Instability of steady states with inhomogeneous field in electron-positron plasma diode

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Received May 13, 2023
Revised September 29, 2023
Accepted October, 30, 2023

Instability of steady-states of the plasma diode with inhomogeneous field without reflection from potential extrema are studied. It is shown that all considered solutions are unstable. We have also confirmed this result when simulating small perturbation evolution of a steady-state solution..

Keywords: plasma diode, electron and positron beams, solution stability, nonlinear oscillations.

DOI: 10.61011/TPL.2023.12.57602.54A

The emission of pulsars in the radio band has been attracting the attention of researchers for more than 50 years, but the mechanism of formation of this emission, as well as the explanation of its features, in particular jumps between modes, still raises questions [1,2]. Relatively recently, it has been suggested that pulsar emission is associated with collective processes in the electron-positron plasma of the pulsar diode [3].

In the work [4], a hypothesis was put forward according to which the pulsar emission is caused by fluctuations of the electric field in the plasma arising from the instability of stationary states. Stationary solutions for a diode with counter electron and positron fluxes are studied in detail in [5]. In [4], the equation for the amplitude of a small perturbation of the potential distribution (PD) is derived and its analytical solution is found in the case of a homogeneous PD. It was shown that the homogeneous stationary solution is stable only at diode length $\delta < \sqrt{2}\pi\lambda_D$, where λ_D — the Debye length of Hückel.

In the present work, the stability of inhomogeneous stationary solutions is studied in the case when all charged particles reach the opposite electrode without reflection from the potential extrema. Note that analytical methods for investigating the stability of diode states with electron beam for inhomogeneous plasma have been used in [6,7]. As in work [4], consider that the monoenergetic electron flux comes from the left electrode with density $n_{e,0}$ and non-relativistic velocity $v_{e,0}$, and the positron flux — from the right electrode with density $n_{p,0} = n_{e,0}$ and velocity $v_{p,0} = -v_{e,0}$. The particle energies are $W_0 \equiv m_e v_{e,0}^2/2 = m_p v_{p,0}^2/2$, where m_e and m_p — the masses of the electron and positron, respectively. We consider that the particles are absorbed at the electrodes. The potential difference between the electrodes U is assumed to be zero.

We convert to dimensionless units by choosing the particle energy W_0 and the Debye length of Hückel $\lambda_D = \left[(2\tilde{\varepsilon}_0 W_0)/(e^2 n_{e,0})\right]^{1/2}$ (e — the electron charge, and $\tilde{\varepsilon}_0$ — the dielectric permittivity of the vacuum) as units of energy and length. For dimensionless coordinate, potential and electric field strength we have: $\xi = z/\lambda_D$, $\eta = e\Phi/(2W_0)$ and $\varepsilon = eE\lambda_D/(2W_0)$.

The stationary solutions are defined by three dimensionless parameters: the interelectrode distance δ, the potential difference between the electrodes V, and the electric field strength at the left electrode ε_0. At a fixed potential difference, these solutions are conveniently represented by points on the plane (ε_0, δ), which form separate curves — branches of solutions [5].

Stationary solutions are characterized by wave-like PDs. The branches of the solutions for the regime without particle reflection from the extrema of the potential are shown in Fig. 1. They are labelled n_k, where the index k — number of extrema on the RD In the case when $V = 0$, and particles enter the plasma from opposite electrodes with identical charges (of opposite sign), masses, and kinetic energies, the RD must have odd symmetry about the center of the gap [4].

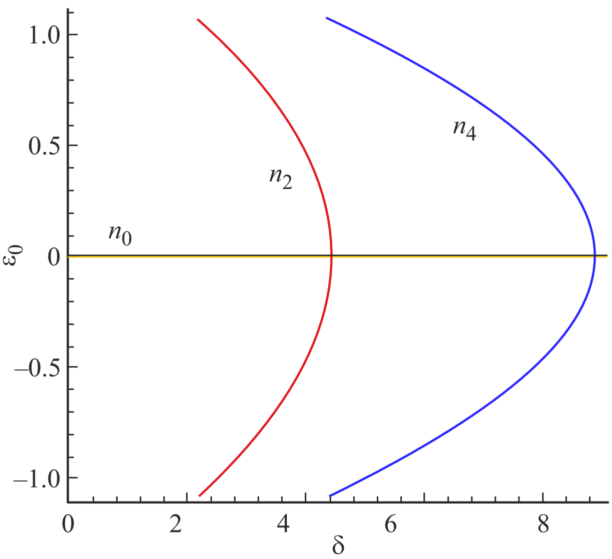

Figure 1. The branches of the stationary solutions n_0, n_2 and n_4 for the case of monoenergetic charged particle beams at $V = 0$.

