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Suppression of exchanged spin waves in a film with a partial magnetic coating

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The paper considers the scattering of an exchanged spin wave in a system consisting of an unlimited magnetic film and a thin magnetic layer located above it in the form of a magnetic stripe of finite or infinite width. It is shown that the main damping mechanism is the excitation of anti-Larmor precession in the stripe during antiferromagnetic interaction with the film magnetization, which corresponds to an evanescent spin wave. The reflection and transmission coefficients and the distribution of the field of the converted spin wave along the film are obtained. The scattering coefficients and distribution of the field of the converted spin wave along the film are obtained.

Keywords: magnon waveguide, chirality of magnetization precession, evanescent spin waves, indirect exchange interaction (RKKY), reflection and transmission of exchange spin waves.

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1. Introduction

To date, in the field of magnonics, extensive theoretical and experimental experience has been gained in studying the properties of spin waves (SW) as potential signal carriers. In particular, [1] describes the main logical modules implemented using spin waves, where the control element in the form of a magnetic strip is oriented perpendicular to the waveguide. Chiral coupling between SW propagating in exchange-coupled or dipole-coupled films or antennas can be used to control the amplitudes and phase shift of SW in chiral magnon resonators. Such resonators can serve both as basic elements for magnon gates, phase shifters, etc. [2,3], and as modules for neural networks due to the presence of a nonlinear response under resonant excitation of SW [4].

When describing wave processes during the propagation of SW, essential is their chirality, which determines the direction of magnetization precession. As follows from the equation for the dynamics of an isolated magnetic moment, it is unambiguously determined by the external magnetic field. The exchange interaction can change the sign of the effective field, and the precession will become anti-Larmor with an imaginary wave number, and the corresponding SW will become evanescent [5]. Therefore, when forming a potential relief for SW, the interaction between spins of the film and strip of exchange or magnetic-dipole nature can be used as a „potential barrier" [6]. The nature of the interlayer exchange interaction can be either ferromagnetic or antiferromagnetic, depending on the thickness of the non-magnetic interlayer [7].

In the existing „scale of spin waves" it is customary to distinguish two ranges. The first is the range of relatively long magnetic-dipole waves (about 10 mkm), the mathematical description of which is complicated due to the nonlocal nature of the magnetic-dipole interaction. Such waves, however, have been in practical use for quite a long time [8–10]. The second is the range of SWs, where the exchange interaction is most significant ($0.1\,\mu$m and shorter). Its use in terahertz devices is seen as a near future and opens up the possibility of miniaturizing magnon devices. Recently, along with magnetic-dipole and exchange waves, a range of exchange-dipole waves ($0.1-10\,\mu$m) has also been distinguished, where both interactions make contributions of the same order of magnitude [11].

In a recent study [12], the scattering of exchange-dipole SWs in a yttrium-iron garnet film coated with a cobalt magnetic strip was experimentally studied and the possibilities of their modulation using both an external field and choice of their geometry were discovered. The practical implementation of such a device seems simpler compared to films made of edge-adjacent magnetic materials described in [13,14]. The latter are convenient in that SWs can be easily excited in them when the interface is exposed to a homogeneous pump field [15].

Controlling the equilibrium magnetization makes it possible to obtain waveguides with non-reciprocal properties [16], which are most pronounced in the Damon–Eshbach geometry [17]. However, it is important that such properties can be observed only in multilayer (in particular, bilayer) structures or in magnon crystals, because the equilibrium magnetizations in the layers can be chosen arbitrarily.

In this study, a theoretical model is built up for the propagation and scattering of the exchange range SWs in a thin film with a thin magnetic strip applied on top.

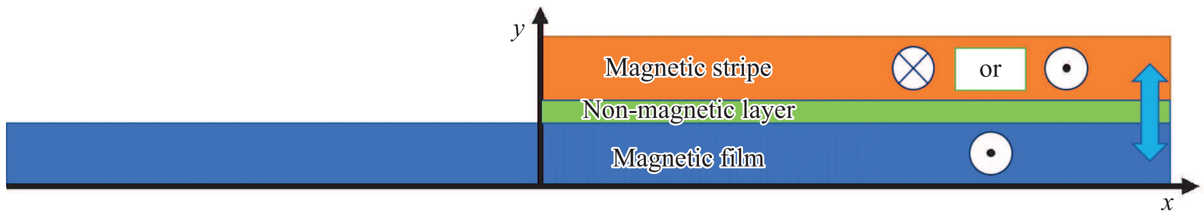

Figure 1. Geometry of the problem. A magnetic strip separated by a thin non-magnetic layer is placed above an infinite film. Depending on the thickness of the interlayer, the exchange interaction with the strip is either ferromagnetic or antiferromagnetic.

The results obtained are also qualitatively valid for longer exchange-dipole SWs.

2. Model and problem statement

A thin magnetic film is located parallel in the zx plane (a). Above it there is a thin semi-infinite magnetic strip (b), occupying the region of $x > 0$. The normal to the layers is parallel to the y axis, and the direction of propagation of the exchange SW is chosen along the x axis.

It is assumed that the components of the structure are separated by a thin non-magnetic layer, and their spins are exchange-interacting. In this case, as noted above, the interlayer exchange interaction constant, depending on the thickness of the interlayer, can be either positive or negative, due to which the mutual equilibrium orientation of spins of the film and strip will be either parallel or antiparallel. In each of them, the presence of easy-axis anisotropy is assumed, that orients the spins in the ground state along z axis. The direction of SW propagation corresponds to the Damon–Eshbach geometry [11].

As for the demagnetizing field, taking it into account does not allow solving the problem accurately. The energy of magnetic-dipole interaction is not local and can be written in a simple form only for an infinite flat structure [11]. In microscopic form, it consists of two parts. The first of them is isotropic, proportional to the scalar product of the spin vectors $(-\mathbf{S}_i\mathbf{S}_j)$ and therefore can be attributed to the exchange energy. Its second, anisotropic part $(\mathbf{S}_i\mathbf{n})(\mathbf{S}_j\mathbf{n})$ determines the selected direction of magnetization.

