Semiconductors, 2023, Vol. 57, No. 7

Multiphoton Franz–Keldysh effect in an armchair graphene nanoribbon

© B.S. Monozon[1], T.A. Fedorova[1,¶], P. Schmelcher[2]

[1] St. Petersburg Marine Technical University,
190121 St. Petersburg, Russia
[2] Zentrum fur Optishe Quanten technologien, The Hamburg Centre for Ultrafast Imaging,
2607 Hamburg, Germany
¶ E-mail: fedorova.tan@gmail.com

Received May 7, 2023
Revised June 21, 2023
Accepted October 30, 2023

We investigate the electron state in an armchair graphene nanoribbon exposed to the time-periodic strong electric field of intense light wave and constant in time weak electric field both polarized to the ribbon axis. The Dirac approach is taken for the investigation of the electron transitions between the size quantized electron and hole subbands. It is shown that the probability of an intersubband multiphoton transition, calculated in the resonant approximation, oscillates in time with the Rabi frequency, which in turn depends on the ribbon parameters and strong periodic electric field magnitude. The oscillations' magnitude is modified significantly by the weak electric field both, below and above the frequency threshold (Franz–Keldysh effect).

Keywords: graphene, ribbon, Franz–Keldysh effect.

DOI: 10.61011/SC.2023.07.57421.4996C

The considered problem goes back to the middle of the last century when Schwinger predicted the decay of the quantum-electrodynamic (QED) vacuum with the birth of an electron-positron pair in a constant electric field $F \geq F_C^{(v)}$, exceeding the critical vacuum breakdown field $F_C^{(v)} \sim 10^{18}$ V/m. Some approximation to $F_C^{(v)}$ values has already been made in modern times by the use of an alternating electric field [1].

At present, the graphene layer ribbon, which was recently obtained experimentally, plays an important role as a model for studying in vitro the problem of the interaction of vacuum QED with the electromagnetic field. The basis of such model is the relativistic dispersion law common for the tape and vacuum. Along with this, for a graphene ribbon, for example, with a width of $d = 2$ nm, the critical constant field turns out to be only $F_c^{(g)} = 2 \cdot 10^8$ V/m. We further study intersubband Rabi oscillations in a graphene crescent ribbon in an electric field $F(t) = F_0 \cos \omega t + F_1$, which is a superposition of a strong electric field of a light wave with amplitude F_0 and frequency ω and a weak constant field $F_1 \ll F_0$. The focus is on the effect of the constant field F_1 on the frequency spectrum of Rabi oscillations (Franz–Keldysh effect) [2]. Since its discovery, this effect has been intensively studied in bulk semiconductors, including those involving excitons, in polymers and perovskites, and subsequently in low-dimensional semiconductor structures. A large number of works are devoted to the Franz–Keldysh effect in quantum wells, filaments, dots and superlattices, in graphene layer and ribbon (see works [3,4]). The combined effect in the presence of DC and AC electric field in graphene ribbon has not been presented in the literature to date. Since the problem of the electronic states in a graphene crescent ribbon in a time-varying strong electric field is sufficiently detailed in the work [5], further, we will follow only the general procedure for its solution. A tape of width d is placed in the plane $x - y$, and the electric field is assumed to be polarized along the y-axis of the tape. The energy spectrum of a free electron in a graphene ribbon with semiconductor dispersion law is a sequence of one-dimensional subbands [5]:

$$\pm E_N(k) = \left(\varepsilon_N^2 + \hbar^2 v_F^2 k^2\right)^{1/2};$$

$$\varepsilon_N = \left|N - \frac{1}{3}\right| \frac{\pi \hbar v_F}{d}; \quad N = 0, \pm 1, \pm 2, \cdots, \tag{1}$$

where ε_N and $\hbar k$ — the energy of dimensional quantized transverse x-motion of the electron and its longitudinal momentum, $v_F = 10^6$ m/c — the Fermi velocity of the electron in graphene. In the Dirac model, the wave function of the longitudinal motion of an electron in A and B sublattices of graphene $\vec{u}(u_A(y,t), u_B(y,t))$ in an electric field $F(t)$ satisfies the equation [5]

$$\left[\left(i\hbar \frac{\partial}{\partial t} - eF(t)y\right)I + \varepsilon_N \sigma_x - i\hbar v_F \frac{\partial}{\partial y} \sigma_y\right] \vec{u}(y,t) = 0, \tag{2}$$

in which I and $\vec{\sigma}$ — the unit matrix and the Pauli matrices. Let us proceed to the k representation with the function $\vec{\eta}(k,t) = [\eta_1(k,t), \eta_2(k,t)]$ together with the Foldy-Wutheisen transformation using the unitary operator U^+, whose explicit form is given in [5]:

$$\vec{u}(y,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{iq(k,t)y} U^+(k,t) \vec{\eta}(k,t) dk, \tag{3}$$

where

$$\eta_{1,2}(k,t) = \exp\left\{\pm 2i \int_0^t \Omega_N(k\tau) d\tau\right\} g_{1,2}(k,t),$$

$$\Omega_N^2(k,t) = \omega_N^2 + v_F^2 q^2(k,t);$$

$$q(k,t) = \frac{e}{\hbar}\int_0^t F(\tau)d\tau + k; \quad \omega_N = \frac{\varepsilon_N}{\hbar}.$$

