05

The effect of paired collisions of charge carriers on electrical conductivity thin conductive layer

© E.V. Zavitaev,[1] O.V. Rusakov,[2] E.P. Chukhleb[3]

[1] Mytischi Branch of Bauman Moscow State Technical University,
141005 Mytischi, Moscow Region, Russia
[2] State University of Humanities and Technology,
142611 Orekhovo-Zuevo, Russia
[3] Extended Education Center „Junior Academy of Sciences Impulse“,
142432 Chernogolovka, Moscow Region, Russia
e-mail: eduardzavitaev@yandex.ru

Received July 5, 2023
Revised August 29, 2023
Accepted September 3, 2023

For the first time, the problem of the effect of pair collisions of charge carriers on the electrical conductivity of a thin conducting layer was solved analytically. Permissible values of the layer thickness are limited by the dimensions at which quantum and skin effects do not appear. However, the ratio of the layer thickness to the free path of electrons can be arbitrary. The conditions of specular-diffuse reflection of charge carriers from the layer surfaces are taken as the boundary conditions of the problem. Limiting cases are considered and the results obtained are discussed.

Keywords: thin layer, local conductivity, integral conductivity.

DOI: 10.61011/TP.2023.11.57495.169-23

Introduction

Extensive development of new microelectronic technologies involves deeper theoretical description of the effect of charge carrier scattering mechanisms on electromagnetic properties of small conducting bodies [1].

In addition to the volume and surface electron (hole) scattering, pair collisions of charge carriers contribute a lot to the conductivity of such bodies [2,3], whereby common physical considerations suggest that their typical rate is limited.

It should be noted that average path of charge carriers in many standard semiconductors and metals at room temperature considerably exceeds the de Broglie wavelength. Thus, the situation occurs, when quantum size effects may be neglected, but classical size effects must be considered [4].

In [1], the problem of static conductivity of thin metal film in a perpendicular magnetic field has been solved considering diffuse and mirror-diffuse boundary conditions of charge carrier reflection.

In [5], a model of dependence of local conductivity of a thin metal layer on the external electric field frequency was built for the first time, and in [6–8], kinetic theory of conductivity and Hall constant of a thin conducting film was built up considering various reflectances of the film surfaces.

This study for the first time discusses the effect of pair collisions of charge carriers on conductivity of a thin conducting layer without considering the skin effect on the assumption that the layer thickness is low compared with the typical skin layer depth [9].

1. Problem formulation

A conducting thin layer with thickness b, length L and cross-section area S, to which AC voltage ω is applied, is discussed herein. The electric field is parallel to the layer and is oriented along the Z axis, the X axis is perpendicular to the layer. Field strength $\mathbf{E}$ may be expressed depending on time t:

$$\mathbf{E} = \mathbf{E_0} \exp(-i\omega t). \tag{1}$$

On the assumption that the non-equilibrium Fermi-Dirac function or electrons (holes) $f(x, \mathbf{v}) = f_0(\varepsilon) + f_1(x, \mathbf{v})$ satisfies the Boltzmann equation [10], we have

$$v_x \frac{\partial f_1}{\partial x} + ev_z E \frac{\partial f_0}{\partial \varepsilon} - i\omega f_1 = -\frac{f_1}{\tau}, \tag{2}$$

where e is the electron (hole) charge, v_z, v_x are the corresponding electron (hole) velocity vector projections on the coordinate axes, τ is the electron (hole) relaxation time, $\varepsilon = mv^2/2$ is the kinetic electron (hole) energy, v is the electron (hole) velocity vector modulus $\mathbf{v}$, m is the effective electron (hole) mass.

Here,

$$f_0 = \frac{1}{\exp[(\varepsilon - \mu)/kT] + 1}, \tag{3}$$

$$\frac{\partial f_0}{\partial \varepsilon} = -\frac{\exp[(\varepsilon - \mu)/kT]}{\{\exp[(\varepsilon - \mu)/kT] + 1\}^2 kT}, \tag{4}$$

where μ is the chemical potential, k is the Boltzmann constant, T is the absolute temperature.

Note that in thermodynamic equilibrium conditions, the chemical potential μ depends on the temperature T, effective mass m and equilibrium electron (hole) gas concentration n, i.e. $\mu = \mu(T, m, n)$. Depending on the specified parameters, μ for the electron (hole) gas may accept values within $\mu \in (-\infty, +\infty)$.

High frequency current density $\mathbf{j}$ induced by the applied voltage is calculated as follows:

$$\mathbf{j} = 2e\left(\frac{m}{h}\right)^3 \int \mathbf{v} f_1 d^3\mathbf{v}. \tag{5}$$

Electron (hole) concentration n calculated using the Fermi–Dirac distribution:

$$n = 2\left(\frac{m}{h}\right)^3 \int f_0 d^3\mathbf{v}, \tag{6}$$

where h is Planck's constant.

At low temperatures, pair collisions of charge carriers are considerable, kinetic equation (2) to consider them will be written as follows [2]:

$$\begin{aligned} &- i\omega f_1 + v_x \frac{\partial f_1}{\partial x} + e v_z E \frac{\partial f_0}{\partial \varepsilon} = \\ &= -\frac{1}{\tau}\left(f_1 - \frac{g_0 m}{\int f_0 d^3\mathbf{v}} v_z \frac{\partial f_0}{\partial \varepsilon} \int v_z f_1 d^3\mathbf{v}\right), \end{aligned} \tag{7}$$

where g_0 is the parameter $(0 \le g_0 \le 1)$ that describes the intensity of pair collisions of charge carriers. Note that at $g_0 = 0$, collisions are not revealed, and when $g_0 = 1$, such collisions are fully present.

