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14

Features of particle trapping in vacuum by an intensifying over time light beam with axial symmetry

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The features of the trapping of particles under high vacuum conditions in a two-dimensional potential well, which is created by an intensifying over time laser beam with axial (cylindrical) symmetry, are theoretically studied. It is shown that, under certain conditions, the kinetic energy of trapped particles can significantly exceed the depth of this light-induced well. The study of such trapping of particles was at first carried out for a Gaussian light beam by numerically solving the corresponding differential equations of particle motion. At the same time, for the beam with a cylindrical intensity profile, detailed analysis of the features of the capture of relatively fast particles was carried out on the basis of the visual model and rather simple mathematical relations. The theoretical results obtained can be used in optics and spectroscopy to establish the optimal conditions for the capture and localization of various particles in the considered light traps in vacuum, including, in certain cases, atoms and molecules.

Keywords: capture and localization of particles, light-induced potential well, laser beam with axial symmetry.

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1. Introduction

The development of efficient methods for slowdown and localization of particles (specifically, atoms and molecules) under high vacuum conditions is crucial for high-resolution spectroscopy [1–3] and optomechanics [3–5] of such particles.

Relatively simple techniques for slowdown and localization of particles of this kind by external electromagnetic fields, which induce potential wells with a fixed spatial distribution and a depth increasing with time up to a certain limit, have been proposed in [6,7]. It is assumed that the considered particles are under high vacuum and the forces acting on them are non-dissipative (i.e., particles move without friction). Depending on whether these particles have an electric or magnetic dipole moment, a controlled electric or magnetic field and nonresonance laser radiation may be used to capture and decelerate them in the proposed methods [6,7]. Of special interest is the possibility of production of such electromagnetic wells with various fixed spatial configurations by transforming a single laser beam, which intensifies over time, with the use of different stationary optical elements (mirrors, prisms, lenses, or polarizers).

In the present study, nontrivial features of particle trapping by an axially (cylindrically) symmetric light beam intensifying over time are identified and analyzed. It is demonstrated that particles with their kinetic energy being significantly greater than the depth of a light-induced potential well may be captured by such a beam.

The general equations of motion of particles in the region of an axially symmetric light beam intensifying over time are presented in the next section. The trapping of particles by a Gaussian beam is analyzed in Section 3 by solving these differential equations numerically. The specifics of trapping and localization of relatively fast particles by a light beam with a cylindrical intensity profile are examined in detail in Section 4 with the use of an illustrative model and simple mathematical relations. Such a profile may be derived, e.g., from a common Gaussian beam transformed by currently available π-shapers [8]. The key findings are summarized in the concluding section.

2. General relations

Following [6,7], we perform a theoretical analysis within the classical mechanics and electrodynamics. Let us assume that a point-like particle with mass m is moving in vacuum within a plane orthogonal to axis z of propagation of a cylindrically symmetric light beam intensifying over time. This beam induces an electromagnetic well for the particle. The potential energy of the well is

$$U(r,t) = s(r)\varphi(t), \tag{1}$$

where function $s(r) \leq 0$ characterizes a fixed spatial configuration of the considered well with r being the distance from the particle to the light beam axis and $\varphi(t)$ is a nondecreasing function of time t. Total energy E of this particle moving with nonrelativistic velocity υ is given by

$$E = 0.5m\upsilon^2 + s(r)\varphi(t). \tag{2}$$

It is assumed that the force of light (laser) beam pressure is negligible compared to the light-induced gradient force acting on the particle. This condition is satisfied for particles that are virtually transparent in the spectral range of their irradiation. We consider radiation of a moderate intensity with the induced dipole moment of the particle being proportional to the light field strength; potential energy (1) of this particle is then proportional to the radiation intensity [3]. Thus, the motion of the particle with coordinate vector $\mathbf{r}$ in potential well $U(r,t)$ (1) is characterized by the following equation [9]:

$$m\,\frac{d^2\mathbf{r}}{dt^2} = -\frac{\partial U}{\partial \mathbf{r}} = -\varphi(t)\,\frac{ds(r)}{dr}\,\frac{\mathbf{r}}{r}. \qquad (3)$$

