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Mathematical modeling of refraction of light in asymmetrical gravitational field

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Instrument of mathematical modeling of trajectory characteristics of light at propagation in gravitational field of group of astrophysical objects has been suggested. The influence of gravitational field has been taken into account by introducing an effective index of refraction of vacuum. It is shown that difficult spatial structure of light forms on picture plane of observer at a asymetrical gravitational field. Estimation of spatial weakening of energy flux has been performed by summing of relative energetic portions of light and with help of calculation of light divergance. Comparison of results of light and partial modeling of light flux in gravitational field with different configuration has been performed.

Keywords: refraction of light, astrophysical objects, geometrical optics, gravitational lensing.

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Introduction

It follows from the general theory of relativity [1–3] that the propagation of light in curved (Riemannian) space can be regarded as a problem of waves in Euclidean space, if consider that the gravitational field changes the refractive index of the vacuum in a certain way. Inasmuch as the spatial scales of the modified refractive index are large compared to the wavelength of light, the calculation of the characteristics of light in the gravitational field of astrophysical objects can be performed in the approximation of geometric optics [4]. For simple gravitational objects, the real ray trajectories of the waves can be replaced by asymptotes [1,2]. However, in the case of a group of objects, the method of asymptotic representation of ray trajectories is not accurate enough, and the calculation of true trajectories is necessary to interpret the measurement data of the new generation of astrophysical precision instruments.

1. Mathematical Simulation Apparatus

Geometrooptical calculation of the direction of light propagation in the gravitational field of a group of astrophysical objects was carried out in the range of angular sighting parameters α_n, β_n based on a system of ray differential equations in the spherical case [4]:

$$\frac{dR}{d\varphi} = R\cot\beta,$$

$$\frac{d\beta}{d\varphi} = (1+\sin^2\beta\tan^2\alpha)\left(\frac{1}{n}\left(\frac{\partial n}{\partial\varphi}\cot\beta - R\frac{\partial n}{\partial R}\right) - 1\right),$$

$$\frac{d\delta}{d\varphi} = \tan\alpha,$$

$$\frac{d\alpha}{d\varphi} = (1+\cos^2\alpha\cot^2\beta)\left(\frac{\partial n}{\partial\delta} - \frac{\partial n}{\partial\varphi}\tan\alpha\right), \qquad (1)$$

where R, δ, φ — radial and angular coordinates of the beam, respectively; α, β — beam refractive angles; n — effective refractive index of the vacuum given the influence of the gravitational field. As is known [3], the effective refractive index of a vacuum can be expressed in terms of the gravitational potential. In particular, for weak gravitational fields we have: $n \approx 1 - 2\Phi(\mathbf{R})/c^2$ where $\Phi(\mathbf{R})$ — gravitational potential, c — speed of light. In the general case, the potential $\Phi(\mathbf{R})$ produced by an arbitrary mass distribution is the solution of the Poisson equation [1]:

$$\Phi(\mathbf{R}) = -G\iiint \frac{\sigma(\mathbf{r})}{|\mathbf{R}-\mathbf{r}|}\, d^3\mathbf{r},$$

where $\sigma(\mathbf{r})$ — mass distribution density, G — gravitational constant. For a single astrophysical object with a mass of M, when $\sigma(\mathbf{r}) = \text{const}$, the potential is simple: $\Phi(R) = -GM/R$ In this case for the effective refractive index, the following should be: $n = 1 + R_g/R$ where $R_g = 2GM/c^2$ — gravitational radius of the object. By specifying various distribution functions $\sigma(\mathbf{r})$, the effective refractive index of a vacuum in the gravitational field of a group of astrophysical objects can be calculated using numerical integration of the Poisson equation. Meanwhile, it is possible to introduce an approximate model of the effective refractive index, which most simply considers the continuity and locality of the effect of gravitational objects on the curvature of space, assuming the additivity of the contributions of gravity from various objects to the general

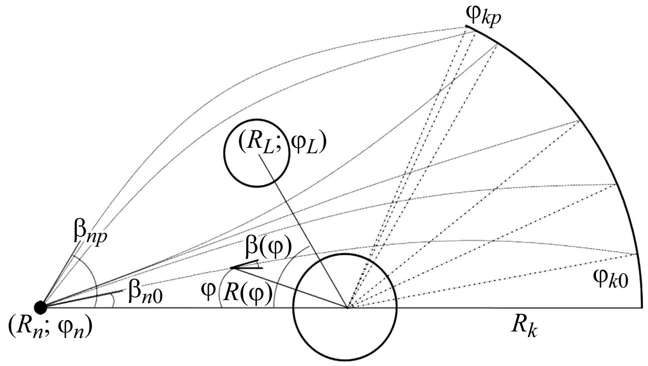

Figure 1. Geometry of the problem. Side view. (R_n, φ_n) — coordinates of the radiation source relative to the center of the main gravitational object; R_k — distance to the observer's picture plane; (R_L, φ_L) — coordinates of the additional gravitational object; $[\beta_{n0}, \beta_{np}]$ — range of initial radiation angles; $[\varphi_{k0}, \varphi_{kp}]$ — range of angular coordinates of rays arriving in the observer's picture plane.