Substituting the function $\vec{u}(y,t)$ (3) into equation (2), we obtain a system of equations for the coefficients $g_{1,2}(k,t)$:

$$i\dot{g}_{1,2}(k,t) = -R_N(k,t)\exp\left\{\pm 2i\int_0^t \Omega_N(k,\tau)d\tau\right\}g_{2,1}(k,t);$$

$$R_N(k,t) = \frac{\omega_N v_F}{2\hbar\Omega_N^2} eF(t). \tag{4}$$

Further consideration is convenient using the coordinate function

$$\vec{\psi}(y,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{iq(k,t)y}\vec{\eta}(k,t)dk. \tag{5}$$

If in equations (4), we neglect the action of the intersubband transition operator R_N, then the solutions of these equations $a_{1,2}(k)$, together with formulas (3) and (5), define the intraband wave function $\vec{\psi}^{(0)}[\psi_1^{(0)}, \psi_2^{(0)}]$. Due to the periodicity of the electric field,

$$F(t) = F\left(t + \frac{2\pi}{\omega}\right)$$

the intraband function must satisfy the condition

$$\vec{\psi}^{(0)}\left(y,\ t + \frac{2\pi}{\omega}\right) = \exp\left\{-i\varepsilon_N\frac{2\pi}{\omega}\right\}\vec{\psi}^{(0)}(y,t)$$

with quasi-energy ε_N. This condition leads to coefficients

$$a_{1,2}(k) = C\exp\left\{-\frac{i}{eF_1}\left[-\varepsilon_N k \pm \left(\Delta_N k + \frac{\hbar^2}{2m_N}\frac{k^3}{3}\right)\right]\right\},$$

$$C = \left[\frac{\hbar}{2\pi eF_1}\sqrt{\frac{2(|\varepsilon_N| - \Delta_N}{m}}\right]^{1/4}, \tag{6}$$

where

$$\frac{\hbar^2}{2m_N} = \frac{s_N E\left(\sqrt{1-s_N^2}\right)\hbar^2 v_F^2}{\pi\varepsilon_N}; \quad \Delta_N = \frac{2\varepsilon_N E\left(\sqrt{1-s_N^2}\right)}{\pi s_N};$$

$$s_N^2 = (1-\gamma_N^{-2})^{-1}; \quad \gamma_N = \frac{\omega\varepsilon_N}{ev_F F_0}.$$

In this formula, $m_N(\gamma_N)$ and $\Delta_N(\gamma_N)$ — the effective mass and extremum of the N-th subband taking into account the field effect F_0, γ_N — the Keldysh parameter [1], C — the normalization factor determined by the limiting transition of the results obtained hereafter to the case $F_1 = 0$ [5].

The weakness of the constant electric field $F_1 \ll F_0$ allows us to neglect its influence on the frequencies Ω_N and R_N and to isolate the quasi-energy of the transition in the exponential factor of equation (4)

$$\tilde{\varepsilon}_N(k) = \frac{\hbar\omega}{\pi}\int_{-\frac{\pi}{\omega}}^{+\frac{\pi}{\omega}} \Omega_N(k,t)dt$$

$$\exp\left\{2i\int_0^t \Omega_N(k,\tau)d\tau\right\} = \exp\left\{\frac{i}{\hbar}\tilde{\varepsilon}_N(k)t\right\}S_N(k,t);$$

$$S_N(k,t) = S_N(k,t+T);$$

$$S_N(k,t)R_N(k,t) = \sum_{l=-\infty}^{+\infty} e^{-il\omega t}A_l(k,\omega);$$

$$A_l(k,\omega) = \frac{\omega}{2\pi}\int_{\frac{-\pi}{\omega}}^{\frac{-\pi}{\omega}} S_N(k,t)R_N(k,t)e^{il\omega t}dt.$$