Under the given conditions, the force at the right-hand side of Eq. (3) exerts no dissipative influence on the particle motion (i.e., the friction force is zero). An important corollary for the time derivative of total particle energy E follows from relations (1)−(3):

$$\frac{dE}{dt} = s(r)\,\frac{d\varphi(t)}{dt} \le 0. \qquad (4)$$

According to inequality (4), the growth of function $\varphi(t)$ with time t leads to a reduction in particle energy E in the region of the potential well where coordinate function $s(r) \le 0$ (1). This enables the capture of particles by an electromagnetic trap of this kind. Similar localization of particles in potential wells type (1) has been demonstrated earlier only by means of examples with particle energy E eventually reaching negative values [6,7]. However, under certain conditions, the considered axially symmetric laser beam may perform such trapping even for particles with their kinetic energy being significantly greater than the depth of a light-induced potential well.

In the case of axial beam symmetry, it is more convenient to solve equations of motion (3) in cylindrical coordinates of the particle: distance r from the beam center to the particle and rotation angle ψ of radius vector $\mathbf{r}$. It is worth noting that angular momentum M of the particle about axis z is conserved under this symmetry [9]:

$$M = mr^2\,\frac{d\psi}{dt}. \qquad (5)$$

Variations of radial particle coordinate r with time t are derived from the following equation of motion that follows from relations (3), (5) and the expression for effective potential energy $U_{\text{ef}} = [U(r,t) + 0.5M/(mr^2)]$ in a central field [9]:

$$m\,\frac{d^2r}{dt^2} = -\frac{\partial U_{\text{ef}}}{\partial r} = -\varphi(t)\,\frac{ds(r)}{dr} + \frac{M^2}{mr^3}. \qquad (6)$$

3. Particle trapping by a Gaussian light beam

Let us apply the general relations from Section 2 to a light beam that intensifies over time and features a transverse Gaussian distribution of intensity. For a particle with an induced dipole moment, this radiation produces potential well type (1) in plane xy with the following coordinate function:

$$s(r) = -J_0 \exp\Bigl(-\frac{r^2}{R^2}\Bigr), \qquad (7)$$

where R is the characteristic light beam radius and $J_0 > 0$ is a fixed quantity (specified by the particle polarizability) with the dimension of energy. Let us examine the following example time dependence $\varphi(t)$ (1) of the beam intensity:

$$\varphi(t) = \Bigl(\frac{t}{T}\Bigr)^n \eta(T-t) + \eta(t-T) \quad (n > 0,\ t \ge 0), \qquad (8)$$

where $\eta(q)$ is a step function ($\eta(q) = 1$ at $q \ge 0$ and $\eta(q) = 0$ if $q < 0$). Function $\varphi(t)$ (8) grows from 0 to 1 within the $0 \le t \le T$ time interval and assumes a value of 1 when $t > T$.

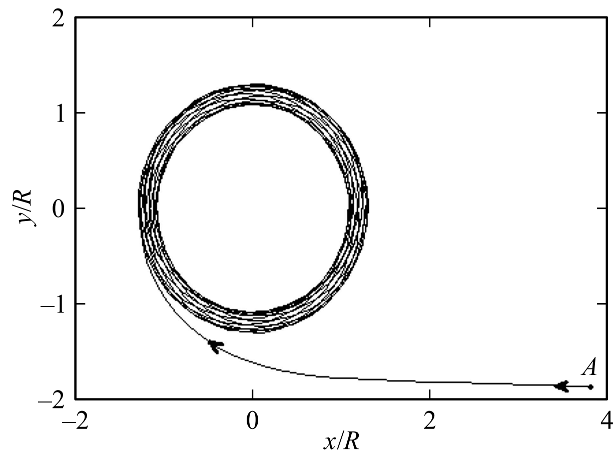

Figure 1. Particle motion trajectory (in plane xy) in the region of a Gaussian light beam intensifying over time. The cylindrical coordinates and velocity components of the particle at initial point A (at time instant $t = 0$) are as follows: $r_0 = 4.23R$, $\psi_0 = -0.456$, $\upsilon_{r0} = -7.18\,R/T$. Calculations were performed at $J_0 = 200mR^2/T^2$, $M = -13.84mR^2/T$ for function $\varphi(t)$ (8) with parameter $n = 0.5$.