Let us consider a simplified model where the above-mentioned interaction is negligible, i.e. the constants of exchange interaction J, G and uniaxial anisotropy B are large enough for the precession of magnetization to be considered circular. Taking into account the demagnetizing field would lead to an elliptical precession of spins [19] and to expressions that are much more cumbersome for analysis, hiding the essence of the effect under consideration. For the same purpose, we will assume the magnetic parameters of the film and strip to be the same.

The energy of the structure presented in Figure 1 includes crystallographic anisotropy (with constant B), exchange intralayer (J) and interlayer (G) interaction. In the lattice model it has the following form:

$$W = -\sum_{j=-\infty}^{\infty}\left(J\mathbf{S}_j^{(a)}\mathbf{S}_{j+1}^{(a)} + \frac{B}{2}\left(\mathbf{S}_j^{(a)}\mathbf{n}_z\right)^2\right)$$

$$-\sum_{o=1}^{\infty}\left(J\mathbf{S}_j^{(b)}\mathbf{S}_{j+1}^{(b)} + \frac{B}{2}\left(\mathbf{S}_j^{(b)}\mathbf{n}_z\right)^2 + \sigma G\mathbf{S}_j^{(a)}\mathbf{S}_j^{(b)}\right), \quad (1)$$

where $\sigma = +1$ corresponds to ferromagnetic (FM), and $\sigma = -1$ corresponds to antiferromagnetic (AFM) cases. The „a" superscripts refer to the spins of the film, the „b" superscripts refer to those of the strip.

Landau–Lifshitz spin dynamics equations

$$\hbar S\frac{\partial \mathbf{S}}{\partial t} = \left[\mathbf{S}\times\frac{\partial W}{\partial \mathbf{S}}\right] \quad (2)$$

for film $(af,\ j \le -1,\ x < 0$ and $a,\ j \ge 2,\ x > 0)$ and strip $(b,\ j \ge 2,\ x > 0)$ taking into account the periodic time dependence of the $e^{-i\omega t}$ variables, have the following form:

$$SE\mathbf{S}_j^{(af)} = \left[\mathbf{S}_j^{(af)} \times \left(J\left(\mathbf{S}_{j+1}^{(af)} + S_{j-1}^{(af)}\right) + BS_{jz}^{(af)}\mathbf{n}_z\right)\right] \quad (3.1)$$

$$SE\mathbf{S}_j^{(a)} = \left[\mathbf{S}_j^{(a)} \times \left(J\left(\mathbf{S}_{j+1}^{(a)} + \mathbf{S}_{j-1}^{(a)}\right) + BS_j^{(a)}\mathbf{n}_z + \sigma G\mathbf{S}_j^{(b)}\right)\right] \quad (3.2)$$

$$SE\mathbf{S}_j^{(b)} = \left[\mathbf{S}_j^{(b)} \times \left(J\left(\mathbf{S}_{j+1}^{(b)} + \mathbf{S}_{j-1}^{(b)}\right) + BS_j^{(b)}\mathbf{n}_z + \sigma G\mathbf{S}_j^{(a)}\right)\right], \quad (3.3)$$

where S is spin value, $E = \hbar\omega$ is magnon energy. In the linear approximation of small deviations from the ground state, spins can be represented as follows

$$\mathbf{S}_j^{(af)} = S\mathbf{n}_z + \mathbf{s}_j^{(af)},\ \ \mathbf{S}_j^{(a)} = S\mathbf{n}_z + \mathbf{s}_j^{(a)},\ \ \mathbf{S}_j^{(b)} = \sigma S\mathbf{n}_z + \mathbf{s}_j^{(b)}. \quad (4)$$

We assume that the dynamic components of the spin have a periodic dependence on the spin number in the lattice j (d being lattice constant):

$$\mathbf{s}_j^{(af)} = \mathbf{s}^{(af)}e^{ik^{(f)}jd},\quad \mathbf{s}_j^{(a/b)} = \mathbf{s}^{(a/b)}e^{ik^{(l)}jd}. \quad (5)$$

Let us find the law of dispersion for SWs for various types of interlayer coupling.

Table 1. Types of waves in the case of ferromagnetic ordering of equilibrium magnetizations of film and strip

Polarization	System of equations	Law of dispersion	Normal modes	Initial variables
s_+ (right-handed)	$\begin{pmatrix} E+E_{k^{(l)}} & -G \\ -G & E+E_{k^{(l)}} \end{pmatrix}\begin{pmatrix} s_+^{(a)} \\ s_+^{(b)} \end{pmatrix}=0$	$2\sin\left(\frac{k_{e1}^{(l)}d}{2}\right)=\sqrt{\frac{-E-B}{J}}$	$s_{+j}^{(a)}+s_{+j}^{(b)}$	$s_{+j}^{(a)}=t_{e1}e^{ik_{e1}^{(l)}d(j-1)}+t_{e2}e^{ik_{e2}^{(l)}d(j-1)}$
		$2\sin\left(\frac{k_{e2}^{(l)}d}{2}\right)=\sqrt{\frac{-E-B-2G}{J}}$	$s_{+j}^{(a)}-s_{+j}^{(b)}$	$s_{+j}^{(b)}=t_{e1}e^{ik_{e1}^{(l)}d(j-1)}-t_{e2}e^{ik_{e2}^{(l)}d(j-1)}$
s_- (left-handed)	$\begin{pmatrix} E-E_{k^{(l)}} & G \\ G & E-E_{k^{(l)}} \end{pmatrix}\begin{pmatrix} s_-^{(a)} \\ s_-^{(b)} \end{pmatrix}=0$	$2\sin\left(\frac{k_{p1}^{(l)}d}{2}\right)=\sqrt{\frac{E-B}{J}}$	$s_{+j}^{(a)}+s_{+j}^{(b)}$	$s_{+j}^{(a)}=t_{p1}e^{ik_{p1}^{(l)}d(j-1)}+t_{p2}e^{ik_{p2}^{(l)}d(j-1)}$
		$2\sin\left(\frac{k_{p2}^{(l)}d}{2}\right)=\sqrt{\frac{E-B-2G}{J}}$	$s_{+j}^{(a)}-s_{+j}^{(b)}$	$s_{+j}^{(b)}=t_{p1}e^{ik_{p1}^{(l)}d(j-1)}-t_{p2}e^{ik_{p2}^{(l)}d(j-1)}$