By applying

$$f_1(x, \mathbf{v}) = g(x, \mathbf{v}) \frac{\partial f_0}{\partial \varepsilon} \exp(-i\omega t),$$

function to equation (7), we get

$$\nu g + v_x \frac{\partial g}{\partial x} + e v_z E_0 = \frac{g_0 m}{\tau \int f_0 d^3\mathbf{v}} v_z \int v_z g \frac{\partial f_0}{\partial \varepsilon} d^3\mathbf{v}, \tag{8}$$

where $\nu = 1/\tau - i\omega$.

2. Distribution function

Equation (8) will be solved using the moment method [10]:

$$g = a_1(x) v_z + a_2(x) v_z \operatorname{sign}(v_x). \tag{9}$$

Taking into account (9), equation (8) is written as

$$\begin{aligned} &\nu(a_1 v_z + a_2 v_z \operatorname{sign}(v_x)) + v_x v_z \frac{\partial a_1}{\partial x} \\ &+ v_x v_z \operatorname{sign}(v_x) \frac{\partial a_2}{\partial x} + e v_z E_0 = \frac{g_0 m}{\tau \int f_0 d^3\mathbf{v}} v_z \\ &\times \int v_z (a_1 v_z + a_2 v_z \operatorname{sign}(v_x)) \frac{\partial f_0}{\partial \varepsilon} d^3\mathbf{v}. \end{aligned} \tag{10}$$

Taking into account (3), integral is as follows

$$\int f_0 d^3\mathbf{v} = 4\pi \int v^2 f_0 dv = 4\pi \int \frac{v^2 dv}{\exp[(\varepsilon - \mu)/kT] + 1}. \tag{11}$$

Introduce relative variables

$$u = \frac{\varepsilon}{kT}, \quad u_\mu = \frac{\mu}{kT}, \tag{12}$$

whence

$$u = \frac{\varepsilon}{kT} = \frac{mv^2}{2kT} \Rightarrow v = \sqrt{\frac{2kTu}{m}}, \quad dv = \sqrt{\frac{kT}{2m}} \frac{du}{\sqrt{u}}.$$

Then for (11), we get

$$\int f_0 d^3\mathbf{v} = 2\pi \left(\frac{2kT}{m}\right)^{3/2} I_0, \tag{13}$$

where

$$I_0 = \int_0^\infty \frac{\sqrt{u} du}{\exp(u - u_\mu) + 1}. \tag{14}$$

Considering (13), expression (6) will be written as

$$n = 2\left(\frac{m}{h}\right)^3 \int f_0 d^3\mathbf{v} = 4\pi \left(\frac{m}{h}\right)^3 \left(\frac{2kT}{m}\right)^{3/2} I_0. \tag{15}$$

To calculate the integral in the right side of equation (10), it is convenient to use the cylindrical coordinate system $(v_\perp, \varphi, v_z)$ in the velocity space:

$$v_x = v_\perp \cos\varphi, \quad v_\perp^2 + v_z^2 = v_L^2, \tag{16}$$

where $v_L = \gamma^{1/2} \cdot v_C$ is the quasi limit velocity of charge carriers (γ is the coefficient of proportionality that will be introduced later), and v_C is the typical charge carrier velocity that is introduced as follows:

$$\begin{aligned} n v_C^2 &= \frac{5}{3} \int v^2 f_0 \frac{2 d^3(m\mathbf{v})}{h^3} = \\ &= 16\pi \frac{5}{3} \frac{k^2 T^2}{h^3} \sqrt{2mkT} \int \frac{u^{3/2} du}{\exp(u - u_\mu) + 1}. \end{aligned}$$

After applying (15), we have

$$v_C^2 = \frac{5}{3} \frac{2kT}{m I_0} \int \frac{u^{3/2} du}{\exp(u - u_\mu) + 1}$$

or in dimensionless form

$$\tilde{v}_C = \sqrt{\frac{m}{2kT}} v_C = I_0^{-1/2} \left(\frac{5}{3} \int_0^\infty \frac{u^{3/2} du}{\exp(u - u_\mu) + 1}\right)^{1/2}. \tag{17}$$

For the case of highly degenerate fermionic gas ($u_\mu \gg 1$) at $T \to 0$, $v_C \to v_\mathrm{F}$, where v_F — is the fermionic velocity calculated using expression (17) for the Fermi function f_0 $(T \to 0)$. For the other limit case of non-degenerate fermionic gas ($u_\mu < 0$) at $T \to \infty$, $v_C \to \sqrt{5kT/m}$, i.e. an order of the average thermal velocity of charge carriers takes place [7].

Note that, for calculation of the effect of only volume and surface charge carrier scattering on the conductivity of the conducting body, the upper boundary of the dimensionless kinetic energy integration may be assumed as formally equal to $u \to +\infty$ [11]. However, in real practice, on the assumption of physical considerations, such quasi constraint exists, because, at some charge carrier velocity v_L, the fracture of electrons with dimensionless velocities exceeding this one and the electron contribution to conductivity are negligibly low at an arbitrary value of the dimensionless chemical potential. Therefore, it is reasonable to introduce such quasi constraint when considering the effect of pair collisions of the conductivity.