gravitational field:

$$n(R,\varphi,\delta) = 1 + R_g/R + \sum_{i=1}^{N} A_i \exp\big[-b_{\varphi i}(\varphi-\varphi_{Li})^2 - b_{\delta i}(\delta-\delta_{Li})^2 - b_{Ri}(R-R_{Li})^2\big], \quad (2)$$

where R_g — gravitational radius of the main object; N — number of additional objects; A_i, φ_{Li}, δ_{Li}, R_{Li}, $b_{\varphi i}$, $b_{\delta i}$, b_{Ri} — respectively the contribution fraction, localization coordinates, and perturbation scales of the refractive index caused by i-m astrophysical object. Fig. 1 illustrates the geometry of the problem.

To calculate the group delay of light τ, the equation was obtained

$$\frac{d\tau}{d\varphi} = \frac{R}{cn\sin\beta}\sqrt{1+\sin^2\beta\tan^2\alpha}. \quad (3)$$

The combined solution of equations (1) and (3) makes it possible to determine the group delay and the direction of light propagation in Euclidean space with an effective refractive index (2). The calculation of the radial divergence of light in the curved Riemannian space relative to the light field in Euclidean space was carried out on the basis of the results of the integration of the system of equations attached to (1) for the derivatives of the initial parameters of the problem from the solutions of the equation (1). In particular, in the case of an asymmetric gravitational field at $\alpha_n = 0$ for the focusing factor, we have:

$$\mathrm{I} = \log_{10}\left|\frac{\sin\beta_n R_0(\varphi_0)\varphi_0}{\sin\varphi\sin\beta(\varphi)R(\varphi)R'_{\beta_n}}\right|,$$

where $R_0(\varphi_0)$, φ_0 — radial and angular coordinates of the ray in Euclidean vacuum space; $R'_{\beta_n} = \partial R/\partial\beta_n$.

2. Calculation results and discussion

Fig. 2, 3 show the results of calculations of trajectories, group delay, and attenuation of the light energy flux using the model (2) at $\delta_n = \delta(\varphi = 0) = 0$, $\alpha_n = 0$, $R_n = 50\,\mathrm{cul}$, $R_k = 50\,\mathrm{cul}$, where cul — conventional unit of length, R_k — distance to the picture plane. Sight parameter β_n varied in the range $[-0.314; -0.114]$ rad in increments of 0.0025 rad, $R_g = 1\,\mathrm{cul}$, $A_1 = 0.5$, $R_{L1} = 10\,\mathrm{cul}$, $\varphi_{L1} = 0.4$, $\delta_{L1} = 0.0$; $A_2 = 0.5$, $R_{L2} = 10\,\mathrm{cul}$, $\varphi_{L2} = 0.6$, $\delta_{L2} = 0.0$, $b_{\varphi 1} = 1.23$, $b_{\delta 1} = 0.0$, $b_{R1} = 1.23\,\mathrm{cul}^{-2}$; $b_{\varphi 2} = 1.23$, $b_{\delta 2} = 0.0$, $b_{R2} = 1.23\,\mathrm{cul}^{-2}$. Fig. 2 shows the trajectories and group delays of light in the gravitational field of three astrophysical objects. From Fig. 2, *a* it follows that the appearance of a group of objects in the path of light propagation leads to significant distortions of trajectories. Trajectory oscillations are associated with the propagation of light in a gravitational waveguide formed under the influence of the gravitational fields of objects. The penetration of light into the waveguide channel leads to an increase in the propagation path. You may also notice the condensation of the trajectories, which causes the light to focus. The effects of gravity are also manifested in the group delay of light for some values of the angular sighting parameter β_n, in particular, for rays that hit the waveguide (Fig. 2, sector of angles $\varphi \in [3.0; 3.5]$ rad). The results of the simulation of the radial divergence of light are presented in Fig. 3, *a*. In a asymmetrical gravitational field, a complex pattern of spatial attenuation of light emerges. A significant increase in the focus factor at some aiming angular parameters β_n is due to the lensing of light in the gravitational field. The different number of sectors of increase in the focus factor is due to the influence of one or more objects on the propagation of light in the gravitational field.

It is also possible to estimate the attenuation of light in the vicinity of the observer using a partial approach. It was assumed that the power of the field emitted by the source $U_0 = 1\,\mathrm{cuef}$, where cuef — is a conventional unit of measurement. Then the power value at each point of the picture plane was equal to the sum of the portions $U = \sum U_0/p\,\mathrm{cuef}$, where p — number of rays. The calculations results are presented in Fig. 3, *b*. It can be noted that in the power distribution in the angle ranges $\varphi_k \in [2.6; 2.8]$ rad and [3.0; 3.2] rad two peaks are formed. Under the influence of a single gravitational object, the length of the path of light propagation decreases, and for a gravitational waveguide — increases. Thus, using the partial approach, it is possible to obtain the distribution of light energy in the vicinity of the observer and estimate the value of the power in the focus areas.