3. Analysis of wave types in the structure

Free film ($x < 0$)

Equation (3.1) for a free film, after substituting (4) into it and linearization, in the cyclic components $s_\pm = s_x \pm is_y$ is reduced to the following form

$$(E_{k^{(f)}} \pm E)s_\pm = 0, \tag{6}$$

where the magnon energy is

$$E_{k^{(f)}} = B + J\left(2\sin\left(\frac{k^{(f)}d}{2}\right)\right)^2. \tag{6}$$

The values of $E_{k^{(f)}} = \pm E$ correspond to the dispersion law

$$2\sin\left(\frac{k_{p/e}^{(f)}d}{2}\right) = \sqrt{\frac{\pm E - B}{J}} \tag{7}$$

and dynamic variables $s_{\mp k}^{(af)} \sim e^{ik_{p/e}^{(f)}d\cdot(-k)}$. Thus, the spectrum at $E > B$ contains one traveling wave and one evanescent wave, which correspond to different circular polarizations, therefore

$$s_{+j}^{(af)} = r_e e^{-iq_e^{(f)}d\cdot(-j)},$$

$$s_{-j}^{(af)} = 1\cdot e^{+iq_p^{(f)}d\cdot(-j)} + r_p e^{-iq_p^{(f)}d\cdot(-j)}. \tag{8}$$

It should be noted that if the SW length is much greater than the lattice constant (continuum approximation), then

$$k_p^{(f)} = \frac{1}{\lambda_0}\sqrt{\frac{\omega}{\omega_0}-1}, \quad k_e^{(f)} = \frac{i}{\lambda_0}\sqrt{\frac{\omega}{\omega_0}+1} \tag{9}$$

$$s_+^{(af)} = r_e e^{-iq_e^{(f)}x}, \quad s_-^{(af)} = 1\cdot e^{+iq_p^{(f)}x} + r_p e^{-iq_p^{(f)}x} \tag{10}$$

where $\lambda_0 = d\sqrt{\frac{J}{B}}$ is exchange length, and $\omega_0 = \frac{B}{\hbar}$.

4. Film-strip ($x > 0$)

FM interlayer coupling ($\sigma > 0$)

The linearized system of equations for coupled oscillations (3.1) and (3.2) when substituting (4) into it takes the following form:

$$iE\mathbf{s}^{(a)} = \left[\left(E_{k^{(l)}}\mathbf{s}^{(a)} - G\mathbf{s}e_k^{(b)}\right)\times\mathbf{n}_z\right]$$

$$iE\mathbf{s}^{(b)} = \left[\left(E_{k^{(l)}}\mathbf{s}^{(b)} - G\mathbf{s}_k^{(a)}\right)\times\mathbf{n}_z\right], \tag{11}$$

where

$$E_{k^{(l)}} = B + G + J\left(2\sin\left(\frac{k^{(l)}d}{2}\right)\right)^2. \tag{12}$$

In cyclic components

$$s_\pm^{(a)} = s_x^{(a)} \pm is_y^{(a)}, \quad s_\pm^{(b)} = s_x^{(b)} \pm is_y^{(b)} \tag{13}$$

system (11) is factorized by polarizations. The result is presented in Table 1 Both waves of right-handed polarization are evanescent, and left-handed waves are traveling under the condition $E \geq B + 2G$, i.e. for magnon energy above the activation frequency. Similar to expressions (9) and (10), the following can be written in the continuum limit

$$k_{e1}^{(l)} = \frac{i}{\lambda_0}\sqrt{\frac{\omega}{\omega_0}+1}, \quad k_{e2}^{(l)} = \frac{i}{\lambda_0}\sqrt{\frac{\omega+2\omega_g}{\omega_0}+1},$$

$$k_{p1}^{(l)} = \frac{1}{\lambda_0}\sqrt{\frac{\omega}{\omega_0}-1}, \quad k_{p2}^{(l)} = \frac{1}{\lambda_0}\sqrt{\frac{\omega-2\omega_g}{\omega_0}-1} \tag{14}$$

$$s_+^{(a)} = t_{e1}e^{iq_{e1}^{(l)}x} + t_{e2}e^{iq_{e2}^{(l)}x}, \quad s_-^{(a)} = t_{p1}e^{iq_{p1}^{(l)}x} + t_{p2}e^{iq_{p2}^{(l)}x}$$

$$s_+^{(b)} = t_{e1}e^{iq_{e1}^{(l)}x} - t_{e2}e^{iq_{e2}^{(l)}x}, \quad s_-^{(b)} = t_{p1}e^{iq_{p1}^{(l)}x} - t_{p2}e^{iq_{p2}^{(l)}x} \tag{15}$$

Table 2. Types of waves in the case of antiferromagnetic ordering of equilibrium magnetizations of film and strip