Considering (4) and couplings (16), for the collision integral in equation (10), we have

$$\int v_z\left(a_1 v_z + a_2 v_z \operatorname{sign}(v_x)\right)\frac{\partial f_0}{\partial \varepsilon}\,d^3\mathbf{v} =$$

$$= -\int v_z^2\left(a_1 + a_2 \operatorname{sign}(v_x)\right)$$

$$\times\frac{\exp[(m(v_\perp^2+v_z^2)-2\mu)/2kT]}{\{\exp[(m(v_\perp^2+v_z^2)-2\mu)/2kT]+1\}^2 kT}\,v_\perp dv_z d\varphi dv_\perp.$$

Proceed to the dimensionless form of the derived expression taking into account (12) and introduce the following variables

$$u_C = \tilde{v}_C^2 = \frac{mv_C^2}{2kT},\quad u_L = \frac{mv_L^2}{2kT},\quad u_z = \frac{mv_z^2}{2kT},$$

$$u_\perp = \frac{mv_\perp^2}{2kT},\quad v_\perp\cos\varphi = \cos\varphi\sqrt{\frac{2kTu_\perp}{m}}. \tag{18}$$

Then the charge carrier collision integral

$$\int v_z\left(a_1 v_z + a_2 v_z \operatorname{sign}(v_x)\right)\frac{\partial f_0}{\partial \varepsilon}\,d^3\mathbf{v} =$$

$$= -\frac{2\sqrt{2}}{m}\left(\frac{kT}{m}\right)^{3/2}\int_0^{u_L}\int_0^{2\pi}\int_0^{u_L-u_z}\left[a_1 + a_2\operatorname{sign}(\cos\varphi)\right]$$

$$\times\frac{\exp(u_\perp+u_z-u_\mu)\sqrt{u_z}}{\{\exp(u_\perp+u_z-u_\mu)+1\}^2}\,du_z d\varphi du_\perp =$$

$$= -\frac{2\pi a_1}{m}\left(\frac{2kT}{m}\right)^{3/2}$$

$$\times\int_0^{u_L}\left(\int_0^{u_L-u_z}\frac{d\{\exp(u_\perp+u_z-u_\mu)\}}{\{\exp(u_\perp+u_z-u_\mu)+1\}^2}\right)\sqrt{u_z}\,du_z =$$

$$= -\frac{2\pi a_1}{m}\left(\frac{2kT}{m}\right)^{3/2} I_1, \tag{19}$$

Parameter value γ

u_μ	$-\infty$	-10	-5	-3	-1	0	1	5	10	∞
γ	6	5.6	5.4	5.2	5	4.8	4.4	2.5	1.8	1

where

$$I_1 = \int_0^{u_L}\int_0^{u_L-u_z}\frac{\exp(u_\perp+u_z-u_\mu)\sqrt{u_z}}{\{\exp(u_\perp+u_z-u_\mu)+1\}^2}\,du_z du_\perp =$$

$$= \int_0^{u_L}\frac{\sqrt{u_z}du_z}{\exp(u_z-u_\mu)+1} - \frac{2u_L^{3/2}}{3(\exp(u_L-u_\mu)+1)}. \tag{20}$$

For further calculations, it is convenient to express the dimensionless quasi limit charge carrier energy u_L in terms their typical dimensionless energy u_C as follows $u_L = \gamma u_C$, where the coefficient of proportionality γ may be separately calculated by numerical methods for each dimensionless chemical potential u_μ. γ values are listed in the Table.

Taking into account (13) and (19), equation (10) will be written as

$$\left(\nu + \frac{g_0 I_1}{\tau I_0}\right)a_1 v_z + \nu a_2 v_z\operatorname{sign}(v_x) + v_x v_z\frac{\partial a_1}{\partial x}$$

$$+ v_x v_z \operatorname{sign}(v_x)\frac{\partial a_2}{\partial x} + e v_z E_0 = 0. \tag{21}$$

Multiplying equation (21) step-by-step, first, by the electron velocity projection v_z, and then by $v_z\operatorname{sign}(v_x)$, and integrating over the whole velocity space, we will derive two equations

$$\begin{cases}\frac{4}{5}\left(\nu + \frac{g_0 I_1}{\tau I_0}\right)a_1 + \frac{v_L}{4}\frac{\partial a_2}{\partial x} = -\frac{4}{5}eE_0,\\ a_2 = -\frac{5v_L}{16\nu}\frac{\partial a_1}{\partial x}.\end{cases} \tag{22}$$

Whence it follows that

$$\frac{\partial^2 a_1}{\partial x^2} - \lambda^2 a_1 = \lambda^2\frac{eE_0}{\nu\beta^2}, \tag{23}$$

where

$$\beta = \sqrt{1 + g_0 I_1/(\nu\tau I_0)} = |\nu\tau = 1 - i\omega\tau = 1 - i\Omega\tau v_C/b$$

$$= 1 - i\Omega/\Delta = \chi/\Delta| = \sqrt{1 + g_0 I_1\Delta/(I_0\chi)},$$

$\lambda = 16\nu\beta/5v_L$, $\chi = \Delta - i\Omega$ — the dimensionless complex carrier scattering frequency .

Moment coefficient $a_1(x)$ will be found by solution of second order non-homogeneous differential equation (23):

$$a_1(x) = A_0 + C_1\exp(\lambda x) + C_2\exp(-\lambda x), \tag{24}$$

where $A_0 = -eE_0/\nu\beta^2$; C_1, C_2 are integration constants.

Then from the second equation of system (22), it follows that

$$a_2(x) = \beta C_2\exp(-\lambda x) - \beta C_1\exp(\lambda x). \tag{25}$$

Applying (24) and (25) to (9), we find the general equation solution view (8), i.e.

$$g = \big(A_0 + C_1 \exp(\lambda x) + C_2 \exp(-\lambda x)\big) v_z + \big(\beta C_2 \exp(-\lambda x) - \beta C_1 \exp(\lambda x)\big) v_z \operatorname{sign}(v_x). \quad (26)$$

Apply the limit conditions at the upper ans lower boundaries of the conducting layer to find coefficients C_1 and C_2:

$$\begin{cases} g(v_x, x) = q_1 g(-v_x, x), & v_x < 0,\ x = b, \\ g(v_x, x) = q_2 g(-v_x, x), & v_x > 0,\ x = 0, \end{cases}$$

where q_1 and q_2 are reflectance coefficients of the layer surfaces.

