The effect of ultrasonic waves on the occurrence of a discharge in a liquid

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An equation similar to the Rayleigh equation is obtained, describing the oscillation of a single small bubble in a liquid, taking into account the compressibility of the liquid when exposed to a sound wave. Calculations of the dynamics of the bubble show that there are two possible variants of such an impact: the convergence of the bubble or its sharp expansion, depending on the frequency of the sound wave. The change in behavior occurs abruptly with a small change in frequency. Theoretically, this can lead to unexpected physical effects, for example, the formation of a high-temperature ionized gas, which causes a discharge in an electric field.

Keywords: bubble in liquid, ultrasound, cavitation, resonance, electric discharge.

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Cavitation phenomena in liquid media have a significant effect on the physical properties of the media. This is due to the change in the structure of the medium during cavitation and the fact that cavitation leads to abrupt bubble convergence when the bubble reaches some maximum size during growth. As Zababakhin [1] showed, this size is determined by viscous properties. Bubble „collapse" is accompanied by cumulative phenomena: an increase in temperature and pressure in the centre of the bubble. This leads to physical phenomena such as luminescence, chemical decomposition of liquid molecules and even ionization [2].

The influence of ultrasound on cavitation phenomena is generally recognized, cavitation of a liquid during expansion in a sound field is called inertial cavitation [2]. To describe the dynamics of a single bubble in an incompressible fluid, the Rayleigh equation is used, which, when ultrasound is taken into account, is transformed into the Nolting–Neppaires equation by adding to the free term of the periodic pressure of the sound wave [3]. If compressibility is taken into account, inside the fluid outside the bubble, the solution has the appearance of a travelling wave, since the conservation of matter equation is transformed into a wave equation. In liquid, the speed of sound wave propagation is much greater than the speed of sound in gas (in water — 1500 m/s). Therefore, delaying the appearance of a wave at a distance much larger than the bubble size will not affect the velocity of the flow, which is integrated in the derivation of the Rayleigh equation. Thus, at these distances, the solution must be sought in the form of a harmonic oscillation. The velocity of the fluid outside the bubble must be of the form $V = V_0\cos(\omega t)$, since the velocity V must continuously transition to the incompressible fluid velocity V_0, since the solution in the incompressible fluid corresponds to the zero frequency of the ultrasonic field ω. The harmonic solution is a simplification of the travelling wave solution: $V = V_0\cos((d - at)/\lambda)$. Here d — the distance from the center of the bubble, a — the speed of sound in the medium, λ — the wavelength of the sound perturbation, the frequency of sound $\omega = 2\pi a/\lambda$.

If the travelling wave propagation is not taken into account, the wave equation turns into an incompressible fluid equation: $\mathrm{div}\mathbf{V} = 0$. So, the expression for the velocity amplitude V_0 under the condition of spherical symmetry will be the same as for an incompressible fluid: $V_0 = U(R/d)^2$, where U — the velocity of the bubble boundary.

In presented setting, a small concentration of bubbles is assumed, which reduces the probability of their merging mechanism. The dynamics of a single bubble are studied. The viscosity of the fluid is also taken into account. The diffusion and evaporation processes on the bubble surface are not taken into account in this formulation. The same derivation of the well-known Rayleigh equation for a single bubble is made with the procedure of integrating three integrals over the entire space surrounding the bubble. In essence, the Rayleigh equation is a balance of pressure in the bubble and pressure due to the inertial forces of the surrounding mass. Then the Rayleigh equation for the bubble radius R (cm [4], equation (1)) takes the form of

$$\cos(\omega t)\left(\frac{R}{R_0}\right)\left(\frac{\ddot{R}}{\omega^2 R_0}\right) + \left[2\cos(\omega t) - \frac{1+\cos(2\omega t)}{4}\right] \times\left(\frac{\dot{R}}{\omega R_0}\right)^2 + \sin(\omega t)\left(\frac{\dot{R}}{\omega R_0}\right) + \left\{\frac{p_0 - p_{vap}}{\omega^2\rho R_0^2} + \frac{2\sigma}{\omega^2\rho R_0^3}\left(\frac{R_0}{R}\right) - \left[\frac{p_0 - p_{vap}}{\rho R_0^2\omega^2} + \frac{2\sigma}{\omega^2\rho R_0^3}\right]\left(\frac{R}{R_0}\right)^{-3\gamma}\right\} + \frac{4\nu}{\omega R_0^2}\cos(\omega t)\left(\frac{R_0}{R}\right)\left(\frac{\dot{R}}{\omega R_0}\right) = 0. \tag{1}$$