Conclusion

On the basis of the radial approximation, an apparatus for mathematical simulation of the refractive characteristics of light in the gravitational field of a group of astrophysical

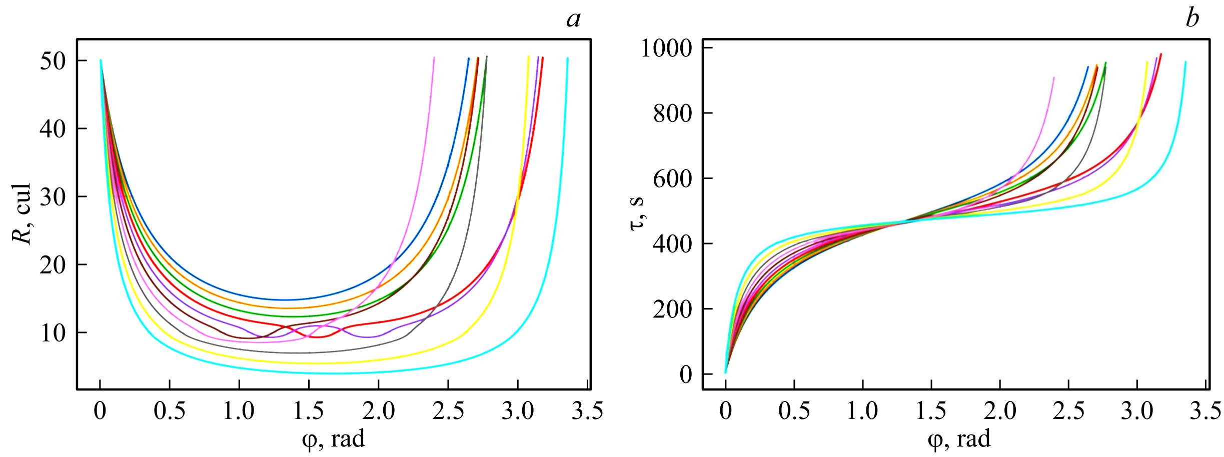

Figure 2. Trajectories (a) and group delays (b) of the light of the gravitational field of three objects.

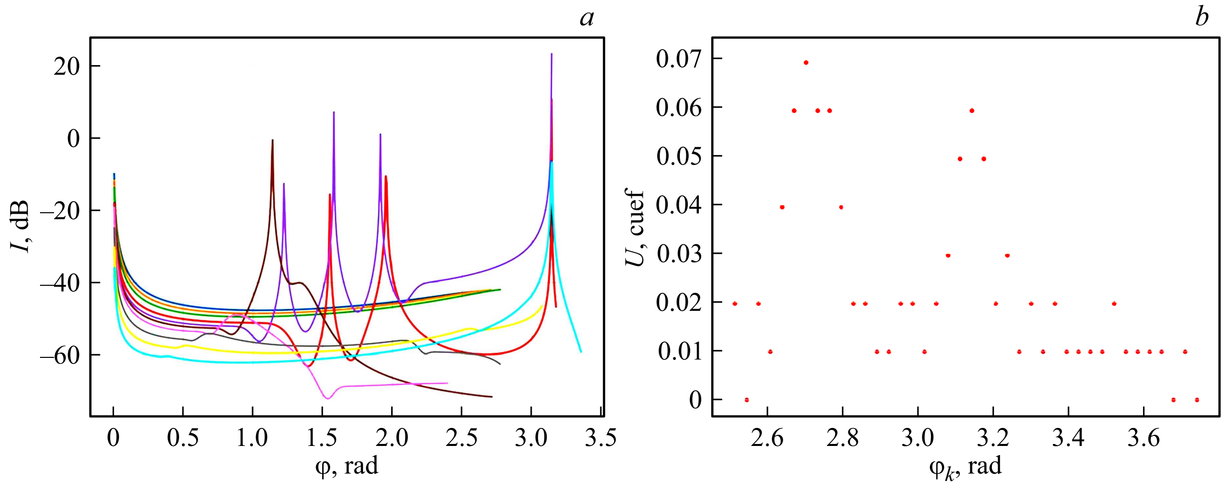

Figure 3. Focus factor (a) and relative power (b) of light propagating in an asymmetrical gravitational field.

objects is proposed. The influence of gravity is considered by using the effective refractive index of the vacuum. An approximate model of the refractive index is proposed, considering the locality and continuity of the effect of gravitational objects on the curvature of space. It is shown that depending on the configuration of the gravitational field, a complex spatial structure of light is formed in the picture plane of the observer. An estimate of the spatial attenuation of the light energy flux is made on the basis of the calculation of the radial divergence and with the help of partial summation of the rays in the region of gravitational focusing.

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Conflict of interest

The authors declare that they have no conflict of interest.

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