrt{h}\,\delta^\alpha_r + B_{\hat\theta} \cdot \frac{1}{r}\,\delta^\alpha_\theta,$$

$$B_{\hat r} = \sum_{l=0}^{+\infty} b_{l0}\,(l+1)\,\left(\frac{r_{ns}}{r}\right)^{l+2} f_l(r)\,Y_{l0},$$

$$B_{\hat\theta} = -\sum_{l=0}^{+\infty} b_{l0}\,\left(\frac{r_{ns}}{r}\right)^{l+2} \sqrt{h(r)}\,g_l(r)\,\frac{\partial Y_{l0}}{\partial\theta}, \qquad (1)$$

where $h(r) = 1 - r_g/r$, $r_g = 2GM_{ns}/c^2$ — the gravitational radius of the star, M_{ns} — the mass of the star, r_{ns} — its radius, Y_{l0} — spherical functions at $m = 0$ and functions $f_l(r)$ and $g_l(r)$ are defined as [7]:

$$f_l(r) = F\left(l,\ l+2\,;\,2(l+1)\,,\ \frac{r_g}{r}\right),$$

$$g_l(r) = F\left(l+1,\ l+2\,;\,2(l+1)\,,\ \frac{r_g}{r}\right). \qquad (2)$$

The components of the angular momentum of the field in the Schwarzschild metric have the form [8]:

$$L_A = -\int \sqrt{-g}\ T^0_m(\vec x, t)\cdot \xi^m_A\, d^3x\,, \quad A = x, y, z\,, \qquad (3)$$

where $g = \det(g_{ik})$, g_{ik} — is the space-time metric, T^n_m — is the energy-momentum tensor of the field, and ξ^m_A — are the Killing vectors corresponding to the spherical symmetry of the metric [9]:

$$\xi^m_x = -\sin\phi\,\delta^m_\theta - \mathrm{ctg}\,\theta\cdot\cos\phi\,\delta^m_\phi,$$

$$\xi^m_y = \cos\phi\,\delta^m_\theta - \mathrm{ctg}\,\theta\cdot\sin\phi\,\delta^m_\phi,$$

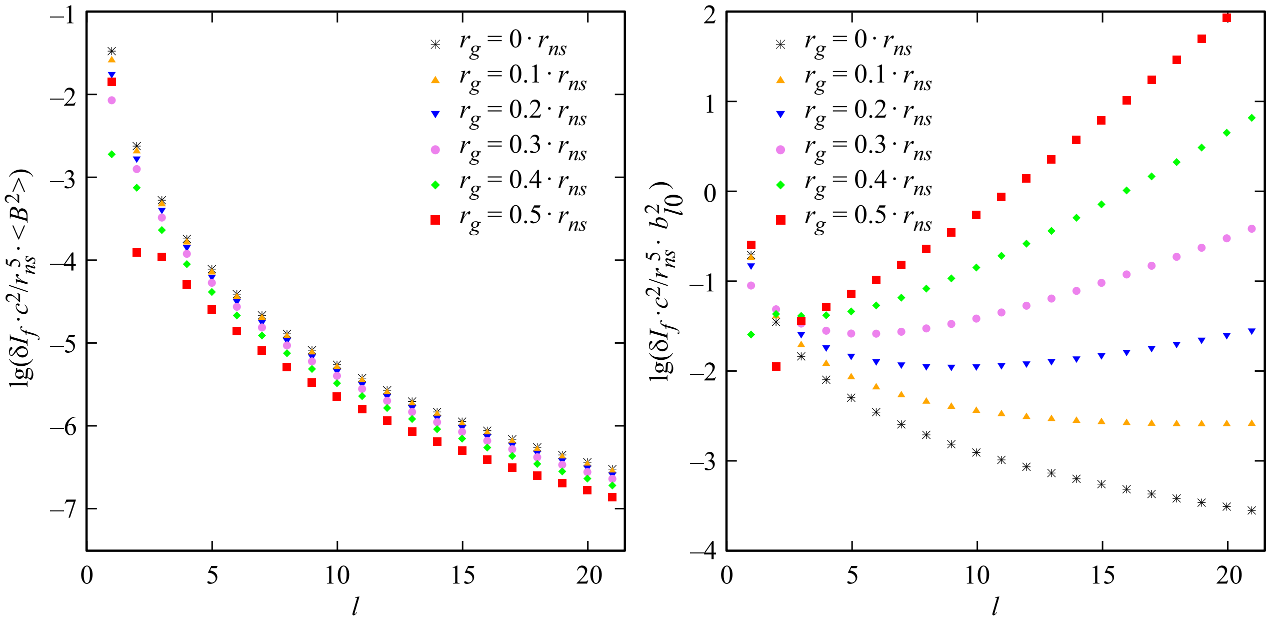

Figure 1. Dependence of δI_f on l for different values r_g/r_{ns}.

