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03

On the negative pressure of light in a dispersing medium

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Dispersing dissipative homogeneous isotropic local (without spatial dispersion) magnetodielectric media are considered, for which, using Maxwell's equations, balance relations for the momentum of the field-matter system are obtained, momentum fluxes are determined, the pressure of a plane monochromatic wave on the medium layer and the forces acting on small bodies in such a medium are determined. The possibility of negative pressure on the layer in the limitless left medium, as well as on the layer in the limitless right medium with low losses in the case when the electrical losses exceed the magnetic ones, is shown. The pressure on a half-plane or plate with any type of dispersion when a plane monochromatic wave falls on them from a vacuum is always positive. Transparent media and structures with different permeabilities do not experience pressure. A model of a hypothetical Veselago medium in the form of a rarefied plasma of electric and magnetic charges is considered, the rate of energy and momentum transfer in it is found.

Keywords: light pressure, photon momentum, dispersion, Abraham-Minkowski contraversion, metamaterial.

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Introduction

The pressure of light in weak light fields has been measured by P.N. Lebedev back in 1899. With the advent of high-power lasers, experiments on measurement of the pressure of light on microparticles in relatively transparent media (liquids and gases) have become feasible. The indicated force may be significantly stronger than the force of gravity. Regardless of the nature of dispersion in a medium (see references in [1] and [2–8]), these measurements yield positive results (i.e., the pressure is directed away from the source). V.G. Veselago was the first to suggest the possibility of negative pressure of light in his study [9] into the properties of media with backward waves (BWs). These media were called left-handed ones in [9]. A negative pressure on particles (i.e., attraction toward the source) in left-handed media (LHM, media with BWs) was assumed, and these media were considered to be homogeneous and isotropic with local dielectric constant (DC) ε and magnetic permeability (MP) μ. Locality implies the lack of spatial dispersion (SD). Phase velocity (PV) vector $\mathbf{v}_\mathrm{p}$ and wave vector $\mathbf{k}$ in such media are antiparallel to Poynting vector $\mathbf{S}$. The contrary case with these vectors being parallel (or with an acute angle between them) corresponds to right-handed media (RHM). Vectors $\mathbf{E}$, $\mathbf{H}$, and $\mathbf{k}$ form a right-hand system in common homogeneous and isotropic RHM; i.e., power flux $\mathbf{S} = \mathrm{Re}(\mathbf{R} \times \mathbf{H}^*)/2$ is directed along $\mathbf{k}$. In LHM, a left-hand system is formed and the power flux is directed opposite to the PV. Backward waves are observed in such media, and the dispersion is anomalous (negative) [10]. Negative refraction (NR) is observed if a plane wave is incident onto the RHM–LHM boundary. Isotropic LHM are infeasible [11], and all known media with BWs are anisotropic or bianisotropic (with a marked SD). NR and BWs are different and independent phenomena [12,13]. NR is observed in anisotropic media and depends strongly on the orientation of the incident wave vector with respect to the isofrequency surface of a crystal or a photonic crystal (PC) [13]. Quasiphotons (polaritons) are quasiparticles of the electromagnetic field in a dispersing medium. Their energy is $\hbar\omega$; however, their momentum is, in contrast to that of photons in vacuum, still a subject of debate. The issues of energy density and the form of the energy–momentum tensor (EMT) [1–8] are also debatable, although the matter of energy density in certain simple model media (e.g., cold plasma) has been settled [14].

The aim of the present study is to consider the possibility of negative pressure of light on the basis of rigorous electrodynamic relations for the model of an isotropic dispersing local medium (without SD) characterized by spectral DC $\varepsilon(\omega) = \varepsilon'(\omega) - i\varepsilon''(\omega)$ and MP $\mu(\omega) = \mu'(\omega) - i\mu''(\omega)$. Even in this simple formulation, the problem is not an easy one to solve. The results may be extended to anisotropic media with SD, but this requires separate consideration.

V.G. Veselago has made a mistake in his proof of negative pressure in [9]. He introduced negative refraction index (RI) $n = \sqrt{\varepsilon\mu} < 0$, having used the Minkowski expression for the momentum density ($\mathbf{g}^M = \mathbf{D} \times \mathbf{B} = n^2\mathbf{S}/c^2$) without any justification. He then introduced energy density w in the Umov formulation ($\mathbf{S} = -w\mathbf{v}_p = -wc\mathbf{k}/|n|$) and defined the momentum of a „photon" (polariton) as $\mathbf{p} = \hbar\mathbf{k}$. Here, $|n|$ should be regarded as a retardation coefficient. These relations were left unsubstantiated. It is demonstrated below that the momentum in the medium considered by Veselago moves in the direction of energy transfer (i.e., away from the source). No homogeneous and isotropic media with the energy moving opposite to the momentum or to the phase have been discovered or produced since 1967, although a great number of studies into NR, metamaterials with a „negative RI," etc., have been published (see, e.g., [15–40] and references therein). Several different designations for such hypothetical (as was noted in [9]) media have been proposed. For example, they were referred to as media with NR and media with a negative group velocity (GV). This is incorrect. NR is not characterized entirely by a medium and depends on the medium from which a wave (ray) is incident and on the incidence angle. Notably, NR is feasible even at $\mu = 1$ (i.e., without BWs). This is also incorrect in regard to a negative GV: the GV does not characterize the rate of energy motion in dissipative media and metamaterials. It has already been established that media with BWs, NR, and a negative GV and NR differ in nature and that these phenomena may exist independently of each other [12,13]. A negative GV is true, e.g., of plasmons (waves at the interfaces of media), and the existence of BWs in this case is not the same as a negative GV [41]. Media with ε and μ being „simultaneously" (at one and the same frequency) negative may also be referred to as media with BWs. The formal introduction of such scalar parameters into Maxwell equations does indeed turn a plane wave into a backward one. There is not need to introduce an RI in this case. However, if infinitely small losses are added to the DC and MP, the ambiguity of the square root vanishes: $n = \sqrt{\varepsilon\mu} < 0$. All metamaterials fabricated to verify the results from [9] are anisotropic with SD. It is impossible to fabricate isotropic metamaterials without SD with the DC and MP being negative at one and the same frequency (i.e., with a negative RI) [11,38]. However, it is instructive to examine a plasma with electric and magnetic monopoles (charges) that has been mentioned in [9]. A medium with $\varepsilon = \mu - 1$ (anti-vacuum), which is called the Veselago medium, has also been introduced there. The aim of the present study is to examine waves and pressure in such a medium, in a medium with $\varepsilon < 0$, $\mu > 0$ (plasma), in a medium with $\varepsilon > 0$, $\mu < 0$, and in a common weakly dissipative medium.