Polarization	System of equations	Law of dispersion	Normal modes	Initial variables
s_+ (right-handed)	$\begin{pmatrix} E_{k^{(l)}}+E & G \\ G & E_{k^{(l)}}-E \end{pmatrix}\begin{pmatrix} s_+^{(a)} \\ s_+^{(b)} \end{pmatrix}=0$	$2\sin\left(\frac{k_p^{(l)}d}{2}\right)=\sqrt{\frac{\sqrt{E^2+G^2}-G-B}{J}}$	$s_{+j}^{(a)}-\eta s_{+j}^{(b)}$	$s_{+k}^{(a)}=t_{e+}e^{ik_e^{(l)}d(k-1)}+t_{p+}e^{ik_p^{(l)}d(k-1)}$
		$2\sin\left(\frac{k_e^{(l)}d}{2}\right)=\sqrt{\frac{-\sqrt{E^2+G^2}-G-B}{J}}$	$\eta s_{+j}^{(a)}+s_{+j}^{(b)}$	$s_{+k}^{(b)}=\frac{1}{\eta}t_{e+}e^{ik_e^{(l)}d(k-1)}-\eta t_{p+}e^{ik_p^{(l)}d(k-1)}$
s_- (left-handed)	$\begin{pmatrix} E_{k^{(l)}}-E & G \\ G & E_{k^{(l)}}+E \end{pmatrix}\begin{pmatrix} s_-^{(a)} \\ s_-^{(b)} \end{pmatrix}=0$	$2\sin\left(\frac{k_p^{(l)}d}{2}\right)=\sqrt{\frac{\sqrt{E^2+G^2}-G-B}{J}}$	$\eta s_{-j}^{(a)}-s_{-j}^{(b)}$	$s_{-j}^{(a)}=t_{e-}e^{ik_e^{(l)}d(j-1)}+t_{p-}e^{ik_p^{(l)}d(j-1)}$
		$2\sin\left(\frac{k_e^{(l)}d}{2}\right)=\sqrt{\frac{-\sqrt{E^2+G^2}-G-B}{J}}$	$s_{-j}^{(a)}+\eta s_{-j}^{(b)}$	$s_{-j}^{(b)}=\eta t_{e-}e^{ik_e^{(l)}d(j-1)}-\frac{1}{\eta}t_{p-}e^{ik_p^{(l)}d(j-1)}$

5. AFM interlayer coupling $\sigma = -1$

Choosing the ground state of the film and strip spins to be antiferromagnetic yields the following:

$$\mathbf{S}_j^{(a)} = S\mathbf{n}_z + \mathbf{s}_j^{(a)}, \quad \mathbf{S}_j^{(b)} = -S\mathbf{n}_z + \mathbf{s}_j^{(b)}. \tag{16}$$

Dynamics equations (3.2) and (3.3) after substituting (16) into them take the following form:

$$iE\mathbf{s}^{(a)} = \left[\left(E_{k^{(l)}}\mathbf{s}^{(a)} + G\mathbf{s}^{(b)}\right)\times\mathbf{n}_z\right]$$

$$-iE\mathbf{s}^{(b)} = \left[\left(E_{k^{(l)}}\mathbf{s}^{(b)} + G\mathbf{s}^{(a)}\right)\times\mathbf{n}_z\right] \tag{17}$$

where $E_{k^{(l)}}$ is determined by formula (12).

This system can also be factorized in cyclic variables. The result is presented in Table 2. In contrast to the case of FM coupling, the waves are now elliptically polarized.

In the continuum limit, the coordinate dependences and wave numbers will be written in the following form:

$$k_p^{(l)} = \frac{1}{\lambda_0}\sqrt{\frac{\sqrt{\omega^2+\omega_g^2}-\omega_g}{\omega_0}-1},$$

$$k_e^{(l)} = \frac{i}{\lambda_0}\sqrt{\frac{\sqrt{\omega^2+\omega_g^2}+\omega_g}{\omega_0}+1}, \tag{18}$$

$$\eta = \frac{\sqrt{\omega^2+\omega_g^2}+\omega}{\omega_g}$$

$$s_+^{(a)} = t_{e+}e^{ik_{e1}^{(l)}x}+t_{p+}e^{ik_{e2}^{(l)}x},\ s_-^{(a)} = t_{e-}e^{iq_e^{(l)}x}+t_{p-}e^{iq_p^{(l)}x}$$

$$s_+^{(b)} = \frac{1}{\eta}t_{e+}e^{ik_e^{(l)}x}-\eta t_{p+}e^{ik_p^{(l)}x},\ s_-^{(b)} = \eta t_{e-}e^{iq_e^{(l)}x}-\frac{1}{\eta}t_{p-}e^{iq_p^{(l)}x}. \tag{19}$$

The value of

$$\eta = \frac{\sqrt{E^2+G^2}+E}{G} > 1$$

is the ratio of SW amplitudes in the film and in the strip.

6. Boundary conditions

As it was shown in [20], boundary conditions are the dynamics equations for the three boundary spins ($af0$, $a1$, and $b1$):

$$iES\mathbf{s}_0^{(af)} = \left[\mathbf{S}_0^{(af)}\times\left(J\left(\mathbf{S}_{-1}^{(af)}+\mathbf{S}_1^{(a)}\right)+BS\mathbf{n}_z\right)\right] \tag{20.1}$$

$$iES\mathbf{s}_1^{(a)} = \left[\mathbf{S}_1^{(a)}\times\left(J\left(\mathbf{S}_2^{(a)}+\mathbf{S}_0^{(af)}\right)+B\mathbf{n}_z\left(\mathbf{S}_1^{(a)}\mathbf{n}_z\right)+G\mathbf{S}_1^{(b)}\right)\right] \tag{20.2}$$

$$iES\mathbf{s}_1^{(b)} = \left[\mathbf{S}_1^{(b)}\times\left(J\mathbf{S}_2^{(b)}+B\mathbf{n}_z\left(\mathbf{S}_1^{(b)}\mathbf{n}_z\right)+G\mathbf{S}_1^{(a)}\right)\right] \tag{20.3}$$

In the continuum approximation, the term J is of the order of $\frac{1}{d^2}$ in terms of lattice constant and therefore the corresponding term is much greater than the others.