Then, in solving the system of equations (4), following the results of [5], the resonance approximation

$$\omega_l \ll \omega \quad \left(\omega_l = \frac{\tilde{\varepsilon}_N}{\hbar} - l\omega, \quad l = 1,\ 2,\ \ldots\right),$$

with initial conditions $\overline{g_{1,2}}(k,0) = a_{1,2}(k)$, in which the functions $a_1(k) \ll a_2(k)$ are defined by equations (6) for the initial state in the valence band ($\varepsilon_N < 0$), and $\bar{g}_{1,2}(k,t)$ — the period-averaged $\frac{2\pi}{\omega}$ values of the functions $g_{1,2}(k,t)$ near the value t. As a result, we obtain

$$\bar{g}_1(k,t) = e^{\frac{i\omega_l t}{2}}$$

$$\times\left[i\left(A_l a_2(k) - \frac{\omega_l}{2}a_1(k)\right)\frac{\sin\Gamma_l t}{\Gamma_l} + a_1\cos\Gamma_l t\right];$$

$$\Gamma_l^2 = \left(\frac{\omega_l}{2}\right)^2 + A_l^2;$$

$$\bar{g}_2(k,t) = e^{-\frac{i\omega_l t}{2}}$$

$$\times\left[i\left(A_l^* a_1(k) + \frac{\omega_l}{2}a_2(k)\right)\frac{\sin\Gamma_l t}{\Gamma_l} + a_2\cos\Gamma_l t\right] \tag{7}$$

$$A_l(\omega,k) = \frac{\omega}{2\pi}\int_{-\frac{\pi}{\omega}}^{+\frac{\pi}{\omega}} \exp\left\{-i\varepsilon_N(k)\frac{t}{\hbar} + 2i\int_0^t \Omega_N(k\tau)d\tau\right\}$$

$$\times R_N(k,t)e^{il\omega t}dt.$$

In further we will use the relation $\tilde{\varepsilon}_N = 2\varepsilon_N$, following from the explicit form of the quasi-energy transition $\tilde{\varepsilon}_N$ [5] and from the formula (6) for the weak electric field F_1. The transition coefficient from the valence subband ($\varepsilon_N = -|\varepsilon_N| < 0$) to the conduction subband ($\varepsilon_N > 0$) can be calculated as

$$c(\varepsilon_N,t) = \int_{-\infty}^{+\infty} \vec{\psi}_{\varepsilon_N}(y,t)\vec{\psi}_{\varepsilon_N}^{(0)*}(y,t)dy, \tag{8}$$

where $\vec{\psi}_{\varepsilon_N}$ — two-subband and $\vec{\psi}^{(0)}_{\varepsilon_N}$ — intra-subband functions defined by formula (5), in which the coefficients $\eta_{1,2}(k,t)$ (3) should be taken with the solutions $\bar{g}_{1,2}(k,t)$ (7) and $a_{1,2}(k)$ (6) ($\varepsilon_N > 0$, $a_1 \gg a_2$) for the functions $\vec{\psi}_{\varepsilon_N}$ and $\vec{\psi}^{(0)*}_{\varepsilon_N}$, respectively. Leaving in formula (8) only the product of $a_2 a_1^*$ „of large" coefficients a_2 in function $\vec{\psi}_{\varepsilon_N}$ and a_1^* in function $\vec{\psi}^{(0)*}_{\varepsilon_N}$, after simple calculations we obtain

$$c(\varepsilon_N, t) = ie^{\frac{i\omega_l t}{2}} A_l(\omega, k) 2\sqrt{\pi} \times \frac{\sin\Gamma_l t}{\Gamma_l} C^2 \left(\frac{2\mu_N G_N}{\hbar^2}\right)^{1/2} Ai\left(\frac{-\beta_N}{G_N}\right), \quad (9)$$

where

$$\beta_N = l\hbar\omega - 2\Delta_N + \hbar\omega_l, \quad G_N = \left(\frac{\hbar^2 e^2 F_1^2}{2\mu_N}\right)^{1/3},$$

$\mu_N = \frac{m_N}{2}$ — the reduced mass of the zones, $Ai\left(\frac{-\beta_N}{G_N}\right)$ — the Airy function. In weak electric field we can consider $A_l(\omega, k) \approx A_l(\omega, 0)$ [5], where

$$A_l(\omega, 0) = \frac{\omega}{3} \exp\left\{-\frac{l}{s_N}\left[K(s_N) - E(s_N)\right]\right\} \sin^2 \frac{l\pi}{2}, \quad l = 1, 3, 5\ldots, \quad (10)$$

and $K(s_N)$ and $E(s_N)$ — complete elliptic integrals of the first and second kind.