$$\xi_z^m = \delta_\phi^m. \tag{4}$$

Thereat, considering that the electric field in the magnetosphere is shielded by plasma, which means that the electric field is equal to zero in the reference system rotating with the star, we get that the angular momentum of the field outside the star is equal to [3]:

$$L_A = I_{AB}\,\Omega_B + O(\Omega^2), \tag{5}$$

where $\Omega = 2\pi/P$ — is the angular velocity of rotation of the star and I_{AB} — moment of inertia of the field outside the star [3]. In the case of $m = 0$, this expression can be written as [3]:

$$\vec{L} = I_f\,\vec{\Omega}_f + \delta I_f \vec{e}_z\,(\vec{e}_z \cdot \vec{\Omega}), \tag{6}$$

where the coefficient δI_f describes the difference between the field inertia tensor and the spherically symmetric case. In the case of one harmonic l at $m = 0$, the expression for δI_f of [3] retains its form in the Schwarzschild metric as well:

$$\delta I_f = \frac{b_{l0}^2\,r_{ns}^4}{c^2} \cdot \frac{l\,(l+1)^3}{(2l+3)\,(2l-1)} \times$$

$$\times \left(\frac{2l^2+2l+3}{2l^2\,(l+1)^2} \cdot Q_\perp - Q_r \right), \tag{7}$$

and only the $Q_r(r)$ and $Q_\perp(r)$ functions of [3] change their appearance slightly due to the curvature of space:

$$Q_r = \frac{1}{l^2} \int_0^{r_{LC}} \frac{1}{h(r)} \left(\frac{r_{ns}}{r}\right)^{2l} f_l^2(r)\,dr,$$

$$Q_\perp = \int_0^{r_{LC}} \left(\frac{r_{ns}}{r}\right)^{2l} g_l^2(r)\,dr, \tag{8}$$

where $r_{LC} = c/\Omega$ — is the radius of the light cylinder.

2. Results

Fig. 1 shows the dependence of δI_f on the number l for different values of the r_g/r_{ns} ratio. Fig. 2 shows the dependence of $\delta I_f/\,\delta I_f|_{r_g=0}$ on the number l , i.e. the difference between the value of δI_f and its value in the case of a flat space $\delta I_f|_{r_g=0}$. Fig. 3 shows the relationship $\delta I_f/\,\delta I_f|_{r_g=0}$ as a function of r_g/r_{ns}. In all the figures on the right graph, the values δI_f were calculated at a constant value of b_{l0}, i.e. at a constant moment at infinity, at the left — with a constant value of the mean square of the field $< B^2 >$ on the surface of the star. It can be seen that in the case of constant $< B^2 >$ the magnitudes δI_f do not change very much due to the curvature of space, decreasing by a factor of only 2–10. In the case of a constant moment at infinity b_{l0} the difference from the case of flat space can reach $10^2 - 10^3$ times, which is a manifestation of the magnetic field magnification noted in [7] in the Schwarzschild metric. We note that the decrease in the value of δI_f in the case of constant $< B^2 >$ is related to the same phenomenon: in the Schwarzschild metric, the high harmonics of the field are pressed more strongly against the neutron star, and accordingly, the contribution of the field at large distances from the star to the inertia tensor decreases. Though, definitely both options are acceptable, in our opinion, it is more reasonable for neutron stars to consider the values of the magnitude δI_f at the given field value on the surface, i.e. with „constant“ value $< B^2 >$. And hence, when considering the contribution of the magnetic field outside the neutron star at the moment of inertia of the pulsar, one can safely neglect the corrections associated with the curvature of space-time. It is also worth noting that the peculiarity in the graph for