Theorem on the momentum balance in a monochromatic field

A homogeneous isotropic medium with a monochromatic wave is characterized by Maxwell equations of the following form:

$$\nabla \times \mathbf{H}(\omega) = i\omega\varepsilon_0\varepsilon(\omega)\mathbf{E}(\omega) + \mathbf{J}(\omega), \tag{1}$$

$$\nabla \times \mathbf{E}(\omega) = -i\omega\mu_0\mu(\omega)\mathbf{H}(\omega), \tag{2}$$

Here, $\varepsilon(\omega) = \varepsilon'(\omega) - i\varepsilon''(\omega)$ and $\mu(\omega) = \mu'(\omega) - i\mu''(\omega)$, and dissipation imposes conditions $\varepsilon''(\omega) > 0$, $\mu''(\omega) > 0$. It follows from (1) that

$$i\omega\varepsilon_0\varepsilon(\omega)\nabla \cdot \mathbf{E}(\omega) = -\nabla \cdot \mathbf{J}(\omega) = i\omega\rho$$

or

$$\nabla \cdot \mathbf{E}(\omega) = \rho/(\varepsilon_0\varepsilon(\omega)).$$

Here, ρ is the density of foreign charges corresponding to the sources of field $\mathbf{J}$. In the electrodynamics of an infinite medium, density J may also correspond to drains (i.e., characterize a certain particle that absorbs the field energy). It is convenient in this case to consider field sources as the ones positioned at infinity. One needs to analyze the momentum conservation law in order to determine forces and pressures. Let us use Eqs. (1) and (2) for this purpose. We multiply the first equation vectorially on the left by $\mu_0\mu^*\mathbf{H}^*$, multiply the complex conjugate of Eq. (2) vectorially by $\varepsilon_0\varepsilon\mathbf{E}$, and combine them:

$$\mu_0\mu^*\mathbf{H}^* \times \nabla \times \mathbf{H} + \varepsilon_0\mathbf{E} \times \nabla \times \varepsilon^*\mathbf{E}^* = i\omega c^{-2}\left(\mu^*\varepsilon\mathbf{H}^* \times \mathbf{E} + \varepsilon\mu^*\mathbf{E} \times \mathbf{H}^*\right) + \mu_0\mu^*\mathbf{H}^* \times \mathbf{J}. \tag{3}$$

The complex conjugate of Eq. (3) is

$$\mu_0\mu\mathbf{H} \times \nabla \times \mathbf{H}^* + \varepsilon_0\mathbf{E}^* \times \nabla \times \varepsilon\mathbf{E} = -i\omega c^{-2}\left(\mu\varepsilon^*\mathbf{H} \times \mathbf{E}^* + \varepsilon^*\mu\mathbf{E}^* \times \mathbf{H}\right) + \mu_0\mu\mathbf{H} \times \mathbf{J}^*. \tag{4}$$

We then find the complex conjugate of Eq. (1) and multiply it by $\mu_0\mu^*$ and vectorially on the left by $\mathbf{H}$, multiply Eq. (2) by $\varepsilon_0\varepsilon$ and vectorially by $\mathbf{E}^*$, and combine them:

$$\mu_0\mathbf{H} \times \nabla \times \mu^*\mathbf{H}^* + \varepsilon_0\mathbf{E}^* \times \nabla \times \varepsilon\mathbf{E} = -i\omega c^{-2} \left(\mu^*\varepsilon^*\mathbf{H} \times \mathbf{E}^* + \varepsilon\mu\mathbf{E}^* \times \mathbf{H}\right) + \mu_0\mu^*\mathbf{H} \times \mathbf{J}^*. \tag{5}$$

The complex conjugate of Eq. (5) is

$$\mu_0\mathbf{H}^* \times \nabla \times \mu\mathbf{H} + \varepsilon_0\mathbf{E} \times \nabla \times \varepsilon^*\mathbf{E}^* = i\omega c^{-2}\left(\mu\varepsilon\mathbf{H}^* \times \mathbf{E} + \varepsilon^*\mu^*\mathbf{E} \times \mathbf{H}^*\right) + \mu_0\mu\mathbf{H}^* \times \mathbf{J}. \tag{6}$$

Let us set $\mu\mathbf{H} = \mathbf{a}$, $\mathbf{H}^* = \mathbf{b}$, $\varepsilon\mathbf{E} = \tilde{\mathbf{a}}$, and $\mathbf{E}^* = \tilde{\mathbf{b}}$ in these four relations and combine them all. The left-hand side of the equality containing magnetic fields takes the form

$$2\mu\mathrm{Re}(\mathbf{a} \times \nabla \times \mathbf{b} + \mathbf{b} \times \nabla \times \mathbf{a}) = 2\mu_0\mathrm{Re}\{-\mu\nabla \cdot \left(\hat{I}|\mathbf{H}|^2 - 2\mathbf{H} \otimes \mathbf{H}^*\right) + 2\mu\mathbf{H}\left(\nabla \cdot \mathbf{H}^*\right)\}.$$

The following well-known vector–tensor identity was used here:

$$\mathbf{a} \times \nabla \times \mathbf{b} + \mathbf{b} \times \nabla \times \mathbf{a} - \nabla \times [\hat{I}(\mathbf{a} \cdot \mathbf{b}) - \mathbf{a} \otimes \mathbf{b} - \mathbf{b} - \mathbf{b} \otimes \mathbf{a}] + \mathbf{a}(\nabla \cdot \mathbf{b}) + \mathbf{b}(\nabla \cdot \mathbf{a}). \tag{7}$$

The part containing electric fields is transformed in a similar fashion. The right-hand side containing sources takes the form

$$-2\mu_0\mathrm{Re}\left(\mathbf{J} \times (\mu^* + \mu)\mathbf{H}^*\right) = -4\mu_0\mu'\mathrm{Re}(\mathbf{J} \times \mathbf{H}^*).$$

The remaining part of the right-hand side is written as

$$2\omega c^{-2}\mathrm{Im}((\varepsilon\mu-\varepsilon^*\mu^*)\mathbf{E}\times\mathbf{H}^*)$$
$$=-2\omega c^{-2}(\varepsilon'\mu''+\varepsilon''\mu')\mathrm{Re}(\mathbf{E}\times\mathbf{H}^*).$$

It is equal to zero in a loss-free medium. In the context of the nonstationary balance equation [1], this part is equal to the time derivative of the period-average momentum density (i.e., the field momentum increment). The momentum is not accumulated or spent in a monochromatic field in a loss-free medium, and this quantity is zero. Let us introduce the following notation for complex quantities:

$$\tilde{\mathbf{S}}=\mathbf{E}\times\mathbf{H}^*/2,\qquad \tilde{\mathbf{F}}_L=\mathbf{J}\times\mu_0\mu'\mathbf{H}^*/2,$$
$$\tilde{\mathbf{g}}^M=\varepsilon\mathbf{E}^*\times\mu^*\mathbf{H}^*/(2c^2),\qquad \tilde{p}_J=\mathbf{J}\cdot\mathbf{E}^*/2,$$

and real quantiles:

$$\tilde{p}_s=\omega\varepsilon''|\mathbf{E}|^2/2,\qquad \tilde{p}_\mu=\omega\mu''|H|^2/2,$$
$$\tilde{w}_e=\varepsilon_0\varepsilon'|\mathbf{E}|^2/4,\qquad \tilde{w}_\mu=\mu_0\mu'|\mathbf{H}|^2/4,$$

with the application of (7) leading to tensor Σ:

$$\hat{\Sigma}=\mathrm{Re}\left(\mu_0\mu\left(\hat{I}|\mathbf{H}|^2-2\mathbf{H}\otimes\mathbf{H}^*\right)\right.$$
$$\left.+\varepsilon_0\varepsilon\left(\hat{I}|\mathbf{E}|^2-2\mathbf{E}\otimes\mathbf{E}^*\right)\right)/4. \tag{8}$$