Taking into account (26), the system of boundary conditions may be written as

$$\begin{cases} A_0 + C_1 \exp(\lambda b) + C_2 \exp(-\lambda b) + \beta C_1 \exp(\lambda b) - \\ \beta C_2 \exp(-\lambda b) = q_1 \big[A_0 + C_1 \exp(\lambda b) + C_2 \exp(-\lambda b) + \\ \beta C_2 \exp(-\lambda b) - \beta C_1 \exp(\lambda b)\big], \\ A_0 + C_1 + C_2 + \beta C_2 - \beta C_1 = \\ q_2 [A_0 + C_1 + C_2 + \beta C_1 - \beta C_2]. \end{cases}$$

Whence

$$\begin{cases} D_1 = \dfrac{(1-q_1)(q_2-\beta q_2-\beta-1) - \exp(-\lambda b)(1-q_2)(\beta q_1+q_1+\beta-1)}{\exp(-\lambda b)(\beta q_1+q_1+\beta-1)(1-\beta-q_2-\beta q_2) - \exp(\lambda b)(1+\beta-q_1+\beta q_1)(q_2-\beta q_2-\beta-1)}, \\ D_2 = \dfrac{(1-q_1)(1-\beta-q_2-\beta q_2) - \exp(\lambda b)(1-q_2)(1+\beta-q_1+\beta q_1)}{\exp(-\lambda b)(\beta q_1+q_1+\beta-1)(1-\beta-q_2-\beta q_2) - \exp(\lambda b)(1+\beta-q_1+\beta q_1)(q_2-\beta q_2-\beta-1)}, \end{cases} \quad (27)$$

where new notations $D_1 = C_1/A_0$, $D_2 = C_2/A_0$ are introduced.

Then write expressions (24) and (26) as follows:

$$a_1(x) = A_0\big(1 + D_1 \exp(\lambda x) + D_2 \exp(-\lambda x)\big), \quad (28)$$

$$g = A_0\Big[\big(1 + D_1 \exp(\lambda x) + D_2 \exp(-\lambda x)\big) v_z + \big(\beta D_2 \exp(-\lambda x) - \beta D_1 \exp(\lambda x)\big) v_z \operatorname{sign}(v_x)\Big]. \quad (29)$$

Relations (29), (27) and (4) fully define deviation $f_1(x, \mathbf{v})$ of the distribution function from the equilibrium one in case of mirror-diffuse reflection of carriers at the upper and lower boundaries of the conducting layer taking into account their pair collisions.

3. Conductivity calculation

Refining the form of $f_1(x, \mathbf{v})$ using (29) and (4), find the current density projection $\mathbf{j}$ inside the layer on the axis Z. Applying equation (5) and considering (1), (12), (18), we have

$$j_z = 2e\left(\frac{m}{h}\right)^3 \int v_z f_1 d^3\mathbf{v} = -2eA_0\left(\frac{m}{h}\right)^3 \exp(-i\omega t) \times \int \frac{v_z^2 \exp[(\varepsilon-\mu)/kT]}{\{\exp[(\varepsilon-\mu)/kT]+1\}^2 kT} \times \Big[1 + D_1 \exp(\lambda x) + D_2 \exp(-\lambda x) + \big(\beta D_2 \exp(-\lambda x) - \beta D_1 \exp(\lambda x)\big)\operatorname{sign}(v_x)\Big] d^3\mathbf{v} =$$

$$= \frac{4\pi e^2 E}{m\nu\beta^2}\left(\frac{m}{h}\sqrt{\frac{2kT}{m}}\right)^3 [1 + D_1 \exp(\lambda x) + D_2 \exp(-\lambda x)] \times \int_0^{u_L}\int_0^{u_L-u_z} \frac{\sqrt{u_z}\exp(u_\perp + u_z - u_\mu)}{\{\exp(u_\perp + u_z - u_\mu)+1\}^2} du_z du_\perp =$$

$$= -\frac{neI_1}{mI_0}\exp(-i\omega t) a_1(x). \quad (30)$$

Here, the integrals I_0, I_1 are defined by equations (14) and (20).

Expression for the local layer conductivity σ is derived as a result of Ohm's law in a differential form

$$\sigma = \frac{j_z}{E} = -\frac{neI_1 a_1(x)}{mI_0 E_0}.$$

Taking into account (15), (17) and (28), we have

$$\sigma = \frac{\sigma_0 I_1}{\chi\beta^2 \tilde{v}_C}\big(1 + D_1 \exp(\lambda x) + D_2 \exp(-\lambda x)\big), \quad (31)$$

where $\lambda = 16\nu\beta/5v_L = 16\nu\tau\beta/5\tau v_L = 16\beta\chi/5\Delta\tau v_L = 16\beta\chi/(5b\gamma^{1/2})$, $\sigma_0 = 8\pi e^2 bmkT/h^3$ — static specific conductivity of substance.

On introducing the dimensionless coordinate inside the layer $\xi = x/b$, write expressions for local conductivity (31) and coefficients D_1, D_2 in the dimensionless form $\sigma^* = \sigma/\sigma_0$, where

$$\sigma^* = \frac{I_1}{\chi\beta^2\tilde{v}_C}\left(1 + D_1^* \exp\left(\frac{16\chi\beta}{5\sqrt{\gamma}}\xi\right) + D_2^* \exp\left(-\frac{16\chi\beta}{5\sqrt{\gamma}}\xi\right)\right), \quad (32)$$

$$D_1^* = \frac{(1-q_1)(q_2-\beta q_2-\beta-1) - \exp(-16\chi\beta/5\sqrt{\gamma})(1-q_2)(\beta q_1+q_1+\beta-1)}{\exp(-16\chi\beta/5\sqrt{\gamma})(\beta q_1+q_1+\beta-1)(1-\beta-q_2-\beta q_2) - \exp(16\chi\beta/5\sqrt{\gamma})(1+\beta-q_1+\beta q_1)(q_2-\beta q_2-\beta-1)},$$

$$D_2^* = \frac{(1-q_1)(1-\beta-q_2-\beta q_2) - \exp(16\chi\beta/5\sqrt{\gamma})(1-q_2)(1+\beta-q_1+\beta q_1)}{\exp(-16\chi\beta/5\sqrt{\gamma})(\beta q_1+q_1+\beta-1)(1-\beta-q_2-\beta q_2) - \exp(16\chi\beta/5\sqrt{\gamma})(1+\beta-q_1+\beta q_1)(q_2-\beta q_2-\beta-1)}.$$