Representing the components of boundary spins in the form of expansions

$$\mathbf{S}_{-1}^{(af)} = \mathbf{S}_0^{(af)} - d\frac{d\mathbf{S}_0^{(af)}}{dx}, \quad \mathbf{S}_2^{(a)} = \mathbf{S}_1^{(a)} + d\frac{d\mathbf{S}_0^{(a)}}{dx},$$

$$\mathbf{S}_2^{(b)} = \mathbf{S}_1^{(b)} + d\frac{d\mathbf{S}_0^{(b)}}{dx} \tag{21}$$

and renominating $\mathbf{S}_0^{(af)} = \mathbf{S}^{(af)}$, $\mathbf{S}_1^{(a)} = \mathbf{S}^{(a)}$ we get

$$-d\left[\mathbf{S}^{(af)}\times\frac{dS^{(af)}}{dx}\right]+\left[S^{(af)}\times S^{(al)}\right]=0,$$

$$d\left[S^{(a)}\times\frac{dS^{(a)}}{dx}\right]+\left[S^{(a)}\times S^{(af)}\right]=0. \tag{22}$$

After combining equations (22) in different orders of smallness in the lattice constant, we get

$$[\mathbf{S}^{(af)}\times\mathbf{S}^{(a)}]=0,$$

$$\left[\mathbf{S}^{(af)}\times\frac{d\mathbf{S}^{(af)}}{dx}\right]=\left[\mathbf{S}^{(a)}\times\frac{d\mathbf{S}^{(a)}}{dx}\right], \quad \left[\mathbf{S}^{(b)}\times\frac{d\mathbf{S}^{(b)}}{dx}\right]=0 \tag{23}$$

Linearizing the resulting expressions after substituting (16) into them yields the following:

$$\left[\mathbf{S}_{(0)}^{(af)} \times \mathbf{s}^{(a)}\right] + \left[\mathbf{s}^{(af)} \times \mathbf{S}_{(0)}^{(a)}\right] = 0 \qquad (24)$$

Due to the fact that equilibrium parameters of the film are the same, the condition is reduced to the continuity of the corresponding dynamic components. Thus, the boundary conditions take the following form:

$$s_{\pm}^{(af)}(0) = s_{\pm}^{(a)}(0), \quad \frac{ds_{\pm}^{(af)}}{dx}(0) = \frac{ds_{\pm}^{(a)}}{dx}(0), \quad \frac{ds_{\pm}^{(b)}}{dx}(0) = 0 \qquad (25)$$

It should be noted that conditions (25) for waves of opposite polarizations are independent. Due to the fact the inhomogeneous term (incident wave) is present only in $s_{-}^{(af)}$, then all amplitude coefficients with the subscript $+$ are equal to zero, because the system corresponding to them is homogeneous:

$$r_e = t_{e1} = t_{e2}(\mathrm{FM}) = t_{e+} = t_{p+}(\mathrm{AFM}) = 0. \qquad (26)$$

7. Scattering coefficients

FM-coupling

After substituting (15) into (25), the system for amplitude coefficients takes the following form:

$$1 + r_p = t_{p1} + t_{p2},$$
$$k_p^{(f)}(1 - r_p) = k_{p1}^{(l)} t_{p1} + k_{p2}^{(l)} t_{p2}, \qquad (27)$$
$$k_{p1}^{(l)} t_{p1} - k_{p2}^{(l)} t_{p2} = 0.$$

It follows herefrom that scattering coefficients for the reflected wave and two traveling waves are

$$r_p = \frac{\lambda_{p1}^{(l)} + \lambda_{p2}^{(l)} - 2\lambda_p^{(f)}}{\lambda_{p1}^{(l)} + \lambda_{p2}^{(l)} + 2\lambda_p^{(f)}}, \quad t_{p1} = \frac{2\lambda_{p1}^{(l)}}{\lambda_{p1}^{(l)} + \lambda_{p2}^{(l)} + 2\lambda_p^{(f)}},$$

$$t_{p2} = \frac{2\lambda_{p2}^{(l)}}{\lambda_{p1}^{(l)} + \lambda_{p2}^{(l)} + 2\lambda_p^{(f)}} \qquad (28)$$

where instead of wave numbers, the corresponding wavelengths $\lambda = 2\pi/k$ are introduced.

AFM-coupling

Unlike the case of FM interlayer coupling, the system for amplitudes includes both traveling waves and evanescent waves

$$1 + r_p = t_e + t_p,$$
$$k_p^{(f)}(1 - r_p) = k_e^{(l)} t_e + k_p^{(l)} t_p,$$
$$\frac{k_e^{(l)}}{\eta} t_e - \eta k_p^{(l)} t_p = 0. \qquad (29)$$

The scattering coefficients following from (29) are:

$$r_p = \frac{\eta^2\lambda_e^{(l)} + \lambda_p^{(l)} - (\eta^2+1)\lambda_p^{(f)}}{\eta^2\lambda_e^{(l)} + \lambda_p^{(l)} + (\eta^2+1)\lambda_p^{(f)}},$$
$$t_e = \frac{2\eta^2\lambda_e^{(l)}}{\eta^2\lambda_e^{(l)} + \lambda_p^{(l)} + (\eta^2+1)\lambda_p^{(f)}}, \qquad (30)$$
$$t_p = \frac{2\lambda_p^{(l)}}{\eta^2\lambda_e^{(l)} + \lambda_p^{(l)} + (\eta^2+1)\lambda_p^{(f)}}.$$

Graphs of frequency dependences in dimensionless variables $K = k\lambda_0$ and $\Omega = \frac{\omega}{\omega_0}$ for $\frac{\omega_g}{\omega_0} = 1$ in the cases of FM and AFM coupling are shown in Figure 2. With increasing frequency, the amplitudes of traveling waves in the FM layer corresponding to different wave numbers are of the same order, and the reflectance decreases monotonically. In the case of AFM coupling, the transmission coefficient of the traveling wave also decreases, which can be interpreted as suppression of the amplitude for the AFM-orientation of spins of the film and strip. As follows from expressions (14), the activation frequency for FM coupling is equal to $\omega_0 + 2\omega_g$, and for AFM coupling (18) it is equal to $\sqrt{\omega_0(\omega_0 + 2\omega_g)}$.

Figure 3 shows the distribution of the spin wave field in the vicinity of the interface between the free film and the bilayer film — strip structure for the model value of $\Omega = 4$. In the FM case, the wave is a superposition of traveling waves, and in the AFM case it is a wave with a reduced amplitude due to the decrease in transmission coefficients with increasing frequency. However, due to the fact that $t_p \neq 0$, the field away from the boundary is not zero.

8. Strip of finite width

Similar results can be obtained when the magnetic coating of the film is limited in width. In this case, the number of waves and boundary conditions will be twice as large as in the case of a semi-infinite strip. The corresponding amplitudes are marked with an additional arrow at the top, indicating the direction of propagation of the forward and backward waves.