$\tilde{\mathbf{S}}$ has the meaning of a complex Poynting vector. It features in the Poynting theorem on complex power [42]. Its real part is the period-average power flux density $\tilde{\mathbf{S}}'=\mathrm{Re}(\mathbf{E}\times\mathbf{H}^*)/2$. The complex balance equation for power takes the form

$$-\nabla\tilde{\mathbf{S}}=\tilde{p}_\varepsilon\tilde{p}_\mu+2i\omega(\tilde{w}_\varepsilon-\tilde{w}_\mu)+\tilde{p}_J. \tag{9}$$

Real and imaginary parts of this equation may be taken. The physically relevant one is equation [42]

$$-\nabla\cdot\tilde{\mathbf{S}}'=\tilde{p}_\varepsilon+\tilde{p}_\mu+\tilde{p}'_J.$$

Here, sum $\tilde{p}_\varepsilon+\tilde{p}_\mu$ characterizes the power of field losses in a medium and $\tilde{P}'_J$ is the active power density of sources focused on producing the field. It is spent on losses and emission. The imaginary part is

$$-\nabla\cdot\tilde{\mathbf{S}}''=2\omega(\tilde{w}'_\varepsilon-\tilde{w}'_\mu)+\tilde{p}''_J.$$

Reactive source power density $\tilde{p}''_J$ specifies the doubled time derivative of the difference between electric and magnetic field energy densities and the reactive power flux. However, quantity $\tilde{w}_\varepsilon+\tilde{w}_\mu$ has the meaning of energy density only in non-dispersing (and, consequently, loss-free) media. It is impossible in the general case to determine the energy density in a monochromatic field in dispersing media. Nonstationary field production processes [39,40] are then required, since one needs to find out how the energy was accumulated. In a high-frequency monochromatic field, the electric field energy is converted into the magnetic field energy (and vice versa), and the energies are not defined exclusively by fields, the DC, and the MP. For example, energy in plasma may be accumulated in the form of kinetic energy of charge oscillations [14]. A theorem on oscillating power [42] may be formulated in order to obtain additional relations. Let us denote vector $\tilde{\mathbf{g}}^M$ as the Minkowski period-average complex momentum density. Its real part characterizes the period-average momentum density in a transparent medium: $\tilde{\mathbf{g}}'^M=\mathrm{Re}(\varepsilon\mathbf{E}\times\mu^*\mathbf{H}^*)/(2c^2)=\varepsilon'\mu'\tilde{\mathbf{S}}'/c^2$. Likewise, $\tilde{\mathbf{F}}'_L=\mu_0\mu'\mathbf{Re}(\mathbf{J}\times\mathbf{H}^*)/2$ -is the period-average density of the Lorentz force with which the field acts on sources (recoil force). Tensor $\hat{\Sigma}$ has the form of the period-average Maxwell stress tensor and characterizes the fluxes of momentum components. Dividing the obtained relation by eight, we find the momentum conservation law in the form

$$-\nabla\cdot\hat{\Sigma}+\mathrm{R}e(\varepsilon_0\varepsilon\mathbf{E}(\nabla\cdot\mathbf{E})+\mu_0\mu\mathbf{H})(\nabla\cdot\mathbf{H})/2$$
$$+\omega c^{-2}(\varepsilon'\mu''+\varepsilon''\mu')\tilde{\mathbf{S}}'/2=-\tilde{\mathbf{F}}'_L. \tag{10}$$

The term at the left-hand side of (10) is equal to zero wherever there are no sources. If a field is produced by a point-like dipole, this is the entire space with the exception of the dipole position (where the field is singular). If sources are distributed, this term assumes the value of $\mathrm{Re}(\rho\mathbf{E})/2$; i.e., this is the period-average density of the electric force with which the field acts on sources. When source currents are solenoidal, it is zero.

Let us examine a plane wave with dependence $\exp(i\omega t-i\mathbf{kr})$. The divergence operation then corresponds to scalar multiplication (e.g., $\nabla\cdot\mathbf{E}=-i\mathbf{k}\cdot\mathbf{E}$), while the curl operation is vector multiplication ($\nabla\times\mathbf{E}=-i\mathbf{k}\times\mathbf{E}$). In the case of a plane wave, there are no sources (they are at infinity) and fields are solenoidal. The balance takes the form

$$-\nabla\cdot\hat{\Sigma}+\omega c^{-2}(\varepsilon'\mu''+\varepsilon''\mu')\tilde{\mathbf{S}}'/2=0. \tag{11}$$

If a medium is non-dissipative, $\nabla\cdot\hat{\Sigma}=0$. Only one component $\hat{\Sigma}_{zz}$ is relevant to a plane wave moving along axis z, and $\partial_z\hat{\Sigma}_{zz}=0$; i.e., $\hat{\Sigma}_{zz}(z)=\mathrm{const}$. If ε' and μ' change sign, it follows from (8) that $\hat{\Sigma}_{zz}$ also changes sign.

Since the motion of a wave along z implies the motion of phase, the sign changes with respect to the motion of phase. The sign of the right term in Eq. (11) also changes; therefore, the momentum transfer is codirectional with the transfer of energy $\tilde{\mathbf{S}}'$ regardless of the type of the medium. If ε' and μ' are of different sign, the sign of $\hat{\Sigma}_{zz}$ in (8) depends on which of the „energies“ is greater and, with respect to $\tilde{\mathbf{S}}'$, on the relation between electric and magnetic losses. This scenario is of little interest, since a wave of this type always decays strongly. In the case of a plane wave in electron plasma ($\mu=1$), $H_y=\eta_0\sqrt{\varepsilon}E_y$, $\eta_0=\sqrt{\varepsilon_0/\mu}$, and the sign in (8) changes at point $\varepsilon'=-1$ under weak dissipation; in addition, the right-hand side of (11) is $\omega c^{-2}\varepsilon''\tilde{\mathbf{S}}'/2$ and does not change sign. The conditions in strongly dissipative media are more complex. Although $\hat{\Sigma}_{zz}$ produces a momentum flux in direction z, one should not suppose that a force acting on a particle in the field has the same direction. This is true only in a first approximation with the particle being so small as to induce a near-zero perturbation of the field. In order to characterize the influence of a particle with DC $\tilde{\varepsilon}$, one needs to present it as polarization current $\mathbf{J}_p = i\omega\varepsilon_0(\tilde{\varepsilon} - \varepsilon)\mathbf{E}$ in the right-hand side of (1). We assume for simplicity that DC $\tilde{\varepsilon}$ remains unchanged within the particle. At its boundary, the DC „drops" sharply to ε. Thus,

$$\nabla \cdot ((\tilde{\varepsilon} - \varepsilon)\mathbf{E}) = (\tilde{\varepsilon} - \varepsilon)\nabla \cdot \mathbf{E} + \mathbf{E} \cdot \nabla(\tilde{\varepsilon} - \varepsilon).$$