Figure 1 presents the two-dimensional motion trajectory of a particle calculated numerically based on Eqs. (5) and (6). It is evident that this particle first moves freely in vacuum from point A and is then captured by Gaussian beam (7) intensifying over time. We consider the scenario where the particle enters the beam region at distance $r \sim R$ with radial velocity component $\upsilon_r = \frac{dr}{dt}$ that is much smaller than angular component $\upsilon_t = r\,\frac{d\psi}{dt}$. Following its capture, the particle is localized at the periphery of a light-induced trap (Fig. 1).

Figure 2, *a* shows the dependence of distance $r(t)$ from the particle to the light beam axis on time t. The conditions are the same as in Fig. 1. It is evident that the trapped particle remains localized in the peripheral region of the Gaussian beam with effective radius $r \sim R$. According to Fig. 2, *b*, total particle energy E (2) decreases from the initial level of $E_0 \approx 0.16J_0$ to constant $E_f \approx 0.633E_0$ in the

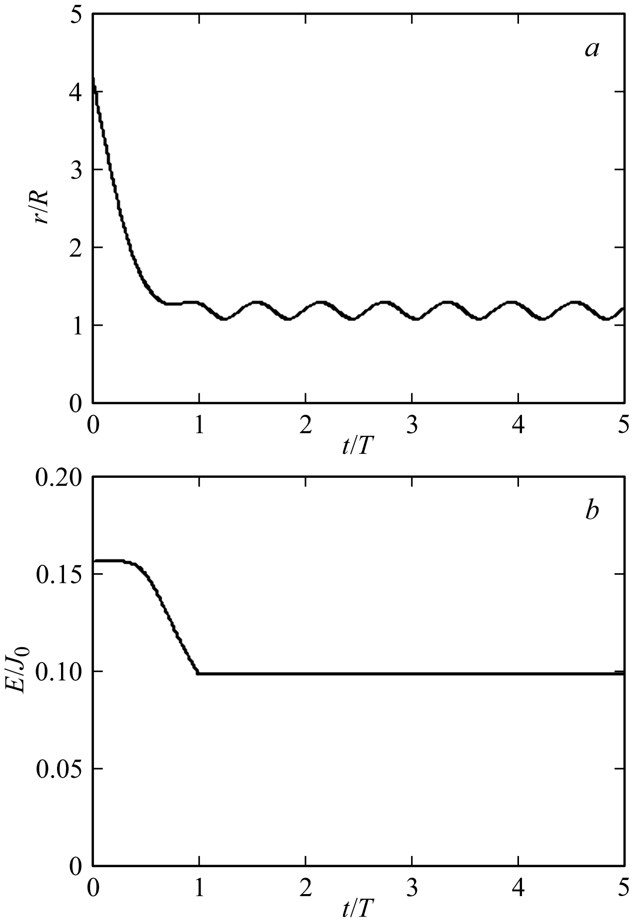

Figure 2. Dependence of radial particle coordinate r (a) in units of R and its total energy E (b) on time t (in units of T) for a Gaussian light beam intensifying over time. The conditions are the same as in Fig. 1; energy E in Fig. 2, b is divided by $J_0 = 200mR^2/T^2$.

course of radiation intensification within the $0 \leq t \leq T$ time interval. Thus, the kinetic energy of the trapped particle in the considered scenario exceeds the maximum possible depth J_0 (7) of potential well (1). This is attributable to the fact that particle velocity $v_0 = (v_r^2 + v_t^2)^{0.5}$ is a combination of its radial component v_r and orthogonal angular component v_t. The trapped particle may escape from the examined electromagnetic trap with cylindrical symmetry only when the value of radial velocity component $|v_r|$ exceeds a certain threshold. Therefore, if condition $|v_t| \gg |v_r|$ is satisfied, this trap may capture particles with their kinetic energy $0.5m(v_r^2 + v_t^2)$ exceeding the trap depth.

In the next section, the mechanisms and specific features of such trapping of relatively fast particles are analyzed in more detail with the use of an illustrative model and fairly simple mathematical relations derived for a light beam that intensifies over time and has a cylindrical intensity profile.