On integrating expression (30), considering (15), (17) and (28), we define the total current through the layer cross-section

$$I = \int_0^b j_z dS = -\frac{neSI_1}{mbI_0}\exp(-i\omega t)\int_0^b a_1(x)dx =$$

$$= \sigma_0\frac{ESI_1}{\chi\beta^2\tilde{v}_C}\left(1 + \frac{D_1}{\lambda b}\big(\exp(\lambda b) - 1\big) + \frac{D_2}{\lambda b}\big(1 - \exp(-\lambda b)\big)\right). \quad (33)$$

Formally from (33), using Ohm's law in the form of $I = GU$, where U is the voltage at the layer ends, we derive the equation for integral conductivity of the conducting layer G (electric field inside the layer is homogeneous, therefore $U = EL$, L is the layer length):

$$G = \frac{G_0 I_1}{\chi\beta^2 \tilde{v}_C}\left(1+\frac{D_1}{\lambda b}\left(\exp(\lambda b) - 1\right)+\frac{D_2}{\lambda b}\left(1 - \exp(-\lambda b)\right)\right), \tag{34}$$

where $G_0 = \sigma_0 S/L$.

Note that in a special case equations (31) and (34) define contribution to the specific and integral conductivities of electron-electron collisions in metal [10], and, when there is no amendment to the Wiedemann-Franz law, when $g_0 = 0$, coincide with findings of [5], where another mathematical approach to the problem was used.

Write expression (34) in the dimensionless form $G^* = G/G_0$, where

$$G^* = \frac{I_1}{\chi\beta^2\tilde{v}_C}\left(1 + \frac{5\sqrt{\gamma}D_1^*}{16\chi\beta}\left(\exp(16\chi\beta/5\sqrt{\gamma}) - 1\right) + \frac{5\sqrt{\gamma}D_2^*}{16\chi\beta}\left(1 - \exp(-16\chi\beta/5\sqrt{\gamma})\right)\right) \tag{35}$$

(dimensionless coefficients D_1^* and D_2^* were defined above).

4. Limit cases

4.1. Degenerate electron gas

Consider the degenerate electron gas case, when $\exp(U_\mu) \gg 1$ (metal layer ($v_L = v_C \to v_F$, since $\gamma \to 1$)). Then from (14) it follows that

$$I_0 = \int_0^\infty \frac{\sqrt{u}\,du}{\exp(u - u_\mu) + 1} \approx \int_0^\infty \sqrt{u}\,\theta(u_\mu - u)du = \frac{2}{3}u_\mu^{3/2},$$

where $\theta(u_\mu - u)$ is the Heaviside function. Thus, electron gas concentration (15) considering (12)

$$n \approx 4\pi\left(\frac{m}{h}\right)^3\left(\frac{2kT}{m}\right)^{3/2}\frac{2}{3}u_\mu^{3/2} = 4\pi\left(\frac{m}{h}\right)^3\left(\frac{2kT}{m}\right)^{3/2}\frac{2}{3}\left(\frac{\mu}{kT}\right)^{3/2}.$$

Since $\mu = mv_F^2/2$, we get

$$n = 4\pi\left(\frac{m}{h}\right)^3\left(\frac{2kT}{m}\right)^{3/2}\frac{2}{3}\left(\frac{mv_F^2}{2kT}\right)^{3/2} = \frac{8\pi}{3}\left(\frac{m}{h}\right)^3 v_F^3.$$

Change the integration order in (20) considering (12)

$$I_1 = \int_0^{u_C}\int_0^{u_C - u_\perp} \frac{\exp(u_\perp + u_z - u_\mu)\sqrt{u_z}}{\{\exp(u_\perp + u_z - u_\mu) + 1\}^2}\,du_\perp du_z$$

$$\approx \int_0^{u_\mu}\int_0^{u_\mu - u_\perp} \delta(u_z - (u_\mu - u_\perp)\sqrt{u_z})du_\perp du_z$$

$$= \int_0^{u_\mu}\sqrt{u_\mu - u_\perp}du_\perp = \frac{2}{3}u_\mu^{3/2} = \frac{2\mu^{3/2}}{3(kT)^{3/2}}.$$

Then local conductivity (32)

$$\sigma \approx \frac{8\pi e^2 bmkT}{h^3\chi\beta^2\tilde{v}_C}\frac{2\mu^{3/2}}{3(kT)^{3/2}} \times \left(1 + D_1^*\exp\left(\frac{16\chi\beta}{5}\xi\right) + D_2^*\exp\left(-\frac{16\chi\beta}{5}\xi\right)\right).$$

Considering (17), we have

$$\sigma = \frac{8\pi m^2 v_F^2 e^2 b}{3h^3\chi\beta^2}\left(1 + D_1^*\exp\left(\frac{16\chi\beta}{5}\xi\right) + D_2^*\exp\left(-\frac{16\chi\beta}{5}\xi\right)\right) = \frac{ne^2\tau}{m}\frac{\Delta}{\chi\beta^2} \times \left(1 + D_1^*\exp\left(\frac{16\chi\beta}{5}\xi\right) + D_2^*\exp\left(-\frac{16\chi\beta}{5}\xi\right)\right). \tag{36}$$