Differentiating the jump from $\tilde{\varepsilon}$ to ε, we obtain a delta function: $\nabla(\tilde{\varepsilon} - \varepsilon) = -\boldsymbol{\nu}(\tilde{\varepsilon} - \varepsilon)\delta(\nu)$. Here, $\boldsymbol{\nu}$ is the outward normal to the particle surface and ν is the coordinate measured along the normal from the surface. Since the field undergoes a jump $\tilde{\varepsilon}E_\nu^- = \varepsilon E_\nu^+$, one may introduce induced surface charge density

$$\sigma = \varepsilon\varepsilon(E_\nu^+ - E_\nu^-) = \varepsilon_0\varepsilon E_\nu^+(1 - \varepsilon/\tilde{\varepsilon}).$$

The divergence is then

$$\nabla \cdot \mathbf{J}_p = i\omega\varepsilon_0\lfloor(\tilde{\varepsilon} - \varepsilon)\nabla \cdot \mathbf{E} + E_\nu^-(\varepsilon - \tilde{\varepsilon})\delta(\nu)\rfloor.$$

If a body is inhomogeneous, $\varepsilon_0\nabla \cdot \mathbf{E}$ is the density of the volume charge interacting with the electric field. In the present case, $\nabla \cdot \mathbf{E} = 0$ within the homogeneous particle, and only the surface divergence and the surface charge density remain: $\nabla \cdot \mathbf{J}_p = -i\omega\sigma\delta(\nu)$. It should be noted that, in accordance with the properties of the delta function, $\nabla \cdot \mathbf{J}_p = 0$ outside the surface. Integrating over the particle volume, we find, by virtue of the Gauss theorem, that the integral of σ over the particle surface (net surface charge) is zero. To factor in the presence of a particle, one needs to introduce current density $\mathbf{J}_p$ into balance (11). The balance then takes the form

$$-\nabla \cdot \hat{\Sigma} = -\mathrm{Re}(\sigma\mathbf{E}\varepsilon/\tilde{\varepsilon})\delta(\nu)/2 - \omega c^{-2}(\varepsilon'\mu'' + \varepsilon''\mu')\tilde{\mathbf{S}}'/2 - \tilde{\mathbf{F}}_p. \quad (12)$$

The first term on the right is the surface Coulomb force density, the second term is the striction force density, and the third term is the density of the volume magnetic Lorentz force acting on the particle:

$$\tilde{\mathbf{F}}_p = \omega c^{-2}\mu'\mathrm{R}e(i(\tilde{\varepsilon} - \varepsilon)\tilde{\mathbf{S}}) = \omega c^{-2}\mu'\lfloor(\tilde{\varepsilon}'' - \varepsilon'')\tilde{\mathbf{S}}' - (\tilde{\varepsilon}' - \varepsilon')\tilde{\mathbf{S}}''\rfloor. \quad (13)$$

The second term is zero in a non-dissipative medium. Integrating over the particle volume, we obtain

$$-\oint_S \boldsymbol{\nu} \cdot \hat{\Sigma}d^2r + \frac{\varepsilon'}{2}\mathrm{Re}\oint_s(\sigma/\tilde{\varepsilon})\mathbf{E}d^2r + \int_V \tilde{\mathbf{F}}_p d^3r = 0. \quad (14)$$

Since the normal is an outward one, the first term on the left is the field momentum inflowing into the particle. It is spent on electric $\mathbf{F}_\varepsilon$ (surface integral in (14)) and magnetic $\mathbf{F}_\mu$ (volume integral) forces with which the field acts on the particle. It is evident that volume density (13) may change sign when the relation between real and imaginary parts of the DC changes. It also changes sign when μ' undergoes sign reversal. However, this scenario is infeasible in optics of common media and metamaterials characterized by local DCs [35]. The electric force changes sign when ε' undergoes sign reversal, which may be observed in plasma. Since the net charge at the particle is zero, the contribution of the second term in (13) is, in a first approximation, the force acting on a dipole. The dipole moment of a unit volume is $\mathbf{P} = \varepsilon_0(\tilde{\varepsilon} - \varepsilon)\mathbf{E}$. The dipole moment for a spherical particle with radius r is $\mathbf{p} = 4\pi r^3\varepsilon_0(\tilde{\varepsilon} - \varepsilon)\mathbf{E}/3$. The particle alters the field; therefore, it is affected by local field [14]

$$\mathbf{E}_l = \frac{3(\tilde{\varepsilon}/\varepsilon)}{2(\tilde{\varepsilon}/\varepsilon) + 1}\mathbf{E}. \quad (15)$$

The energy of the dipole in the field is $W_d = -\mathbf{p} \cdot \mathbf{E}_l$, and the corresponding force is

$$F_d = 2\pi^3\varepsilon_0\nabla|\mathbf{E}|^2\mathrm{Re}\left(\frac{\tilde{\varepsilon}(\tilde{\varepsilon}/\varepsilon - 1)}{2(\tilde{\varepsilon}/\varepsilon) + 1}\right). \quad (16)$$

Local force (16) estimates the second surface integral in (14). We averaged it over a period. In a plane wave,

$$|\mathbf{E}|^2 = |E_x|^2\exp(-2k_z''z),$$

and

$$\nabla|\mathbf{E}|^2 = -2k_z''|E_x|^2\exp(-2k_z''z).$$

The following relations hold true for a plane wave:

$$\mathbf{E} = \mathbf{x}_0E_x\exp(-ik_zz),$$

$$\mathbf{H} = \mathbf{y}_0\eta_0E_x\exp(-ik_zz),$$

$$k_z = k_0\sqrt{\varepsilon\mu},$$

$$\tilde{\mathbf{S}} = |E_x|^2\eta_0\eta,$$

$$\eta = \sqrt{\varepsilon/\mu},$$

where

$$k_z = k_0\sqrt{\varepsilon\mu} = k_z' - ik_z'',$$

$$\eta = \sqrt{\varepsilon/\mu} = \eta' + i\eta',$$

$$k_z' = \pm k_0 \times\sqrt{\frac{\sqrt{(\varepsilon'\mu' - \varepsilon''\mu'')^2 + (\varepsilon'\mu'' + \varepsilon''\mu'')^2} + (\varepsilon'\mu'' + \varepsilon''\mu'')^2}{2}},$$

$$k_z'' = \pm k_0 \times\sqrt{\frac{\sqrt{(\varepsilon'\mu' - \varepsilon''\mu'')^2 + (\varepsilon'\mu'' + \varepsilon''\mu'')^2} - (\varepsilon'\mu'' + \varepsilon''\mu'')}{2}} \geq 0,$$

$$\eta=\sqrt{(\varepsilon'\mu'+\varepsilon''\mu'')+i(\varepsilon'\mu''-\varepsilon''\mu')}/|\mu|=\eta'+i\eta',$$

$$\eta'=\sqrt{\frac{\sqrt{(\varepsilon'\mu'+\varepsilon''\mu'')^2+(\varepsilon'\mu''-\varepsilon''\mu')^2}+(\varepsilon'\mu'+\varepsilon''\mu'')}{2|\mu|^2}},$$

$$\eta''=\sqrt{\frac{\sqrt{(\varepsilon'\mu'+\varepsilon''\mu'')^2+(\varepsilon'\mu''-\varepsilon''\mu')^2}-(\varepsilon'\mu'+\varepsilon''\mu'')}{2|\mu|^2}}.$$