Here, the dot above the letter stands for time differentiation, R_0 — initial radius of the bubble, $\omega = 2\pi f$ (f — frequency), ρ — density of the fluid, p_0 — external pressure (atmosphere), p_{vap} — vapor pressure in the bubble, σ — surface tension of the medium, γ — ratio of specific heat capacities of the gas inside the bubble. Note that the equation goes continuously to the Rayleigh equation at $\omega = 0$. The third term in (1) is a new differential term. The last term in (1) describes the effect of the kinematic viscosity of the medium ν. It is equation (1) that is then numerically solved. In Nolting's–Neppaires equation, the fluctuations of the bubble boundary are not taken into account, so an $p_m \sin(\omega t)$ term appears in the static pressure balance term outside and inside the bubble (curly brackets). In the proposed formulation, oscillations are taken into account, and this term is reduced in the derivation process. To find out the limits of this approach, let us substitute the expression for the velocity V into the wave equation. We obtain the equation for the velocity amplitude v

$$v'' + \frac{4}{d}v' + \left[\frac{2}{d^2} + \frac{\omega^2 R_0^2}{a^2}\right] v = 0. \qquad (2)$$

If the second summand in square brackets is negligibly small, then the velocity amplitude v will not depend on frequency, and the dependence remains only in the second multiplier in the expression for V, i.e., the application of the simplified form of the solution is possible if we consider the bubble radius R_0 to be much smaller than the sound wavelength a/ω (for water at a frequency of 20 kHz — 14 cm). Thus, for small bubbles (10^{-2}, 10^{-3} cm) equation (1) can be applied since integration to infinity can be considered reasonable. The proposed formulation means that the bubbles in the fluid experience oscillations together with the entire fluid column, „breathing" with the medium. In some cases, these oscillations, made together with the medium, may enter into some resonance with the bubble's own oscillations, there is not a convergence, but a sharp increase in the size of the bubble.

The external stretching or compression (initial velocity of the bubble boundary) is the forcing force of the bubble oscillation within the chosen formulation instead of the periodic pressure term in the Nolting–Neppaires equation. For dynamic effects to occur, the calculations show that the initial velocity of the boundary u_0 must be set.

However, in a sound wave, the initial velocity of the u_0 boundary occurs together with the very stretching or compression of the fluid in the sound field. The velocity of the boundary is related to the intensity of the wave, since the volume energy of the expansion and contraction of the fluid at the bubble boundary $\rho u_0^2/2$ must be equal to the pressure amplitude in the sound wave p_m. So in the proposed formulation, the intensity of the sound wave is taken into account.

In equation (1), the second-order senior derivative is preceded by a multiplier $\cos(\omega t)$, which goes to zero at $\omega t = \pi/2$. This is a classical problem with the multiplier in front of the senior derivative going to zero.

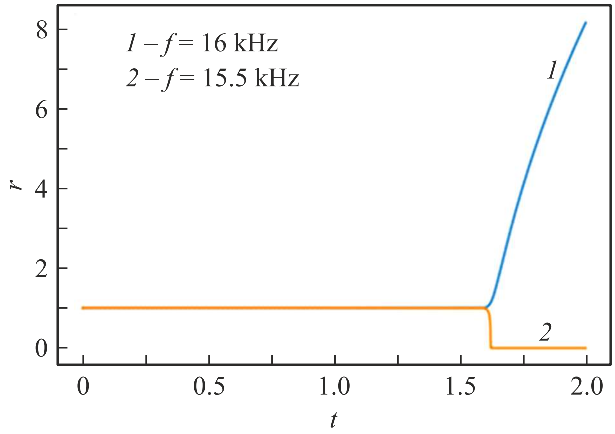

Figure 1. Qualitatively different behavior of the dimensionless bubble radius as a function of ultrasound frequency when the critical dimensionless time $\pi/2$ is reached.