Within this limit, integral layer conductivity (35) is defined using expression

$$G = \frac{ne^2\tau S}{mL}\frac{\Delta}{\chi\beta^2}\left(1 + \frac{5D_1^*}{16\chi\beta}\left(\exp(16\chi\beta/5) - 1\right) + \frac{5D_2^*}{16\chi\beta}\left(1 - \exp(-16\chi\beta/5)\right)\right).$$

4.2. Non-degenerate electron gas

Now, consider the non-degenerate electron gas case, when $\exp(-U_\mu) \gg 1$ ($\mu < 0$, $v_C \to \sqrt{5kT/m}$, $\gamma \to 6$). From (14) , it follows that

$$I_0 = \int_0^\infty \frac{\sqrt{u}\,du}{\exp(u - u_\mu) + 1} \approx \int_0^\infty \sqrt{u}\exp(u_\mu - u)du = \frac{\sqrt{\pi}}{2}\exp(u_\mu).$$

Then electron gas concentration (15) considering (12)

$$n \approx 4\pi\left(\frac{m}{h}\right)^3\left(\frac{2kT}{m}\right)^{3/2}\frac{\sqrt{\pi}}{2}\exp\left(\frac{\mu}{kT}\right).$$

Using (20), the following can be written

$$I_1 = \int_0^{u_L} \frac{\sqrt{u_z}du_z}{\exp(u_z - u_\mu) + 1} - \frac{2u_L^{3/2}}{3(\exp(u_L - u_\mu) + 1)}$$

$$\approx \exp(u_\mu)\left(\int_0^{u_L} \sqrt{u_z}\exp(-u_z)du_z - \frac{2}{3}u_L^{3/2}\exp(-u_L)\right) =$$

$$= \left| \begin{array}{c} \int_0^{u_L} \sqrt{u_z}\exp(-u_z)du_z = \\ -\sqrt{u_z}\exp(-u_z)\big|_0^{u_L} + \frac{\sqrt{\pi}}{2}\operatorname{erf}(\sqrt{u_z})\big|_0^{u_L} = \\ -\sqrt{u_L}\exp(-u_L) + \frac{\sqrt{\pi}}{2}\operatorname{eft}(\sqrt{u_L}) \end{array} \right| =$$

$$= \frac{\sqrt{\pi}}{2}\exp(u_\mu)\Big(\operatorname{erf}(\sqrt{u_L}) - \frac{2}{\sqrt{\pi}}\sqrt{u_L}\exp(-u_L)$$

$$- \frac{2}{\sqrt{\pi}}\frac{2}{3}u_L^{3/2}\exp(-u_L)\Big),$$

where $\operatorname{erf}(\sqrt{u_L})$ is the error function.

Considering (18), for

$$u_C = \tilde{v}_C^2 = mv_C^2/2kT = |v_C \to \sqrt{5kT/m}| = 5/2,$$

$$u_L = \tilde{v}_L^2 = \frac{mv_L^2}{2kT} = |v_L = \sqrt{\gamma}v_C| = \frac{\gamma m v_C^2}{2kT} = \frac{5\gamma}{2} = 15,$$

we have

$$I_1 \approx \frac{\sqrt{\pi}}{2}\exp(u_\mu)\Big(\operatorname{erf}(\sqrt{15}) - \frac{2}{\sqrt{\pi}}\sqrt{15}\exp(-15)$$

$$- \frac{2}{\sqrt{\pi}}\frac{2}{3}(15)^{3/2}\exp(-15)\Big) \approx \frac{\sqrt{\pi}}{2}\exp(u_\mu).$$

Then according to (31), considering (12) and (18), we have

$$\sigma \approx \frac{8\pi e^2 bmkT}{h^3\chi\beta^2}\sqrt{\frac{2}{5}}\frac{\sqrt{\pi}}{2}\exp\left(\frac{\mu}{kT}\right)$$

$$\times\left(1 + D_1^*\exp\left(\frac{16\chi\beta}{5\sqrt{6}}\xi\right) + D_2^*\exp\left(-\frac{16\chi\beta}{5\sqrt{6}}\xi\right)\right)$$

$$= \frac{ne^2b}{\chi\beta^2\sqrt{5mkT}}\Big(1 + D_1^*\exp\left(\frac{16\chi\beta}{5\sqrt{6}}\xi\right)$$

$$+ D_2^*\exp\left(-\frac{16\chi\beta}{5\sqrt{6}}\xi\right)\Big)$$

$$= \frac{ne^2\tau\Delta}{m\chi\beta^2}\left(1 + D_1^*\exp\left(\frac{16\chi\beta}{5\sqrt{6}}\xi\right) + D_2^*\exp\left(-\frac{16\chi\beta}{5\sqrt{6}}\xi\right)\right).$$

In this limit, integral layer conductivity (35) is defined using expression

$$G = \frac{ne^2\tau S}{mL}\frac{\Delta}{\chi\beta^2}\Big(1 + \frac{5\sqrt{6}D_1^*}{16\chi\beta}(\exp(16\chi\beta/5\sqrt{6}) - 1)$$

$$+ \frac{5\sqrt{6}D_2^*}{16\chi\beta}(1 - \exp(-16\chi\beta/5\sqrt{6}))\Big).$$

5. Comparison with the experiment and discussion of findings

Study of thin conducting layers is of interest from the experimental point of view. This is supported by numerous research publications on the topic [12–15].

Figure 1 shows the comparison of the analytical calculation of the thin metal layer conductivity modulus (36), considering the pair collisions of electrons (curve *1*) and without considering such collisions (curve *2*), with the experimental data for gold in case of a stationary electric field ($\Omega = 0$) at absolute temperature 4 K [13].