The minus sign should be chosen for k_z' when both DC and MP have negative real parts. In the case of a plane wave in a non-dissipative medium,

$$k_z=\pm k_0\sqrt{\varepsilon'\mu'},\qquad \tilde{\mathbf{S}}'=0,$$

$$\eta=\eta'=\sqrt{\varepsilon'/\mu'},$$

and the magnetic force is written as

$$\mathbf{F}_\mu=\omega c^{-2}\mu'\tilde{\varepsilon}''\int\limits_V\tilde{\mathbf{S}}'d^3r. \tag{17}$$

Since $\nabla|\mathbf{E}|^2=0$, electric force (16) is zero. In a Veselago medium, $k_z=-k_0$; i.e., there is no need to introduce a negative RI, but the introduction of infinitely small losses is required in order to determine the sign of wave admittance:

$$\eta=\sqrt{\frac{\varepsilon'-i\delta_\varepsilon}{\mu'-i\delta_\mu}}=\sqrt{\frac{\varepsilon'\mu'+\delta_\varepsilon\delta_\mu-i(\varepsilon'\delta_\mu-\delta_\varepsilon\mu')}{\mu'^2+\delta_\mu^2}}$$

$$=\sqrt{\frac{1+\delta_\varepsilon\delta_\mu+i(\delta_\mu-\delta_\varepsilon)}{1+\delta_\mu^2}}.$$

The complex number under the radical sign is in the first quadrant if magnetic losses are greater than electric ones (first case). In the contrary (second) case with magnetic losses being smaller than electric ones, the point is in the fourth quadrant. Physical considerations need to be taken into account in order to resolve the ambiguity of the square root. In the present case, this condition is $\mathrm{Im}(\eta)<0$ (or $\mathrm{Im}(\eta^{-1})>0$ for impedance). Contrary conditions correspond to an active medium. In the first case, η should be located in the third quadrant of the complex plane; at infinitely small losses, $\eta=-1$ (i.e., the force does not change sign when magnetic losses exceed electric ones). In the second case, $\eta=1$. The force then does change sign. The following holds true for a body in the form of a layer with thickness t and a large area S:

$$F_{z\mu}=\omega c^{-2}\mu'\tilde{\varepsilon}''SE_0^2\int\limits_0^t\exp(-2k''z)dz\approx\omega c^{-2}\mu/\tilde{\varepsilon}''StE_0^2,$$

i.e., pressure $F_{z\mu}/S$ may be defined. It is easy to demonstrate that $k_z=-k_0$ always holds at infinitely small losses and $\mathrm{Re}(k_z)>0$. It follows from (17) that the force for a small spherical particle is

$$\mathbf{F}_\mu=\pm4\pi r^3\omega c^{-2}\tilde{\varepsilon}''|E_x|^2\eta_0/3,$$

where plus and minus signs correspond to the first and the second cases, respectively. The force is undefined in a loss-free medium.

Let us consider a possible approach to the problem in anisotropic media. Any body (including bodies with tensor DC and MP and even bianisotropic ones) may be characterized by its polarization currents in vacuum. This method is fairly efficient in terms of producing relations based on the momentum balance theorem [1], since the EMT for vacuum is known. The approach provides an opportunity to determine the force acting on any small body within a large one excited by certain specified sources (including a plane wave incident from vacuum). However, volume integral equations need to be solved in the case of a body of an arbitrary shape. Formulating the theorem for, e.g., a homogeneous layer thick along z and positioning the sources at infinity, we obtain a model of motion of a plane wave upon diffraction at the layer. Wave propagation constant k_z is derived from Fresnel equations of the fourth order [37]. Setting $k_x=k_y=0$ (i.e., a wave moving normally to the boundary) and assuming for simplicity that $\hat{\mu}=1$ (magnetic properties are lacking), we find two solutions

$$k_z=k_0\sqrt{\varepsilon_{xx}(\omega)}\quad\text{and}\quad k_z=k_0\sqrt{\varepsilon_{yy}(\omega)}$$

for two wave polarizations. It is also assumed for simplicity here that the tensor is reduced to principal axes. The wave acts on a medium with Lorentz force densities related to polarization current densities. In the case of the considered polarization, these are

$$J_{px}=i\omega\varepsilon_0(\varepsilon_{xx}(\omega)-1)E_x\quad\text{and}\quad E_{Lz}=\mu_0\mathrm{Re}(J_{zx}H_y^*)/2.$$

Using the Maxwell equation, we find force density

$$F_{Lz}=\varepsilon_0|E_x|^2\mathrm{Re}(ik_z^*(\varepsilon_{xx}(\omega)-1))/2.$$

The lack of dissipation translates into the lack of local densities of such forces. A plane wave acts on the layer and transfers momentum to it via reflection and via the indicated local density. The total force is always directed away from the source. If a small particle with DC $\tilde{\varepsilon}(\omega)$ is located within the layer, the problem may be solved using the perturbation method. It is evident that the nature of the force is specified by the sign of

$$k_x'(\tilde{\varepsilon}''(\omega)-\varepsilon_{xx}''(\omega))-k_x''(\tilde{\varepsilon}'(\omega)-\varepsilon_{xx}'(\omega)).$$

This force may be either attractive or repulsive. It acts relative to the medium. If the medium is sufficiently transparent and $\tilde{\varepsilon}'(\omega)\approx\varepsilon_{xx}'(\omega)$, repulsion is observed at $\tilde{\varepsilon}''(\omega)>\varepsilon_{xx}''(\omega)$ (a greater momentum is transferred to the particle than to the corresponding volume of the medium), while attraction is found at $\tilde{\varepsilon}''(\omega)<\varepsilon_{xx}''\omega$.

Model of a Veselago medium and plasma

A model of a Veselago medium should be examined in order to analyze in more detail the wave electrodynamics in it. The only legitimate model mentioned (but not considered) in [1] is the model of a rarefied cold plasma of electric and magnetic monopoles. A magnetic monopole, which was characterized theoretically by Dirac in 1931, has not been discovered yet, although attempts at detecting it are still being made. For example, measurements at collision energies of 8 and 13 TeV have been performed in 2012 and 2015, respectively, with the MoEDAL detector at the Large Hadron Collider. The existence of magnetic monopoles would introduce symmetry into electrodynamics. It specifies the conditions of charge quantization: the elementary (minimum) magnetic charge should be equal to $g_\mu = \hbar c/(2e) = 137e/2$ in the Gaussian system where charges have the same dimension. In the International System of Units, $e = 1.6 \cdot 10^{-19}$ K, $g = h/e$ in the Weber convention, and $g_\mu = 2\pi\hbar c^2\varepsilon$ in the Ampere-meter convention. The existence of magnetic monopoles does not contradict quantum mechanics and electrodynamics, since all electric charges are quantized. Magnetic monopoles interact with each other in accordance with the Coulomb law. A moving monopole produces an electric field and may interact with an electric charge. Hypothetical particles with both electric and magnetic charges (Schwinger dyons) are also subject to the quantization condition. If a dyon with charges e_2 and g_2 is positioned at the origin of coordinates and another dyon with mass m and charges e_1 and g_1 is moving at point $\mathbf{r}$ with velocity $\mathbf{v}$ ($v \ll c$), the force acting on it is [43,44]

$$\mathbf{F} = \frac{(e_1e_2 + g_1g_2)\mathbf{r} + (e_1g_2 - e_2g_1)\mathbf{v}\times\mathbf{r}/c}{r^3}. \quad (18)$$