Due to the symmetry in the case of $V = 0$, only solutions corresponding to the branches of n_k at $k = 0, 2, 4, \ldots$ can exist. A typical distribution of a potential of the form n_2 is shown in Fig. 2.

Let us consider the evolution of small perturbations of the solutions corresponding to these branches. Let's represent the RD as

$$\eta(\xi,\tau) = \eta_0(\xi) + \tilde{\eta}(\xi)\exp(-i\Omega\tau), \quad |\tilde{\eta}(\xi)| \ll |\eta_0(\xi)|. \tag{1}$$

Here $\eta_0(\xi)$ — unperturbed PD, $\tilde{\eta}(\xi)$ — the amplitude of the PD perturbation, τ — dimensional time, and $\Omega = \omega + i\,\Gamma$ — complex frequency.

The equation for $\tilde{\eta}(\xi)$ is obtained by linearizing the Poisson equation, into which are substituted expressions for the concentrations of charged particles movingin field (1).

For the case of absence of reflection of charged particles in [4], the following is obtained

$$\tilde{\eta}'(\xi) + \int_0^{\xi} dx[u_{e,0}(x)]^{-3} \int_0^{x} dy\, \tilde{\eta}'(y) \exp\{i\Omega[\sigma_e(\xi) - \sigma_e(y)]\}$$
$$+ \int_{\xi}^{\delta} dx[u_{p,0}(x)]^{-3} \int_x^{\delta} dy\, \tilde{\eta}'(y) \exp\{i\Omega[\sigma_p(\xi) - \sigma_p(y)]\}$$
$$= \tilde{\eta}'(\delta) + \int_0^{\delta} dx[u_{e,0}(x)]^{-3} \int_0^{x} dy\, \tilde{\eta}'(y)$$
$$\times \exp\{i\Omega[\sigma_e(\delta) - \sigma_e(y)]\}. \tag{2}$$

Here,

$$u_{e,0}(\xi) = [1 + 2\eta_0(\xi)]^{1/2}, \quad u_{p,0}(\xi) = [1 - 2\eta_0(\xi)]^{1/2},$$
$$\sigma_e(\xi) = \int_0^{\xi} dx\, [u_{e,0}(x)]^{-1}, \quad \sigma_p(\xi) = \int_{\xi}^{\delta} dx\, [u_{p,0}(x)]^{-1} \tag{3}$$

are the velocities of the electron and positron at the point ξ and the travel times of these particles from the corresponding electrode to this point in the unperturbed field.

Solving equation (2), we can find an expression for the perturbation potential $\tilde{\eta}(\xi)$. The boundary condition at the right electrode $\tilde{\eta}(\delta;\Omega) = 0$ gives a dispersion equation whose solutions determine the relationship between the natural frequency Ω and the dimensionless diode length δ (called dispersion branches). In the case where the increment $\Gamma > 0$, the stationary solution is unstable.

Let's introduce a new function $\varphi(\xi) = \tilde{\eta}'(\xi)$. Since the solution of equation (2) is defined to the multiplicative constant proportional to the value of the electric field perturbation at the right boundary $-\tilde{\eta}'(\delta)$, let us put $\varphi(\delta) = 1$. After changing the order of integration in the double integrals, equation (2) reduces to the Fredholm integral equation of the second kind with respect to the function $\varphi(\xi)$:

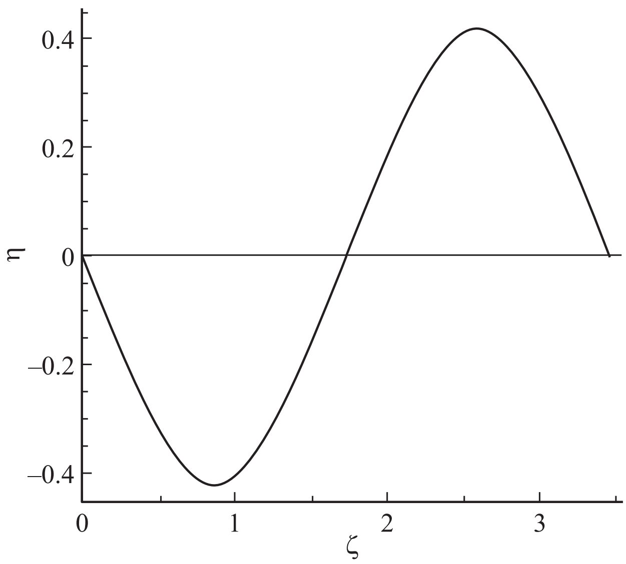

Figure 2. Typical RD for the n_2 branch at $V = 0$.

$$\varphi(\xi;\Omega) + \int_0^{\delta} dy K(\xi,y;\Omega)\,\varphi(y;\Omega)$$
$$- \int_0^{\delta} dy\, K(\delta,y;\Omega)\,\varphi(y;\Omega) = 1, \tag{4}$$

where

$$K(\xi,y;\Omega) = P(\xi,y)\,Q(\xi,y;\Omega),$$
$$P(\xi,y) = \begin{cases} \int_y^{\xi} dx\, [u_{e,0}(x)]^{-3}, & y \le \xi, \\ \int_{\xi}^{y} dx\, [u_{p,0}(x)]^{-3}, & y \ge \xi, \end{cases}$$
$$Q(\xi,y;\Omega) = \begin{cases} \exp\{i\Omega[\sigma_e(\xi) - \sigma_e(y)]\}, & y \le \xi, \\ \exp\{i\Omega[\sigma_p(\xi) - \sigma_p(y)]\}, & y \ge \xi. \end{cases} \tag{5}$$

Given that $\varphi(\xi) = \tilde{\eta}'(\xi)$, on the right boundary we obtain

$$\tilde{\eta}(\delta;\Omega) = \int_0^{\delta} dy\, \varphi(y;\Omega) + \tilde{\eta}(0;\Omega). \tag{6}$$

Since the value of the potential at the electrodes is given, then $\tilde{\eta}(0;\Omega) = 0$, $\tilde{\eta}(\delta;\Omega) = 0$, and from (6) we obtain the dispersion equation

$$\int_0^{\delta} dy\, \varphi(y;\Omega) = 0. \tag{7}$$

Analysis of solutions of equation (7) allows us to determine the stability of stationary solutions with inhomogeneous PDs. Thus, to obtain the dispersion equation, it is

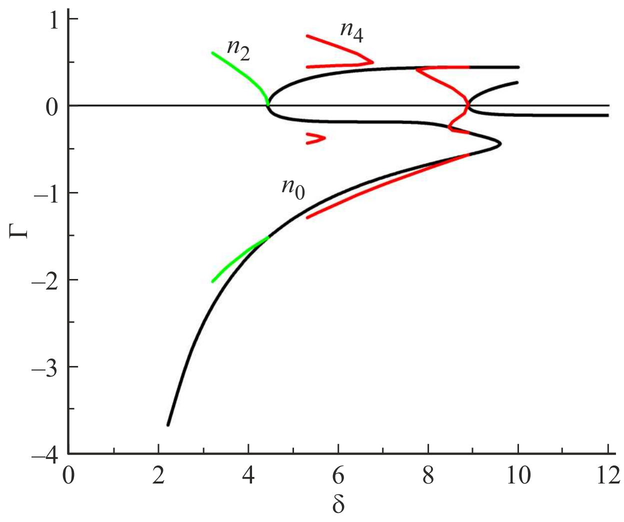

Figure 3. Aperiodic variance branches for solutions corresponding to the branches n_0, n_2 and n_4.

necessary to solve the integral equation (4), i.e. i.e. determine the function $\varphi(\xi;\Omega)$.

In the general case, it is not possible to find the solution of this equation analytically. Therefore, we propose a numerical method for its solution. To do this, we divide the entire interval $[0, \delta]$ into N intervals and replace the integrals in (5) by sums using the trapezoidal method. For the values of the unknown function at the nodes we obtain a system of linear equations. Substituting the solutions of this system into the dispersion equation (7) allows us to determine the natural frequencies and find both aperiodic and oscillatory dispersion branches.