FM-coupling

$$1 + r_p = t_{p1}^{\rightarrow} + t_{p1}^{\leftarrow} + t_{p2}^{\rightarrow} + t_{p2}^{\leftarrow},$$
$$k_p^{(f)}(1 - r_p) = k_{p1}^{(l)}(t_{p1}^{\rightarrow} - t_{p1}^{\leftarrow}) + k_{p2}^{(l)}(t_{p2}^{\rightarrow} - t_{p2}^{\leftarrow}),$$
$$k_{p1}^{(l)}(t_{p1}^{\rightarrow} - t_{p1}^{\leftarrow}) - k_{p2}^{(l)}(t_{p2}^{\rightarrow} - t_{p2}^{\leftarrow}) = 0,$$
$$t_{p1}^{\rightarrow} e^{ik_{p1}^{(l)}h} + t_{p1}^{\leftarrow} e^{-ik_{p1}^{(l)}h} + t_{p2}^{\rightarrow} e^{ik_{p2}^{(l)}h} + t_{p2}^{\leftarrow} e^{-ik_{p2}^{(l)}h} = t_p,$$
$$k_{p1}^{(l)}\left(t_{p1}^{\rightarrow} e^{ik_{p1}^{(l)}h} - t_{p1}^{\leftarrow} e^{-ik_{p1}^{(l)}h}\right) + k_{p2}^{(l)}\left(t_{p2}^{\rightarrow} e^{ik_{p2}^{(l)}h} - t_{p2}^{\leftarrow} e^{-ik_{p2}^{(l)}h}\right) = k_p^{(f)} t_p,$$

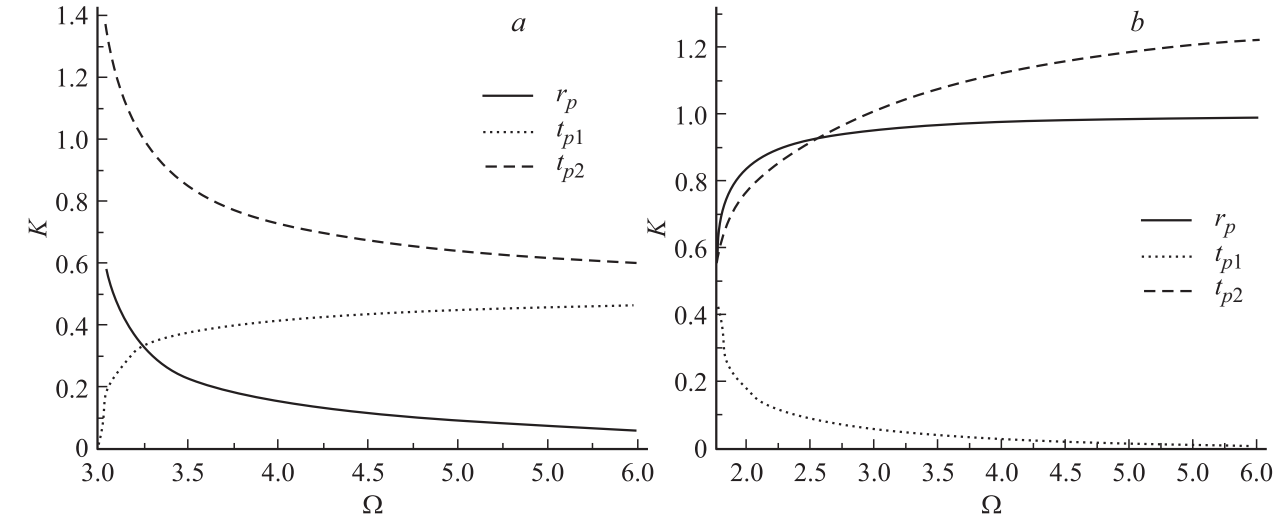

Figure 2. Frequency dependences of scattering coefficients for the cases of FM (*a*) and AFM (*b*) interlayer coupling. The lower frequency corresponds to the activation frequency in each case.

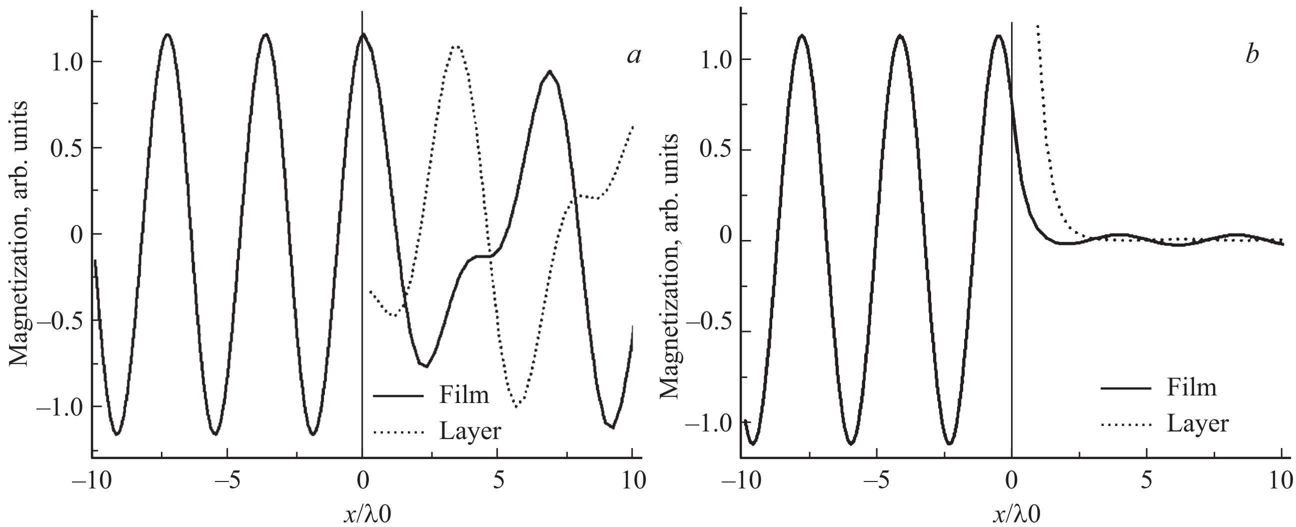

Figure 3. SW field distribution near the boundary $x = 0$ for the cases of FM (*a*) and AFM (*b*) coupling at $\Omega = 4$.