Experimental data does not contain the volume collision frequency (free path length) of electrons in the metal layer. However, comparing the theoretical result for the specific conductivity (curve *1*) with the experimental data, we

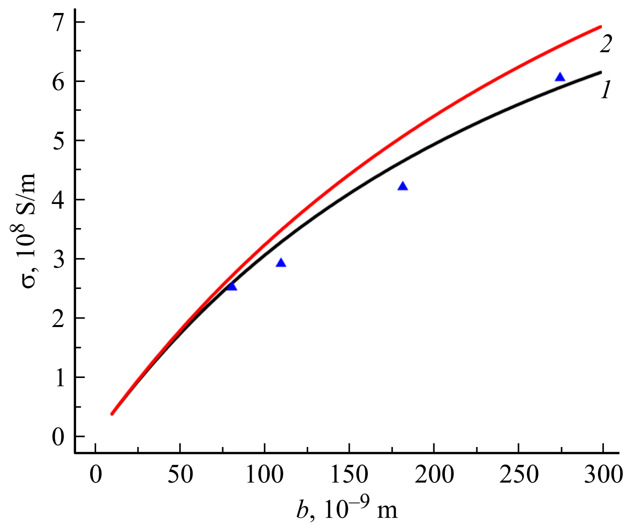

Figure 1. Comparison of the specific conductivity of a thin metal layer with experimental data.

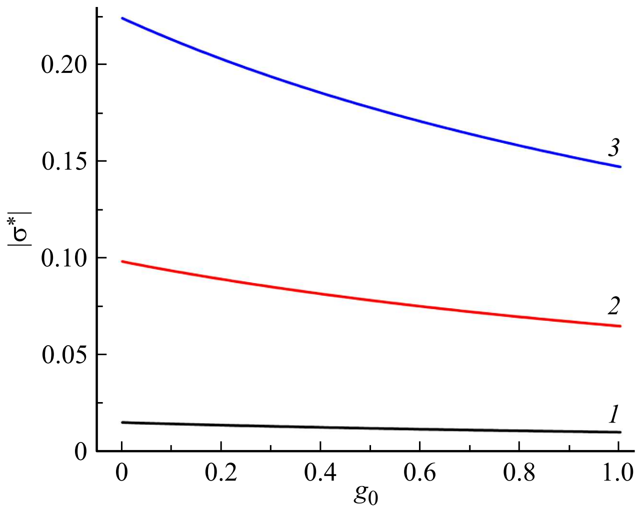

Figure 2. Dependence of the dimensionless specific conductivity modulus of a thin conducting layer on the numeric variable g_0 ($\Delta = 1$; $\Omega = 0.1$; $q_1 = q_2 = 0.1$; $\xi = 0.5$), U_μ: *1* — -3, *2* — -1, *3* — 0.

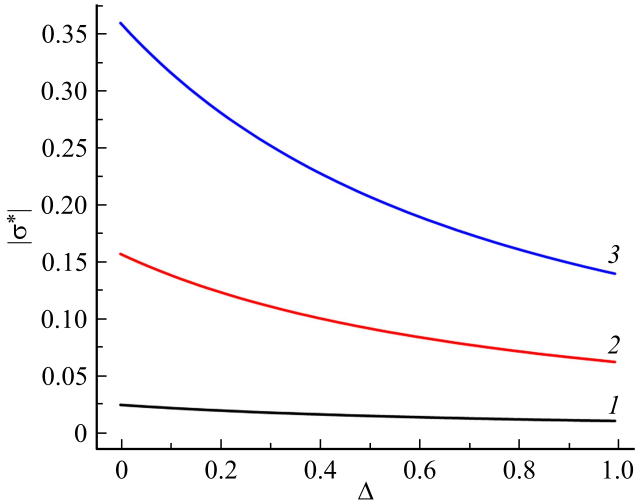

Figure 3. Dependence of the dimensionless specific conductivity modulus of the thin conducting layer on dimensionless inverse length of free electron path Δ ($\xi = 1$; $\Omega = 0.1$; $q_1 = q_2 = 0.1$; $g_0 = 0.5$), U_μ: *1* — −3, *2* — −1, *3* — 0.

can define this value with the given value of reflectance coefficients of metal q_1 and q_2. Assuming for gold $q_1 = q_2 = 0.35$, we get for the free electron path length $\Lambda = 1450\,\text{nm}$, and the numeric variable characterizing the pair collision intensity of carriers, $g_0 = 0.3$.

Comparison of experimental and theoretical data (curve *1*) shows that the calculation of specific conductivity of thin metal layer performed by the kinetic method agrees with the experimental data with accuracy at least 89%, when the pair collisions of charge carriers and the effect of metal layer surface reflectance on electron reflection.

Without considering such collision (curve *2*), when $g_0 = 0$, the derived theoretical dependence is in worse agreement with the experimental data.

Figure 2 shows the curves of the dimensionless specific conductivity modulus of the thin conduction layer (32) vs. numeric variable g_0 for the case when all curves are built with the same dimensionless coordinate values inside the layer ξ, dimensionless inverse free electron path length Δ, dimensionless electric field frequency Ω and reflectance coefficients q_1 and q_2. In this case, the dimensionless chemical potential U_μ takes various values.

The course of curves in the Figure shows that the dimensionless specific conductivity modulus of the layer (32) depends to a considerable extent on the numeric variable g_0, that characterizes the pair collision intensity of charge carriers. For any values of g_0, the dimensionless specific conductivity dominates at layers with high chemical potential.

Figure 3 shows the curve of the dimensionless specific conductivity modulus (32) on the dimensionless inverse length of free charge carrier path Δ. Whereby the dimensionless chemical potential U_μ still varies for each curve.