This relation is written in the Gaussian system where charges have the same dimension. Since $\mathbf{F} = m d\mathbf{v}/dt$, a conserved moment of momentum $\mathbf{L} = \mathbf{r}\times m\mathbf{v} - (e_2g_2 - e_2g_1)\mathbf{r}/(cr)$ and its quantization condition $e_1g_2 - e_2g_1 = \nu\hbar c$ are obtained at $\mathbf{v}\perp\mathbf{r}$. Likewise, the equation of motion of a magnetic monopole in the field of an electric charge is

$$\mathbf{F} m\frac{d\mathbf{v}}{dt} = -\frac{g_1}{c}\mathbf{v}\times\mathbf{E} = -\frac{g_1e_2}{r^3c}\mathbf{v}\times\mathbf{r}. \quad (19)$$

The conserved moment of momentum for Eq. (19) takes the form

$$\mathbf{L} = \mathbf{r}\times m\mathbf{v} + e_2 g\mathbf{r}/(cr),$$

and quantization yields $e_2g_1 = \nu\hbar c$. The Dirac quantization for a monopole yields condition $eg = \nu\hbar c/2$, where ν is an integer number.

A moving electric charge and a moving magnetic monopole produce intrinsic magnetic and electric fields, respectively [43,44]. At nonrelativistic velocities, these fields are weak relative to the Coulomb one in (19). In the International System of Units, the Lorentz force of interaction of the field with monopoles e and g in the Weber convention takes the form

$$\mathbf{F}_L = e(\mathbf{E} + \mathbf{v}\times\mathbf{B}) + g\left(\frac{\mathbf{B}}{\mu_0} - \frac{\mathbf{v}\times\mathbf{E}}{\mu_0 c^2}\right)$$
$$= e(\mathbf{E} + \mathbf{v}\times\mathbf{B}) + g(\mathbf{H} - \mathbf{v}\times\mathbf{D}). \quad (20)$$

This result follows from the momentum conservation law with electric and magnetic sources. Given that $\mathbf{E} = e_2\mathbf{r}/(4\pi\varepsilon_0 r^3)$, $\mathbf{F}_L = -\frac{g_1e_2}{4\pi r^3}\mathbf{v}\times\mathbf{r}$ for Lorentz force (19). The dimension of a magnetic monopole here is V·s. In the case of elementary charges, $g/e = h/e^2 = 2.56\cdot 10^4\,\Omega$. The field of a magnetic monopole moving with an arbitrary velocity is defined in the same way as the field of a moving electric charge [45]. Thus, the mentioned forces may be generalized to arbitrary velocities. We consider a plasma of electric and magnetic monopoles in the field of a weak monochromatic electromagnetic wave. If a wave is weak and its frequency is sufficiently high, charges are first accelerated in a certain direction and then (when the wave sign changes) in the opposite one. The maximum velocity is well below the speed of light (especially at higher frequencies). Therefore, the $v \ll c$ approximation may be used. The following is true under this condition for a common plasma of electric charges:

$$\varepsilon(\omega) = 1 - \frac{\omega_{\varepsilon p}^2}{\omega^2 - i\omega\omega_{\varepsilon c}}. \quad (21)$$

The plasma is considered to be electrically neutral (i.e., consisting of heavy and light electric charges of different signs). The plasma frequency is then governed by light charges and takes the form $\omega_{\varepsilon p}^2 \approx N_\varepsilon e^2/(m_\varepsilon\varepsilon_0)$. Result (21) was obtained without account for the force of interaction between an oscillating charge and the magnetic field of a wave (this force is weak relative to the electric Coulomb one). If this interaction is taken into account, SD emerges. Let us then assume that an electric plasma is combined with a neutral magnetic plasma. A similar equation may be written:

$$\mu(\omega) = 1 - \frac{\omega_{\mu p}^2}{\omega^2 - i\omega\omega_{\mu c}}. \quad (22)$$

Here, $\omega_{\mu o}^2 \approx N_\mu g^2/(m_\mu\mu_0)$. The motion of monopoles under the influence of a wave produces additional contributions to polarization that induce SD. These contributions are neglected due to the fact that a plasma is strongly rarefied and particle velocities are low. Collision rates $\omega_{\varepsilon c}$ and ω_μ may also be neglected in this case. The period-average energy density of the field–matter system may be determined in a closed form for the considered cold rarefied plasma without SD (just as for a rarefied gas of Lorentz oscillators). The energy of oscillations of charges should be taken into account here, and the formula takes the form $w = w_\varepsilon + w_\mu$, where [14]

$$w_\varepsilon = \left[1 + \frac{\omega_{\varepsilon p}^2}{(\omega^2 + \omega_{\varepsilon c}^2)}\right]\frac{\varepsilon_0|E_0|^2}{4},$$

$$w_\mu = \left[1 + \frac{\omega_{\mu p}^2}{(\omega^2 + \omega_{\mu c}^2)}\right] \frac{\mu_0 |H_0|^2}{4}. \quad (23)$$

Squared absolute amplitudes of fields of a plane wave are given here. Since $H_0 = \eta\sqrt{\varepsilon/\mu}E_0$ in a plane wave,

$$\frac{\mu_0 |H_0|^2}{4} = \frac{\varepsilon_0 |E_0|^2}{4} \left|\frac{\varepsilon}{\mu}\right|.$$

However, relations (23) in a plane wave hold true at a fixed point. If the wave decay is to be taken into account, factor $\exp(-2k_z'' z)$ should be introduced into the squared absolute amplitudes. Since

$$\tilde{S}_z' = \eta_0 |E_0|^2 \exp(-2k_z'' z) \mathrm{Re}(\sqrt{\varepsilon/\mu})/2,$$

the energy transfer rate may be determined:

$$v_e = \frac{\tilde{S}_z'}{w} = 2c \frac{\mathrm{Re}(\sqrt{\varepsilon'\mu})}{\left[1 + \frac{\omega_{\varepsilon p}^2}{(\omega^2 + \omega_{\varepsilon c}^2)}\right] + \left[1 + \frac{\omega_{\mu p}^2}{(\omega^2 + \omega_{\mu c}^2)}\right] \left|\frac{\varepsilon}{\mu}\right|}. \quad (24)$$