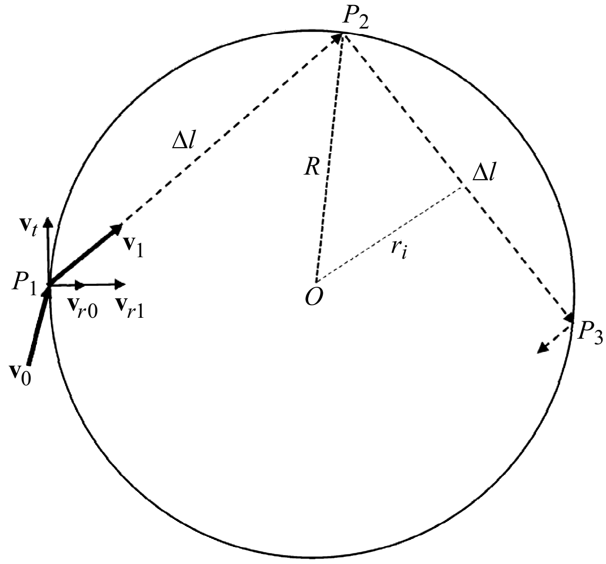

Figure 3. Diagram of trapping and subsequent localization of a particle by a laser beam that intensifies over time and has a cylindrical intensity profile (see text).

4. Particle trapping by a cylindrical light beam

Instead of (7), the following coordinate function $s(r)$ should be used in expression (1) for the potential energy in the case of a cylindrical light beam with radius R:

$$s(r) = -J_0 \eta(R - r), \tag{9}$$

where $\eta(R - r)$ is a step function. The process of particle trapping by this beam may be analyzed without the use of general equations of motion given in Section 2.

Indeed, consider a particle with mass m that moves freely in vacuum and eventually (at time instant t) reaches point P_1 (Fig. 3) at the boundary of light-induced potential well (1), (9) with its depth increasing over time up to a certain limit value $J_0 > 0$ (9). Initial particle velocity v_0 at point P_1 has radial component v_{r0} directed toward center O of the light beam and orthogonal transverse (angular) component v_t. This transverse component v_t remains unchanged when the particle enters the potential well at point P_1 at time instant t, while radial component v_{r0} increases to a certain value v_{r1} (Fig. 3). The relation between v_{r0} and v_{r1} is derived from formula (2) for total particle energy E:

$$0.5mv_{r0}^2 = 0.5mv_{r1}^2 - J_0\varphi(t). \tag{10}$$

According to equation of motion (3) with coordinate function $s(r)$ (9), the considered particle should move in a straight line within the potential well with a constant absolute value of velocity $v_1 = (v_{r1}^2 + v_t^2)^{0.5}$ until it travels over distance Δl and reaches another point P_2 at the boundary of the light-induced well (Fig. 3) at the next time