The aperiodic dispersion branches corresponding to the solution branches n_0, n_2 and n_4, are shown in Fig. 3. The branch for n_0 is exactly the same as that obtained in work [4], where equation (4) was solved analytically. As can be seen from Fig. 3, the stationary solutions corresponding to the branch n_2, are aperiodically unstable (increment $\Gamma > 0$). On the branch n_4, there is a window of values $6.766 < \delta < 7.752$, in which the stationary states are aperiodically stable. However, an examination of the oscillatory branches in this interval shows that one of them has a positive increment; hence, these stationary solutions are unstable with respect to small oscillatory perturbations.

The dependences of Γ on δ for RDs corresponding to the same branch n_k, but with different signs of the tensions at the left boundary ε_0, in the case of $V = 0$ coincide. Moreover, this fact is true for both aperiodic and oscillatory branches.

Thus, it is shown that the inhomogeneous stationary solutions corresponding to the branches $n_2$and n_4 are unstable, i.e., such stationary states cannot exist. We can assume that the stationary solutions corresponding to the n_{2s} with $s \geq 3$ branches are also unstable with respect to small perturbations.

In a numerical study of the stability of stationary solutions in an electron-positron diode, we study the evolution of a small perturbation of the stationary electric field distribution. We select velocity distribution functions (VDFs) on electrodes in the form of small-width„gates“ of $\Delta \ll 1$: $f_0^{(\pm)}(u) = (2\Delta)^{-1}$ in the velocity interval $u \in [\mp 1-\Delta, \mp 1 + \Delta]$ and $f_0^{(\pm)}(u) = 0$ outside this interval (here the signs „minus“ and „plus“ correspond to electrons and positrons).

Stationary solutions should be slightly different from those forδ-shaped VDF. For solutions belonging to the n_{2i} branches (where $i = 0, 1, \ldots$), the particle concentrations are determined using the following formulae:

$$n_\pm(\eta) = \frac{1}{2\Delta}\left[\sqrt{(1+\Delta)^2 \mp 2\eta} - \sqrt{(1-\Delta)^2 \mp 2\eta}\right]. \quad (8)$$

Substituting the particle concentrations (8) into the Poisson equation, multiplying both parts of it by η' and integrating the result over the potential, we obtain the equation for η', from which we can find the relationship between ε_0 and the minimum potential η_m. Integrating the obtained equation for η', we find the distributions of the potential and electric field along the coordinate. As a parameter we use the electric field strength at the left boundary ε_0. The position of the minimum ξ_m is found by integrating from η_m to 0.

Since the RD has symmetry about the point $\xi = \xi_m$, the center of the gap is at the point $\xi = 2\xi_m$, and the value of the gap δ for the branch n_2 is $4\xi_m(\varepsilon_0)$.

We used two different numerical codes to model the evolution of the electric field distribution and the VDF of charged particles: PIC code and EK code.

In the PIC code, the VDF modelling considers individual particles moving in an electric field defined at grid nodes. The„cloud-in-a-cell“ (linear particle contribution to the density at neighboring nodes) [8] model is used to find the charge density at grid nodes. To calculate the electric field at the grid nodes, the Poisson equation is solved and a linear approximation [8] is used between the nodes. To find the position of the particles at the next instant of time, the„method of stepping over“ [8] with step h_τ. At the end of each step, particles that hit the electrodes are removed from the calculation and particles that arrived from the electrodes with random velocities uniformly distributed over the interval $[\mp 1 - \Delta, \mp 1 + \Delta]$, at a random time instant distributed uniformly over the interval h_τ, are added.

The numerical algorithm implemented in the EK code is described in detail in works [9,10]. The particle trajectories are traced back in time up to the moment of departure from the electrode. The field distribution is found from the Poisson equation. The calculations are iterated at each step to ensure self-consistency.

Calculations were performed at values of the interelectrode gap $\delta = 3$ and 4 for stationary solutions with both positive and negative values of ε_0. After a short transient, the maximum disturbance value $\tilde{\eta}_M(\tau)$ grows exponentially with increments of $\Gamma = 0.7$ for $\delta = 3$ and $\Gamma = 0.33$ for $\delta = 4$. This agrees well with the increment values given by the semi-analytical method for monoenergetic beams ($\Gamma = 0.68$ and 0.31), as well as with the result of the calculation using the PIC code ($\Gamma = 0.31$ in the case of $\delta = 4$).

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by J.Deineka