Let particle concentrations N_ε and N_μ be such that the plasma frequencies match. The collision rates are also assumed to be equal. Then, $\varepsilon = \mu$ and $v_e = c/(1 + \omega_p^2/(\omega^2 + \omega_c^2))$. The energy transfer rate is fairly low at low frequencies, is approximately equal to $c/2$ around the plasma frequency, and tends to c at high frequencies. Let us consider a Veselago medium with $\varepsilon = \mu = -1$. It is modeled by a cold collisionless dimonopole plasma at frequency $\omega = \omega_p/\sqrt{2}$. An intriguing result is obtained: $v_e = c/3$. It may be interpreted in the following way: the energy of the field–matter system is distributed completely between the kinetic energy of oscillations of electric and magnetic particles and the electromagnetic field energy, but only the third part is carried by the field. As for the Maxwell tensor,

$$\hat{\Sigma} = -\mathrm{Re}\left(\mu_0 \frac{\hat{I}\mathbf{H}^2 - 2\mathbf{H} \otimes \mathbf{H}^*}{4} + \varepsilon_0 \frac{\hat{I}\mathbf{E}^2 - 2\mathbf{E} \otimes \mathbf{E}^*}{4}\right),$$

i.e., it does indeed change sign in a left-handed medium, although with respect to the direction of phase motion. In a plane wave,

$$\hat{\Sigma}_{zz} = -\varepsilon_0 E_0^2 \mathrm{Re}(\varepsilon/\mu + 1)/4.$$

It is assumed here that the initial phase of the electric field amplitude is zero. $\hat{\Sigma}_{zz} = -\varepsilon_0 E_0^2/2$ for a Veselago medium. This holds true in an infinite medium. The momentum transfer rate may be determined if the momentum density is known. In a Veselago medium, the momentum should not be transferred from the field to matter. The Poynting vector has component $\tilde{S}_z' = \eta_0 E_0^2/2$. Since the momentum is not transferred to the medium, it may be contained only in the field; i.e.,

$$g = g_z^M = n^2 \tilde{S}'/c^2 = \eta_0 E_0^2/(2c^2).$$

In order to identify the direction of momentum motion, one needs to examine the momentum balance in the case of incidence of a plane wave from vacuum onto an ideal medium layer. Since nothing is reflected off it and the momentum is not transferred to the medium, vacuum density $\eta_0 E_0^2/(2c^2)$ enters the layer. The same density emerges from it. Although fluxes change sign at the boundaries, the sign of normals also changes. Thus, the rate of momentum motion is $v_{mom} = c$. Just as the energy transfer, it is driven exclusively by field polaritons. The phase of these polaritons moves in an opposite way, thus distinguishing them from photons in vacuum. Significant differences emerge upon diffraction of strongly nonstationary short wave packets by an arbitrary dispersing medium layer. Any medium is characterized by a certain polarization settling time. In plasma, this is the time needed to accumulate the energy of kinetic oscillations, and the duration of a quasi-monochromatic wave packet should be considerably longer.

Let us examine a more familiar medium: neutral cold electron–ion plasma. It has $\mu = 1$. The electric field accelerates electrons, and the magnetic field transfers momentum to them. In a dissipative plasma, electrons transfer this momentum to heavy ions. The transfer of momentum from the field to matter is observed, and the latter starts moving. The contribution of the momentum transfer by matter in common relatively weak fields is negligible compared to the rate of momentum transfer by the field. It is often assumed that matter is stationary. In a collisionless plasma, a wave transfers momentum to electrons within one half-period and takes this momentum away within the other half-period. Therefore, the momentum is not accumulated in matter, and it does not move. The momentum density may then be determined in the Abraham form. $\hat{E}_{zz} = \varepsilon_0 \varepsilon' E_0^2$ in plasma; i.e., the sign changes approximately at the plasma frequency. The flux is negative below it; this is observed, e.g., when plasmons move along the metal surface with an energy flux into the bulk of a metal. This is relating to the Poynting vector component directed toward the interface: its flux in a metal is opposite to the flux in vacuum, and all fluxes are directed toward the interface [46]. The fluxes along the boundary are always unidirectional, and a plasmon is always a forward one. A metal layer with the flux in it being opposite to the flux in vacuum is needed to support a backward plasmon [46]. In the case of electron plasma,

$$w = \frac{\varepsilon_0 E_0^2}{4} \left[1 + |\varepsilon|^2 + \frac{\omega_p^2}{(\omega^2 + \omega_c^2)}\right], \quad (25)$$

$$\tilde{S}_z' = \nu_0 E_0^2 \mathrm{Re}(\sqrt{\varepsilon})/2. \quad (26)$$

Let us examine the correlation between these formulae and the Brillouin formula [14,47]. Thus,

$$\partial_\omega(\omega\varepsilon(\omega)) = 1 + \frac{\omega_p^2 (1 + i\omega_c/\omega)^2}{\omega^2 (1 + \omega_c^2/\omega^2)},$$

$$\mathrm{Re}(\partial_\omega(\omega\varepsilon(\omega))) = 1 + \frac{\omega_p^2}{(\omega^2 + \omega_c^2)} - \frac{\omega_p^2 \omega_c^2}{\omega^4 (1 + \omega_c^2/\omega^2)^2},$$

$$\partial_\omega(\omega\varepsilon'(\omega)) = 1 + \frac{\omega_p^2}{(\omega^2+\omega_c^2)} - \frac{2\omega_p^2\omega_c^2}{\omega^4(1+\omega_c^2/\omega^2)^2}.$$

At $\omega_c/\omega \ll 1$, two formulae yield results that are different, but similar to (25) within ω_c (i.e., at sufficiently high frequencies) and match in loss-free conditions. With weak dissipation, $\sqrt{\varepsilon} = \sqrt{\varepsilon'}(1 - i\varepsilon''/2\varepsilon')$ at frequencies significantly higher and lower than the plasma one. In the first case, $\mathrm{Re}(\sqrt{\varepsilon}) \approx \varepsilon''/(2\sqrt{|\varepsilon'|})$. In the second case, $\mathrm{Re}(\sqrt{\varepsilon}) \approx \sqrt{\varepsilon'}$. A fairly low energy transfer rate corresponds to the first case. The momentum flux is given by $\hat{\Sigma}_{zz} = \varepsilon_0\varepsilon' E_0^2/2$. This quantity changes sign when ε' undergoes sign reversal. If one assumes that $g_z^M = \eta_0\varepsilon' E_0^2/(2c^2)$, a momentum transfer rate of $v_{mpm} = c$ is obtained. For the Abraham momentum, we find $v_{mom} = c\varepsilon'$. This rate goes to zero at frequency $\omega = \sqrt{\omega_p^2 - \omega_c^2}$ and changes sign at the same frequency. At high frequencies, it approaches the speed of light from below. At ultralow frequencies, $\varepsilon' \approx - - \omega_p^2/\omega_c^2$ assumes a large negative value, and the rate may be substantially greater in magnitude than the speed of light. It should be noted here that losses cannot be neglected at frequencies this low (since $|\varepsilon''| \gg |\varepsilon'|$); therefore, the Abraham formula is inapplicable. While the oscillatory energy of electrons may be accumulated in the $\omega_c \ll \omega < \omega_p$ plasmonics region, it is not accumulated (i.e., is dissipated only) at $\omega < \omega_c$. Since the momentum is not transferred by a stationary medium, $g^M = \eta_0\varepsilon'|\mathbf{E}|^2/(2c^2)$ should be set for its density, and the transfer rate should be assumed equal to the speed of light A phase shift exists between the electric and magnetic fields in plasma; there is no power flux at $\mathrm{Re}(\sqrt{\varepsilon}) = 0$ and no flux and momentum at $\varepsilon' = 0$.