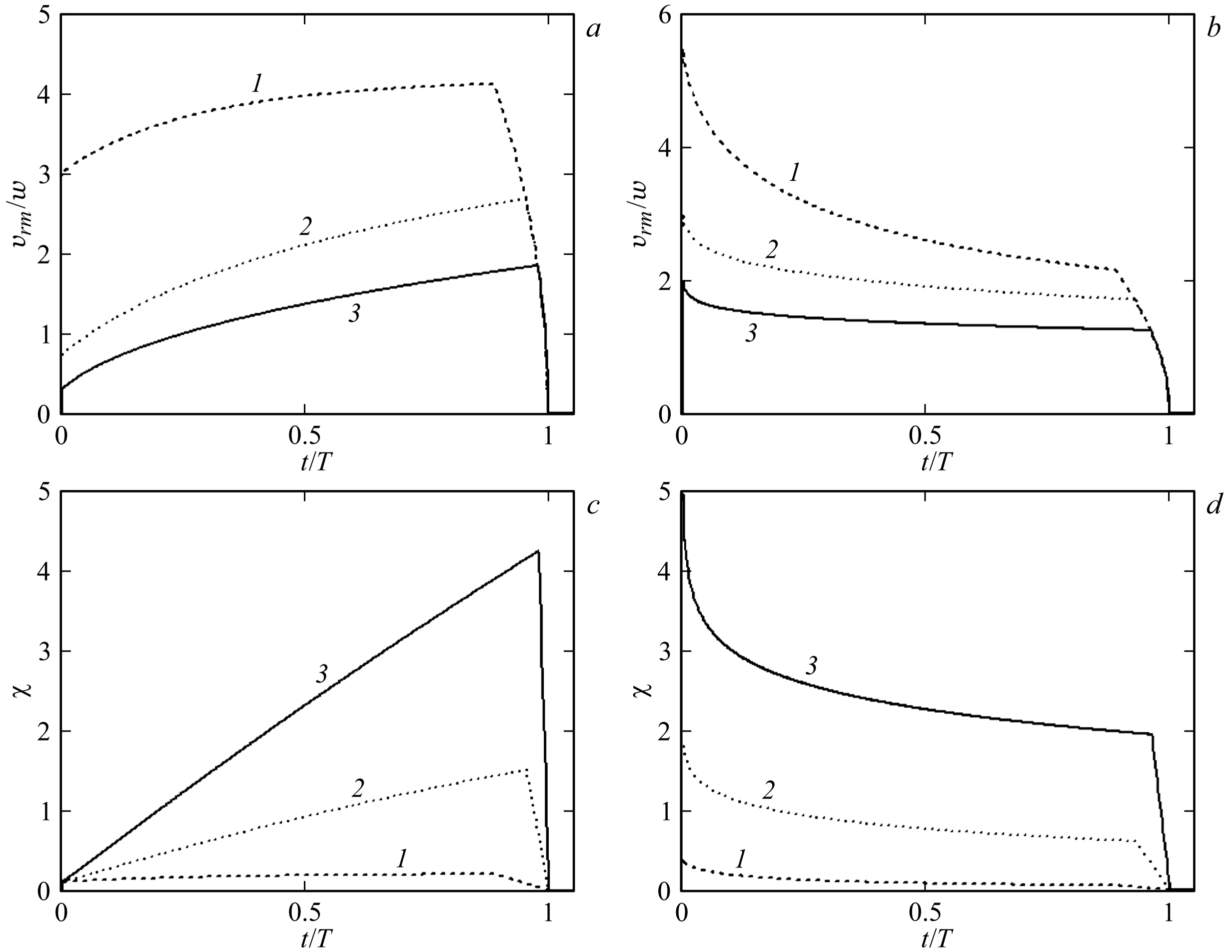

Figure 4. Dependences of radial component v_{rm} of the particle velocity in units of $w = 2R/T$ (a, b) and quantity χ characterized by formula (17) (c, d) on time t (in units of T) for a laser beam with a cylindrical intensity profile, $J_0 = 162mR^2/T^2$, coefficient $p = 0$ (curve *1*), 4 (*2*), and 10 (*3*), and parameter $n = 2$ (a, c) and 0.5 (b, d) in function $\varphi(t)$ (8).

instant t_1:

$$t_1 = t + \frac{\Delta l}{v_1} = t + \frac{2R|v_{r1}|}{(v_{r1}^2 + v_t^2)}. \tag{11}$$

This particle does not cross the light beam boundary at point P_2 if its radial velocity component v_{r1} satisfies condition

$$0.5mv_{r1}^2 < J_0\varphi(t_1), \tag{12}$$

where time t_1 was defined in (11). Having undergone specular reflection from the boundary of the potential well at point P_2, the particle travels again over distance Δl , reaching the next boundary point P_3 (Fig. 3) at time instant $t_2 = t + 2\Delta l/v_1$. Relation $\varphi(t_2) \geq \varphi(t_1)$ for nondecreasing function of time $\varphi(t)$ (1) ensures that the particle continues its movement within the confines of the light beam. The region of particle localization is bounded by outer radius R (9) of the cylindrical light beam and inner radius r_i that is indicated in Fig. 3:

$$r_i = \frac{|v_t|}{v_1} R = \frac{|v_t|}{(v_{r1}^2 + v_t^2)^{0.5}} R. \tag{13}$$

Note that absolute value $|v_t|$ of the angular component of the particle velocity (Fig. 3) at the boundary of the considered cylindrical potential well remains unchanged owing to the conservation of angular momentum M (5) of the particle. The following relation between components $|v_t|$ and $|v_{r0}|$ of initial particle velocity v_0 at point P_1 (Fig. 3) is used below:

$$|v_t| = p|v_{r0}|, \tag{14}$$

where factor p may assume any predetermined value. The maximum possible initial radial velocity component $(v_{rm} \geq v_{r0})$ of trapped particles is determined based on the following equations derived from relations (10)−(14):

$$v_{rm}^2 = v_{r1}^2 - \frac{2J_0}{m}\varphi(t), \tag{15}$$

$$v_{r1}^2 = \frac{2J_0}{m}\varphi\Big(t + \frac{2R|v_{r1}|}{(v_{r1}^2 + p^2 v_{rm}^2)}\Big). \tag{16}$$

In what follows, numerical calculations are performed based on Eqs. (15), (16) with function of time $\varphi(t)$ (8).