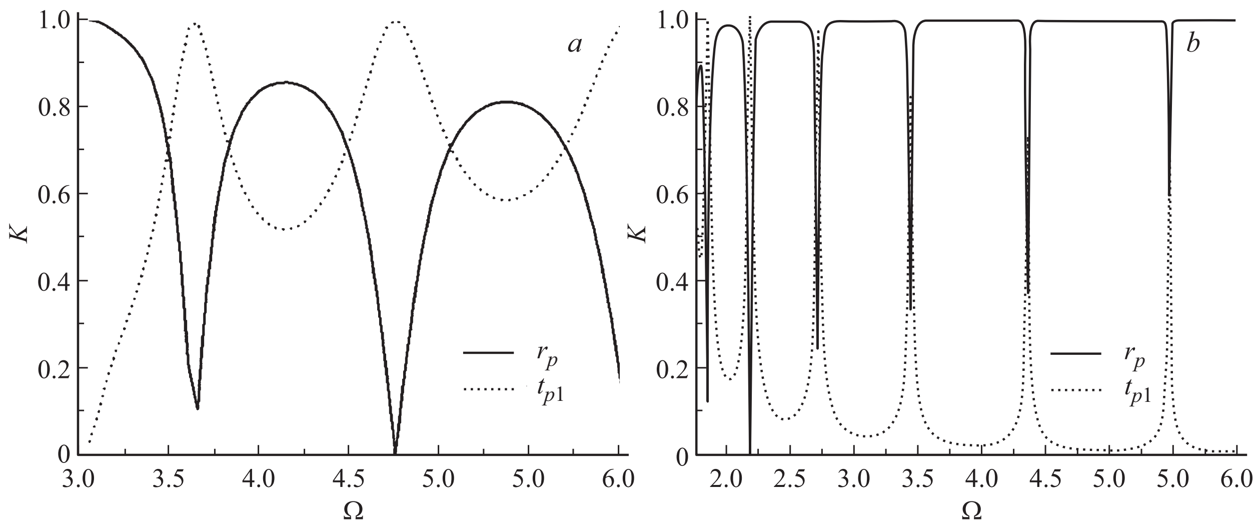

Figure 4. Frequency dependences of scattering coefficients for the cases of FM (*a*) and AFM (*b*) interlayer coupling. The lower frequency corresponds to the activation frequency in each case.

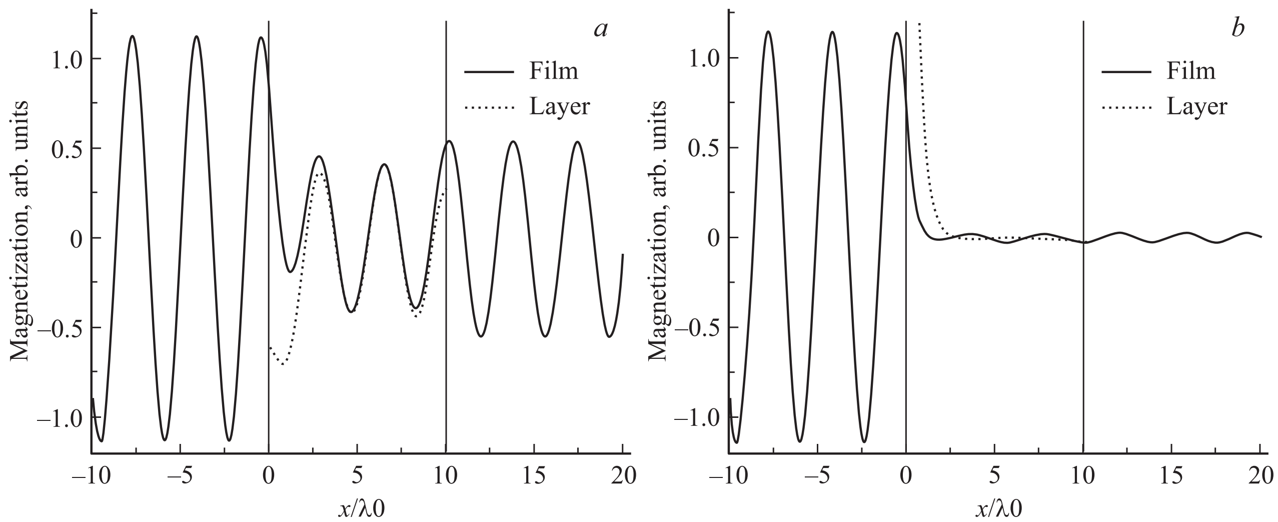

Figure 5. SW field distribution near the boundary $x = 0$ for the cases of FM (a) and AFM (b) coupling at $\Omega = 4$.

$$k_{p1}^{(l)}\left(t_{p1}^{\rightarrow}e^{ik_{p1}^{(l)}h}-t_{p1}^{\leftarrow}e^{-ik_{p1}^{(l)}h}\right)-k_{p2}^{(l)}\left(t_{p2}^{\rightarrow}e^{ik_{p2}^{(l)}h}-t_{p2}^{\leftarrow}e^{-ik_{p2}^{(l)}h}\right)=0.$$