Under the condition of exact resonance $\omega_l = 0$, $\Gamma_l = A_l$, $\beta_N = \beta_{0N} = l\hbar\omega - 2\Delta_N$, the differential probability of the resolved odd-photon intersubband N-transition $w_N l(t) = |c(\varepsilon_N, t)|^2$ takes the form of Rabi oscillations

$$w_{Nl}(\omega,\ F_1; t) = \mathscr{P}_{Nl}(\omega,\ F_1) \sin^2(|A_l(\omega, 0)| t) \quad (11)$$

at frequency

$$\Omega^R_{Nl}(\omega, F_0) = 2|A_l(\omega, 0)|$$

and amplitude

$$\mathscr{P}_{Nl}(\omega, F_1) = 2\left(\frac{\beta_{0N}}{G_N}\right)^{1/2} Ai^2\left(-\frac{\beta_{0N}}{G_N}\right).$$

The Rabi frequency $\Omega^R_{Nl}(\omega, F_0)$ increases with increasing electric field amplitude F_0 and ribbon width d [5]. The constant electric field F_1 significantly affects the oscillation amplitude of $\mathscr{P}_{Nl}(\omega, F_1)$. Above the transition edge $\beta_{0N} > 0$, the amplitude becomes an oscillating function of displacement β_{0N} with period $2.78G_N$, while below the edge $\beta_{0N} < 0$, the transition probability $\sim \mathscr{P}_{Nl}$ decreases as $\exp\left\{-\frac{4}{3}\left(\frac{\beta_{0N}}{G_N}\right)^{3/2}\right\}$. In the absence of a constant field F_1, the amplitude of $\mathscr{P}_{Nl}(\omega, 0)$ above and below the edge of $\beta_{0N} = 0$ becomes equal to $\mathscr{P}_{Nl} = 1$ and $\mathscr{P}_{Nl} = 0$ [5]. In the field F_1, the effective edge of the transition shifts to the long-wavelength side by an amount of $\Delta\beta^{(-)}_{0N} = -1.60G_N$, and the main oscillation peak shifts to the short-wavelength band by a distance of $\Delta\beta^{(+)}_{0N} = 1.02G_N$.

In the multiphoton limit $\gamma_N \gg 1$ energy $G_N \sim (F_1^2 d)^{1/3}$. Estimates made for the main intersubband transition $N = 0$ in a ribbon of width $d = 2$ nm under the influence of micrometre band radiation with field $F_0 = 360$ kV/cm and frequency $\omega = 3.3 \cdot 10^{14}\,\mathrm{c}^{-1}$ ($\lambda = 5.7\,\mu$m, $l = 3$, $\gamma_0 = 3$), give for the Rabi frequency $\Omega^R_{Nl} = 2.54 \cdot 10^{12}\,\mathrm{c}^{-1}$. In field $F_1 = 40$ kV/cm characteristic energy $G_N = 12.5$ MeV. In [6] work, the relaxation time for electron scattering on τ_{ph} phonons in a graphene ribbon over a wide range of widths and at different temperatures was calculated. The case discussed above corresponds at temperature $T = 30$ K to time $\tau_{\mathrm{ph}} = 1.03 \cdot 10^{-11}$ s. The parameter determining the influence of the phonon mechanism of suppression of Rabi oscillations is found to be equal to $\Omega^R_{Nl}\tau_{\mathrm{ph}} \approx 26$ and indicates that Rabi oscillations in the crescent ribbon of graphene at relatively low temperatures are quite observable under laboratory conditions. Of course, in addition to scattering on phonons, one should also keep in mind scattering on defects, the main ones being charged impurities, chaotic voltages, and resonant scatterers. Their influence can be effectively reduced both by lowering the temperature and by rapidly advancing technology to fabricate graphene-based structures [7].

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] H. Taya. Phys. Rev. D, **99**, 056006 (2019). https://doi.org/10.1103/PhysRevD.99.056006

[2] L.V. Keldysh. ZhETF **34**, 1138 (1958). (in Russian).

[3] R.E. Putnam, jr., M.E. Raikh. Solid State Commun., **353**, 114861 (2022).

[4] Y. Zhou, M.W. Wu. Phys. Rev. B, **83**, 245436 (2011).

[5] B.S. Monozon, P.S. Schmelcher. Rev. B, **105**, 115435 (2022). https://doi.org/10.1103/PhysRevB.105.11543

[6] A. Betti, G. Fiori, G. Iannaccone. 2010 Int. Electron Devices Meeting (San Francisco, CA, USA, 2010) p. 32.2.1 DOI: 10.1109/IEDM.2010.5703462

[7] Z.H. Ni, L.A. Ponomarenko, R.R. Nair, R. Yang, S. Anissimova, I.V. Grigorieva, F. Schedin, P. Blake, Z.X. Shen, E.H. Hill, K.S. Novoselov, A.K. Geim. Nano Lett., **10**, 3868 (2010).

Translated by Ego Translating