The dependences of the maximum possible radial component v_{rm} of the trapped particle on time t are shown in Figs. 4, *a, b*. Figures 4, *c, d* present the corresponding time t dependences for the following ratio χ between the initial kinetic energy of the trapped particle and the maximum possible depth J_0 of cylindrical potential well (9):

$$\chi = \frac{0.5m(1+p^2)v_{rm}^2}{J_0}. \tag{17}$$

It is evident that these dependences $v_{rm}(t)$ and $\chi(t)$ are largely governed by coefficient p, which relates velocity components (14), and parameter n of function $\varphi(t)$ (8), which characterizes the rate of light beam intensification. Crucially, ratio χ (17) may assume a value well above unity when parameter $p \gg 1$ in (14) (see curves *3* in Figs. 4, *c, d*). If this is the case, an intensifying cylindrical light beam may capture particles with a kinetic energy that exceeds considerably the maximum possible depth J_0 of the light-induced potential well. This electromagnetic trap loses its ability to capture particles when the beam intensification ceases at $t \geq T$ (8), since $v_{rm}(t) = 0$ and $\chi(t) = 0$ under these conditions (Fig. 4). However, particles captured prior to $t = T$ remain localized inside the potential well in finite motion within the region between beam radius R (Fig. 3) and radius $r_i < R$ (13).

The data obtained in this section based on the illustrative model (Fig. 3) and relatively simple mathematical relations are also verified by the results of much more demanding numerical calculations for the corresponding equations of motion (5), (6) for a light beam with cylindrical intensity profile (9).

5. Conclusion

It was established in the present study that an axially symmetric laser beam intensifying over time may capture point-like particles in vacuum even if the kinetic energy of these particles exceeds considerably the maximum possible depth of the light-induced potential well. Once captured, these localized and fairly fast particles move within a narrow peripheral region of the beam (Fig. 1). It is evident that this particle trapping is more efficient when the radiation intensity in the mentioned peripheral region is higher. This is the reason why, e.g., a light beam with a cylindrical intensity profile examined above has the capacity to capture much faster particles than a Gaussian beam.

Simple analytical relations were formulated for the limiting initial velocities and kinetic energies of particles that may be trapped by a laser beam with a cylindrical intensity profile (see Section 4). They may be used to obtain numerical estimates at the design and execution stages of experiments with specific particles and lasers. In order to produce such estimates, the values of the following parameters known to experimenters should be inserted into these relations: mass of the examined particle, possible components of its initial velocity, characteristic radius of the used laser beam with predetermined dynamics of its intensification, and the maximum effective depth of the potential well induced by this beam. Figure 4 illustrates the results of numerical calculations of this kind performed with certain predetermined relations between the mentioned quantities.

It should be noted that the trapping mechanism considered above is feasible not only for classical particles, but also for atoms and molecules in the ground quantum state irradiated by a travelling laser beam that intensifies over time and has a significant frequency offset from optical transitions from the ground level [6,7].

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] W. Paul, Nobel Lecture, December 8, 1989.
[2] W. Demtroder. *Laser Spectroscopy: Basic Concepts and Instrumentation* (Berlin-Springer, 2003).
[3] A. Ashkin. *Optical Trapping and Manipulation of Neutral Particles Using Lasers* (World Scientific Publishing, 2006).
[4] V.A. Soifer, V.V. Kotlyar, S.N. Khonina. Phys. Part. Nucl., **35** (6), 733 (2004).
[5] J. Millen, T.S. Monteiro, R. Pettit, A.N. Vamivakas. Reports on Progress in Physics, **83** (2), 026401 (2020).
[6] A.Ch. Izmailov. Opt. Spectrosc., **119** (5) 883 (2015). DOI: 10.1134/S0030400X15110107.
[7] A.Ch. Izmailov. Opt. Spectrosc., **122** (2), 315 (2017). DOI: 10.1134/S0030400X17020126.
[8] F.M. Dickey, T.E. Lizotte. *Laser Beam Shaping Applications* (CRC Press, Taylor@Francis Group, 2019).
[9] L.D. Landau, E.M. Lifshitz. *Mechanics* (Pergamon Press, 1969).

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