Backward fluxes and negative pressure

It is of interest to note that backward power and momentum fluxes in a monochromatic wave are observed not only in media (existing and hypothetical), but also in vacuum. A nonplane wave is needed for this. More specifically, a wave beam should be transversally bounded (or even unbounded, but decaying strongly in the transverse direction). The Poynting vector then does not form a laminar flow; instead, it swirls in a manner similar to flow lines in a turbulent jet. A large number of studies intro wave beams (including optical laser vortex wave beams) with backward energy fluxes (even in vacuum) have recently been published [48–99]. The Poynting vector in such wave beams is swirling in nature, allowing for backward energy transfer. The transfer of energy and momentum to the source, including the action of forces of „attraction" to the source or „negative pressure" on a nanoparticle in such a flux, and the potential to construct „optical tweezers" are considered. In contrast to a hypothetical infinite plane electromagnetic wave, these wave beams may have a moment of momentum of the field and flux singularities $\tilde{\mathbf{S}}'$. A negative force acting on a nanoparticle does not necessarily correspond to a negative flux. The inverse is also true: a negative flux does not always yield a negative force. Negative fluxes have been observed in acoustic beams [100]. They are feasible even in X-ray beams [101]. The following types of monochromatic beams with different transverse field distributions (unbounded ones included) are being examined at present: Gaussian, Bessel, Hermite−Gaussian, and Laguerre−Gaussian beams. A transverse confinement or a field intensity reduction necessitates the emergence of backward fluxes, since solenoidal lines of flux are closed. Monochromatic waves are examined in the majority of studies. Such a wave implies infinite operation of sources with a harmonic dependence on time. The transition to it from the nonstationary case is not a trivial one. The transition from dynamics to statics is also challenging. An example here is provided by the endless discussion on the possible existence of electromagnetic energy, momentum, and moment of momentum fluxes in static fields (including fluxes circulating along closed trajectories) that has been ongoing since the days of Poynting. All processes in nature are nonstationary to a certain extent, and all of them had their beginning. In quasistatics, the energies of quanta are fairly low, but the number of these quanta needed to obtain the final energy is fairly high. The impact of a wave packet is substantially nonstationary at the initial moment, and the fluxes are then positive on the average. Negative fluxes may emerge with quasi-monochromatic processes. It is of interest that such fluxes also exist in the near field of a point-like dipole in the vicinity of the field singularity if one considers the real Poynting vector $\mathbf{S} = \mathrm{R}e(\mathbf{E}) \times \mathrm{Re}(\mathbf{H})$; notably, they oscillate in time. Let p_z be the dipole current moment. We then obtain [102]

$$E_r' = \frac{p_z\cos(\theta)}{2\pi\omega\varepsilon_0}\left[\frac{k_0\cos(k_0r)}{r^2} - \frac{\sin(k_0r)}{r^3}\right],$$

$$E_\theta' = \frac{p_z\sin(\theta)}{4\pi\omega\varepsilon_0}\left[\frac{k_0\cos(k_0r)}{r^2} - \frac{\sin(k_0r)(1-k_0^2r^2)}{r^3}\right],$$

$$H_\varphi' = \frac{p_z\sin(\theta)}{4\pi\omega\varepsilon_0}\left[\frac{\cos(k_0, r)}{r^2} - \frac{k_0\sin(k_0r)}{r^3}\right].$$

In addition, $S_r = E_\theta' H_\varphi'$ and $S_\theta = -E_r' H_\varphi'$. The period-average flux is zero: $\tilde{S}_r' = 0$. The front flattens progressively in the far field, and negative fluxes gradually vanish. All fields may be expanded in plane waves. The fluxes in a solitary plane wave are always forward ones. However, it is interesting to note that back in 1997 B.Z. Katsenelenbaum has discovered a backward Poynting vector flux in several plane waves moving at an angle in different directions [103].

Conclusion

The issue of „negative pressure" was highlighted in the title of the paper. However, while the term „pressure" is well-established, „force" is a word more fitting to the examination of the influence of the field on a particle. „Pressure" is an apt term for analyzing the impact of a plane wave on an interface or striction distributed pressure on a plane matter layer; it is also applicable when the wave front and the surface on which it acts are locally planar. The force acting on a particle in a dispersing medium depends essentially on the relation between dissipation in the medium and in the particle. Dissipation in the medium leads to the transfer of momentum to it and to a local force density. The force acting on a particle is negative when a smaller momentum is transferred to the particle than to the corresponding volume of the medium. Thus, it is a force relative to the medium. In loss-free conditions, this force is always positive (i.e., directed away from the source) for a plane wave.

The issues examined above are related to a more than 110-year-long discussion regarding the EMT form in electrodynamics of continuous media for the field–matter system [1–8,104–114]. The Abraham and Minkowski EMT forms corresponding to non-dispersing media are not the only ones available; other known forms include those proposed by Polevoi and Rytov [115] and Pitaevskii [116], the canonical EMT, etc. Several forms have been examined in [108,109,112–114]. Forces in dispersing but non-dissipative media have been analyzed in [113]. The EMT in moving media has been considered in [109].

We will not dwell here upon the issue of correctness or incorrectness of certain EMT forms. Note only that a classical field acts on a dispersing medium in such a way that the medium gets accelerated; the EMT of the closed field–matter system should then be symmetrical and factor in the transfer of energy and momentum by both subsystems (field and matter) and their interaction [117,118]. The field momentum in weak fields is tiny, and the medium and the process are often considered to be stationary. In the general case, one should turn to nonstationary electrodynamics of moving dispersing media.

The Minkowski relations [41,47] without dispersion are still being used in the electrodynamics of moving media. A system may be closed only if sources producing the field are taken into account. Free waves in a medium without sources are considered most often. Nonstationary field-producing processes [39,40] need to be examined in order to determine the energy density of the field–matter system and the momentum density; heating and acceleration of matter, which complicate the analysis, are possible in this case.

We have examined weak monochromatic fields that do not accelerate matter and do not raise its temperature. A rigorous determination of the energy and momentum density in this approximation necessarily involves analyzing the transient process of production of a quasi-monochromatic field in a dispersing dissipative medium with this process reaching the set monochromatic field amplitudes [39]. As oscillations settle, such a wave packet eventually acquires a narrow instantaneous spectrum $\Delta\omega \ll \omega_0$. The commonly used Brillouin formula for the approximate energy density of a narrow wave packet in a non-dissipative medium was derived by expanding the quasi-stationary quadratic integral asymptotically to the first order in $\Delta\omega_0$ [47]. The group velocity for a narrow wave packet is determined in a similar fashion. In loss-free conditions, the omission of all other orders actually implies the lack of dispersion. This may be illustrated using the example of the Lorentz dispersion formula with several resonance frequencies: an infinite separation of frequencies in needed to achieve zero losses. Note that forces under the impact of wave packets have been considered in [113]. It was demonstrated there that, depending on the parameters of LHM–RHM interfaces, both „light pressure" and „light attraction" are possible at such interfaces. As for the assertion regarding the negative pressure on a particle in a left-handed medium made in [9], it is erroneous: the pressure in a plane wave is always directed away from the field source. Note also that the impact of the field on a medium may be analyzed rigorously if the latter is characterized by its polarization with respect to vacuum. The Maxwell equations are then considered in vacuum, and the medium is characterized as secondary field sources in the form of polarization currents [1]. This approach leads to the Einstein–Laub force [119]. It is applied most easily to bodies of a finite volume.

The approach used above is applicable to anisotropic media and media with SD, but requires a separate intricate analysis. As for a hypothetical Veselago medium, its existence would be problematic even if magnetic charges do exist. Rarefaction and the lack of SD require low concentrations and plasma frequencies (i.e., low frequencies at which the effect could be manifested).

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Conflict of interest

The authors declare that they have no conflict of interest.

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