Indeed, at a large step (five hundred steps on unit dimensionless time), instability is demonstrated: the numerical solution of (1) under the same conditions gives qualitatively different results. When the step is reduced by a factor of 2, the solution stabilizes but reveals different behavior depending on the ultrasound frequency. In some frequency ranges there is only convergence at $\omega t = \pi/2$, and in other — only a sharp increase in bubble radius R. But in some cases there is a dramatic change in the behavior of the solution at $\omega t = \pi/2$. Thus, for the dimensionless tensile velocity $u_0 = 10^{-4}$ and bubble size $R_0 = 10^{-5}$ m, a sharp convergence occurs at frequency $f = 15.5$ kHz. At frequency $f = 16$ kHz the bubble expands sharply (Fig. 1). A critical frequency value of 15.674 kHz has been found, beyond which the behavior changes dramatically. For a bubble an order of magnitude larger beyond the $\omega t = \pi/2$ point, the calculations show oscillations. For the smaller bubble size (10^{-5} m), the drop in size to zero is due to the fact that the rate of drop is so large that the program passes the „gap" of the solution by extrapolating beyond the „gap" limit. This may be why fluctuations are not observed.

Calculations near the critical momentum (Fig. 2) show only natural vibrations of the bubble with frequency $\omega_0^2 = [3\gamma p + 2\sigma(3\gamma - 1)/R_0]/(\rho R_0^2)$ [5] of small amplitude. At the critical moment, which is shown by the vertical line, for an expanding bubble the acceleration of the boundary is directed outward and for a collapsing bubble — inward. To fulfil this condition, the period of sound must be a multiple of the quadruple period of natural vibrations. This is the criterion for resonance.

Analytical studies near the critical time were carried out for three small parameters: δr — small deviation of the radius from unity; $\tau = \pi/2 - t$ — small time to critical moment; $\dot{u}$ — small acceleration of the boundary (assumed to be a constant value). The results of the calculations are confirmed: $\delta r = \dot{u}t/\Delta_\sigma p$, where $\Delta_\sigma p = 2\sigma(3\gamma - 1) + 3\gamma(p_0 - p_{vap})$ (here the values are dimensionless). The sign of the deviation of

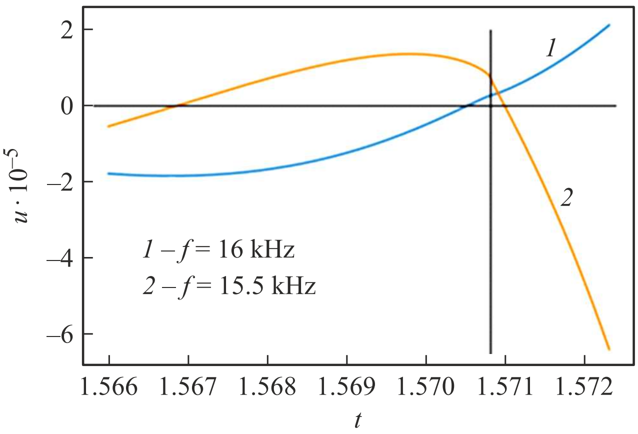

Figure 2. Calculations in the vicinity of the critical moment. At the critical moment (shown by the vertical line), the behavior of the solutions differs in the sign of the boundary acceleration: for a collapsing bubble, the acceleration is directed inwards, while for an expanding — outwards.

the bubble radius δr is equal to the sign of the time derivative of the velocity, i.e., the acceleration of the bubble boundary. It should be noted that the increase in bubble size or collapse is proportional to the second degree of frequency, since both surface tension σ, and static pressure p are inversely proportional to the second degree of frequency when returning to dimensional values.

Thus, on the basis of an equation similar to Rayleigh's equation but derived for a compressible viscous fluid for a bubble of small size (10^{-5}–10^{-4} m), it was possible to find in calculations and theoretically explain a new effect. It manifests itself in the fact that the action of ultrasound in expanding or contracting a sample as a function of frequency can change dramatically with a very small critical change in frequency near the time critical value. A peculiar resonance of sound vibrations and natural vibrations of the bubble occurs. This resonance belongs to a class of hydrodynamic instabilities that are used to explain astrophysical phenomena [6]. It can be called inertial because it is related to the expansion and contraction of the medium in a sound wave. Since sound waves are a set of waves with close frequencies, unstable behavior is possible: either simultaneous contraction and expansion of bubbles of different sizes (the natural frequency of bubble oscillation strongly depends on the size), or abrupt oscillations of the bubble with large amplitude (the rate of size change is shown to be proportional to the square of the ultrasound frequency). The simultaneous expansion and heating of the gas inside the converging bubbles leads to the formation of gaseous substances with high temperature — ionized gases. When an electric field is applied in such a medium, an electric discharge occurs, which is caused precisely by the ultrasonic action. There are reports that such experiments have been done, although they have not been explained [7].

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Conflict of interest

The authors declare that they have no conflict of interest.

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