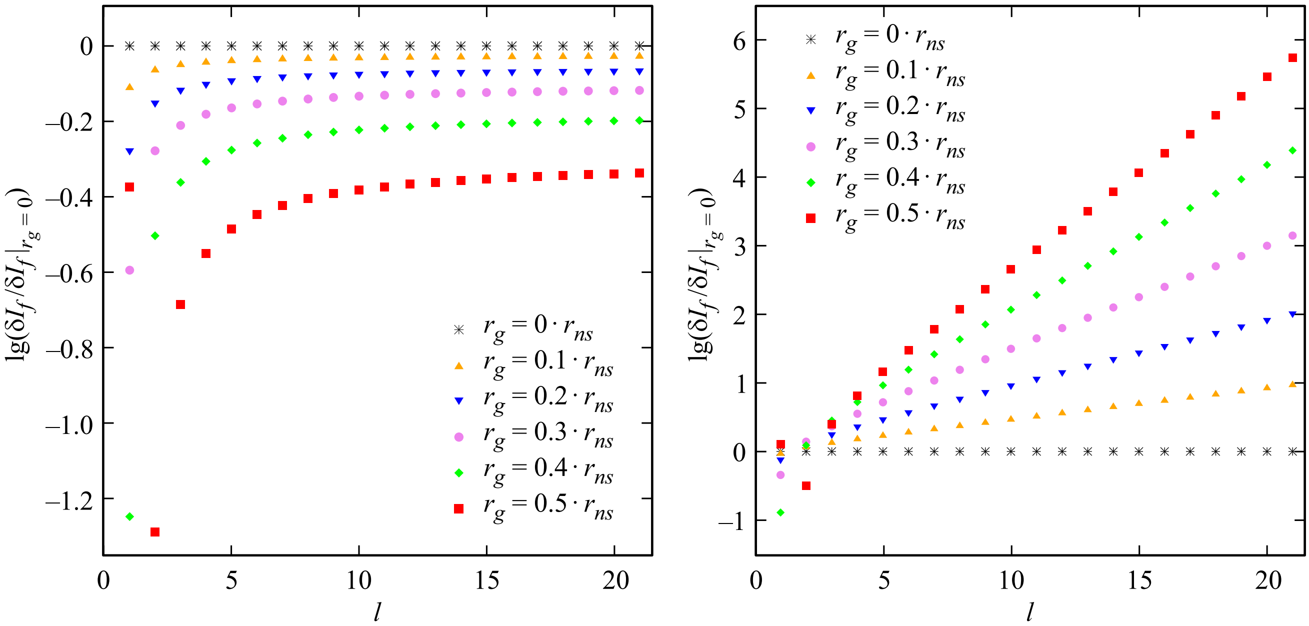

Figure 2. Relationship of $\delta I_f / \delta I_f|_{r_g=0}$ on l for different values r_g/r_{ns}.

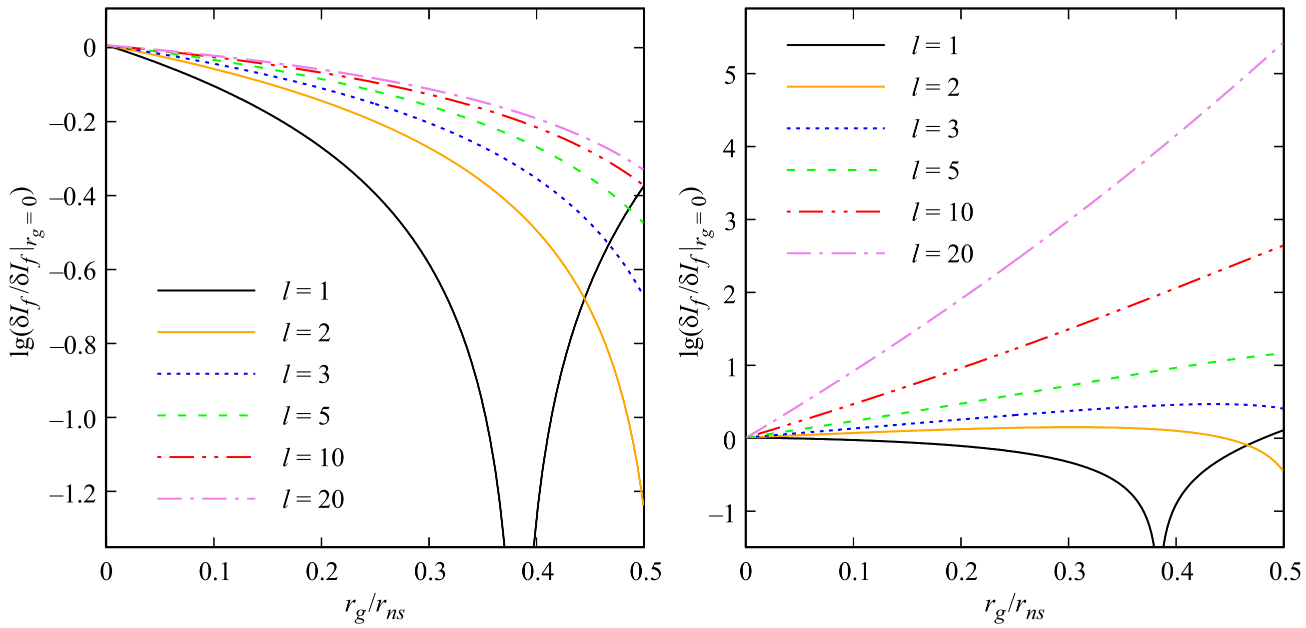

Figure 3. Dependence of the relation $\delta I_f / \delta I_f|_{r_g=0}$ on the value of r_g/r_{ns} for different l.

the case of the dipole field $l = 1$ at $r_g \approx 0.38\, r_{ns}$ is due to the fact that at this point the value of δI_f changes the sign. In the paper, we used the approximation (5) and neglected the corrections $\sim (\Omega r_{ns}/c)^2$. This is valid for slowly rotating pulsars with periods of $P \gtrsim 100$ ms. At the same time, for such pulsars, one can neglect the corrections for the difference between the space-time metric and the Schwarzschild metric due to the rotation of the neutron star, since this gives a correction at the angular momentum of the order of magnitude of the discarded terms $\sim (\Omega r_{ns}/c)^2$.

In the case of rapidly rotating pulsars, the task becomes much more complicated even in the case of a flat metric, since in this case it is necessary to correctly consider the deceleration of the pulsar, considering the influence of the pulsar's magnetosphere on it (see, e.g., in [2] comments about the difficulty of calculating these corrections), as well as considering the processes of particle acceleration in the magnetosphere (see e.g. [10]). And, accordingly, the rotation of the pulsar will be extremely complicated and will not be described by simple precession.

Conflict of interest

The authors declare that they have no conflict of interest.

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