Analysis of the course of curves suggests that achievement of asymptotic values by these dependences at particular values of Δ depends on the value of U_μ.

Figure 4 shows the curve of the dimensionless specific conductivity modulus (32) vs. dimensionless coordinate inside the layer ξ. The dimensionless chemical potential U_μ takes the same values as in Figure 3.

The course of curves in Figure 4 shows that, at various reflectance coefficients of the layer surfaces, the dimensionless specific conductivity modulus has oscillations whose amplitude depends on the values of the chemical potential U_μ. Such nontrivial behavior of the curves

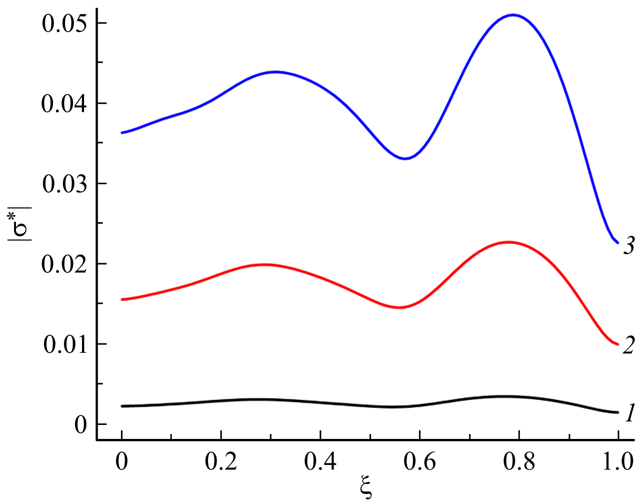

Figure 4. Curve of the dimensionless specific conductivity modulus of the thin conducting layer vs. dimensionless coordinate inside the layer ξ. ($\Delta = 1$; $\Omega = 10$; $q_1 = 0.1$; $q_2 = 0.7$; $g_0 = 0.5$), U_μ: *1* — −3, *2* — −1, *3* — 0.

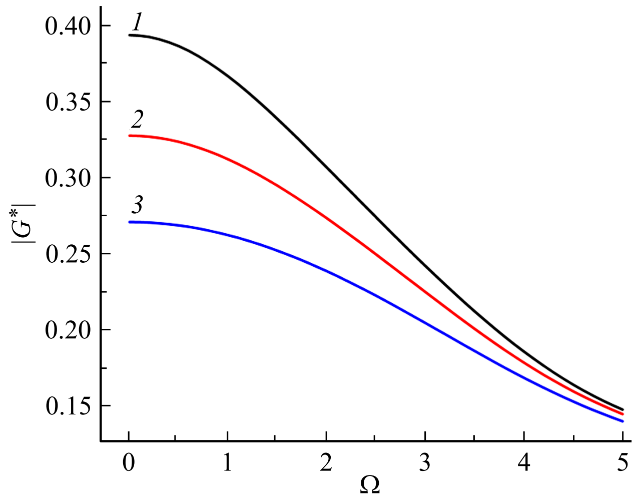

Figure 5. Curve of the dimensionless integral conductivity modulus of the thin conducting layer vs.dimensionless electric field frequency Ω ($U_\mu = 1$; $\Delta = 1$; $q_1 = q_2 = 0.1$), g_0: *1* — 0.1, *2* — 0.5, *3* — 1.

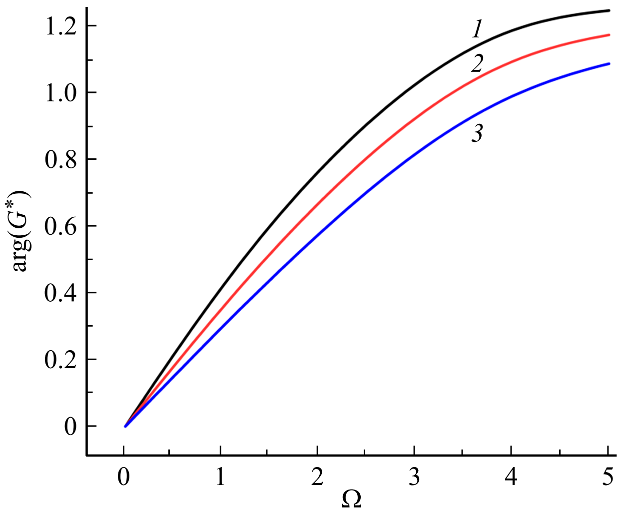

Figure 6. Curve of the argument of dimensionless integral conductivity of the thin conducting layer vs.dimensionless electric field frequency Ω ($U_\mu = 1$; $\Delta = 1$; $q_1 = q_2 = 0.1$), g_0: *1* — 0.1, *2* — 0.5, *3* — 1.

with increasing dimensionless electric field frequency Ω is probably associated with the manifestation of resonance effects observed in charge carrier reflection from the top and bottom surfaces of the conducting layer.

When plotting the dimensionless integral layer conductivity curves (35), a special focus shall be made on the dependence of its modulus and argument on the dimensionless electric field frequency Ω, that are shown in Figures 5 and 6. Whereby the numeric variable g_0 takes various values.

The course of curves in Figure 5 shows that the maximum difference of the integral layer conductivity moduli with growing intensity of the charge carrier pair collisions is observed in the static case, while further increase in the dimensionless electric field frequency results in merging of these curves.

As shown in Figure 6, all curves of the dimensionless integral conductivity argument defined in similar conditions come from the origin of coordinates and grow monotonously with increasing dimensionless electric field frequency.

Conclusion

Since the findings of the study show that the pair collisions of charge carriers have a considerable effect on layer conductivity, it is necessary to apply this theory to the direct calculation of conductivity of such bodies in practical and technical applications, e.g. in commercial manufacture of integrated circuits.

Conflict of interest

The authors declare that they have no conflict of interest.

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