AFM-coupling

$$1+r_p=t_p^{\rightarrow}+t_p^{\leftarrow}+t_e^{\rightarrow}+t_e^{\leftarrow},$$

$$k_p^{(f)}(1-r_p)=k_p^{(l)}(t_p^{\rightarrow}-t_p^{\leftarrow})+k_e^{(l)}(t_e^{\rightarrow}-t_e^{\leftarrow}),$$

$$k_p^{(l)}(t_p^{\rightarrow}-t_p^{\leftarrow})-k_e^{(l)}(t_e^{\rightarrow}-t_e^{\leftarrow})=0,$$

$$t_p^{\rightarrow}e^{ik_p^{(l)}h}+t_p^{\leftarrow}e^{-ik_p^{(l)}h}+t_e^{\rightarrow}e^{ik_e^{(l)}h}+t_e^{\leftarrow}e^{-ik_e^{(l)}h}=t_p,$$

$$k_p^{(l)}\left(t_p^{\rightarrow}e^{ik_p^{(l)}h}-t_p^{\leftarrow}e^{-ik_p^{(l)}h}\right) + k_e^{(l)}\left(t_e^{\rightarrow}e^{ik_e^{(l)}h}-t_e^{\leftarrow}e^{-ik_e^{(l)}h}\right)=k_p^{(f)}t_p,$$

$$k_p^{(l)}\left(t_p^{\rightarrow}e^{ik_p^{(l)}h}-t_p^{\leftarrow}e^{-ik_p^{(l)}h}\right)-k_e^{(l)}\left(t_e^{\rightarrow}e^{ik_e^{(l)}h}-t_e^{\leftarrow}e^{-ik_e^{(l)}h}\right)=0$$

Omitting final cumbersome expressions for the amplitude coefficients, the corresponding frequency dependences (Figure 4) and field distributions (Figure 5) are plotted. As in the case of semi-infinite strip, the AFM interaction results in suppression of the spin wave as it passes through the bilayer structure. In addition, as can be shown, the effect of an external field applied along the easy axis results in an increase in the length of the traveling wave and, on the contrary, localization of the field for the evanescent wave.

9. SW interference in magnon logic modules

Exchange interaction with the spins of the strip results in the possibility of using the interference effects described in [1,2]. In particular, in the XNOR logic module, the magnitude of the phase shift is determined by both the SW frequency and the layer thickness. As a result of adding

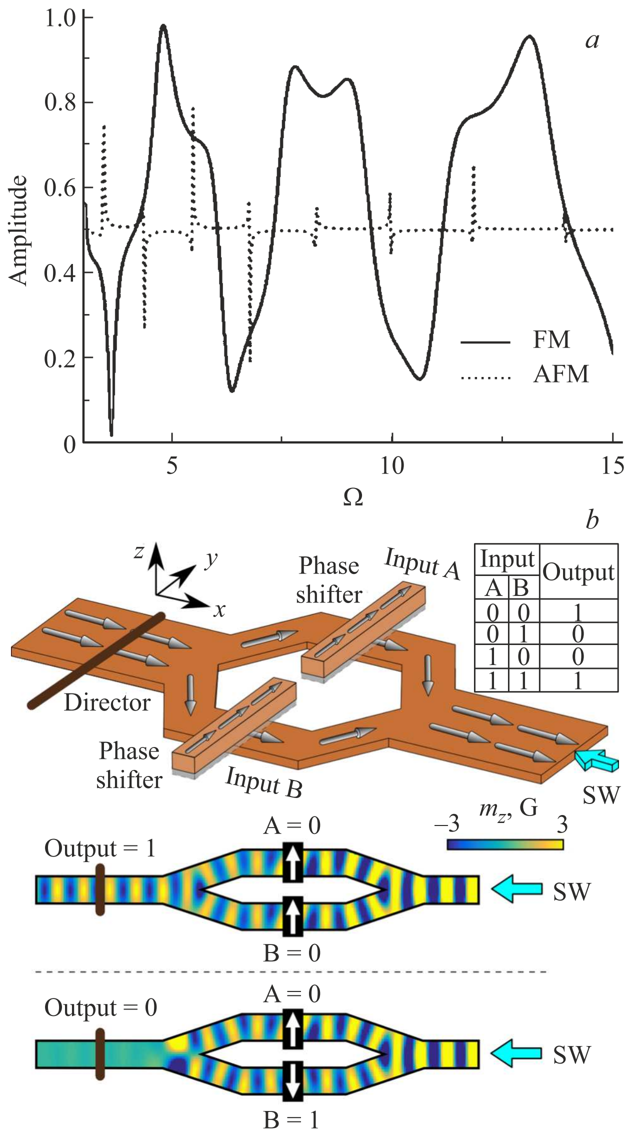

Figure 6. Amplitude of interfering waves in the XNOR logic module (a). Figure (b) is taken from [1].

a reference wave of unit amplitude to that passed through a bilayer system, the output signal has the following form: $\frac{t_p+e^{ik_f h}}{2}$. For the model values of the parameters used in this study, this dependence in the cases of FM and AFM interlayer coupling is shown in Figure 6.

In the case of AFM coupling, the amplitude of the output signal decreases by half over almost the entire frequency range, with the exception of small vicinities of points near the resonant passage through the region with the strip. In the case of FM coupling, the interference turns out to be more effective: amplitudes take on values in almost the entire range of [0,1].

10. Conclusion

In the case of AFM interaction, the chiral coupling of magnetizations of the film and strip requires their precession in the same direction. However, if for the traveling wave in the film such a precession is Larmor precession, then for the strip it is an anti-Larmor precession, and the corresponding wave is evanescent [5]. This will lead to attenuation of the SW amplitude in the film. At relatively high frequencies, the main contribution to the SW field in the region with the strip comes from the evanescent wave, the field of which is localized near the boundary.

The transmission coefficient for AFM-interacting films and strips decreases rapidly with increasing frequency. However, for a layer of finite width near the length of the traveling SW equal to a multiple of twice the width of the layer, a brightening effect is observed in a relatively narrow frequency region, which can be seen from a comparison between Figure 2, *b* and Figure 4, *b*.

Thus, the main mechanism of suppression of SWs is the emergence of evanescent SWs in addition to the traveling ones due to the anti-Larmor precession, which leads to a decrease in the amplitude of the travelling waves. It should be noted that this mechanism is applicable to explain not only the behavior of exchange SWs but also the behavior of dipole-interacting films and strips experimentally studied in [12]. Such a problem was considered in [11] for a bilayer structure.

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Conflict of interest

The author declares that he has no conflict of interest.

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