12

# Frequency dependence measuring of skin-effect on metal wires with circular cross-section

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The results of measurements of the frequency dependence of the active resistance of a circular cross-section of aluminum and copper conductors with a various diameter in a wide frequency range from 20 Hz to 2 MHz are presented. Using the skin effect simulation we show that for all types of wires an increased active resistance observed, compared to the theoretical values in the frequency range above 200 kHz, where the skin layer thickness becomes less than 200 $\mu$m. This phenomenon may be associated with the manufacturing process of a metal wire by drawing through a die, when defects are formed in the near-surface layer, leading to its increased resistivity.

**Keywords:** electrical resistance, aluminum wires, copper wires.

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## Introduction

It is known that with increase in frequency, electric alternating current in a solid metal conductor is distributed unevenly over the cross-section, and flows mainly in the near-surface layer [1]. Since the cross-section area where current flows decreases, the active resistance of the conductor increases with frequency. This phenomenon is called the surface effect or skin effect [2]. We have previously shown that this effect can be used for diagnostic purposes to determine the wear of high-voltage power lines, when the near-surface layer of the wire is modified over time during operation, which has a significant impact on its resistivity [3,4]. This paper describes a technique for measuring and analyzing the frequency dependence of the active resistance of aluminum and copper conductors of various diameters in a wide frequency range from 20 Hz to 2 MHz.

## 1. Methods and equipment

According to the skin effect model [5,6], when electric alternating current flows in an ideal cylindrical conductor, its density decreases from the surface to its center, and at a depth $\delta_s$ from the conductor surface the current density is 0.368 of the current density on the surface. Therefore, we can assume that the current predominantly flows in skin layer of thickness $\delta_s$:

$$\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}}, \tag{1}$$

where $\omega$ — angular frequency of alternating current, $\mu$ — magnetic permeability of the conductor material, $\sigma$ — specific conductivity of the conductor material (in $\Omega^{-1}\cdot\mathrm{m}^{-1}$).

For practical use it is most convenient to use the following expression:

$$\delta_s\,[\mathrm{mm}] = 503\cdot 10^3\sqrt{\frac{\rho}{f\mu_r}}, \tag{2}$$

where $\rho$ — electrical resistivity of the conductor, [$\Omega$·m]; $f = \frac{\omega}{2\pi}$ — alternating current frequency, [Hz]; $\mu_r = \frac{\mu}{\mu_0}$ — relative magnetic permeability of the conductor material, $\mu_0 = 4\pi\cdot 10^{-7}$ — magnetic permeability of vacuum.

For a cylindrical wire with a length $l$, which is significantly greater than its radius $r$, the dependence of the active resistance $Rs$ on the frequency $f$ is divided into two ranges and is described by the following expressions depending on the ratio of the radius of the conductor $r$ and the thickness of the skin layer $\delta_s$ [5,6]:

$$Rs(f) = \frac{\rho l}{\pi r^2} \text{ for } \delta_s > r, \tag{3}$$

$$Rs(f) = \frac{\rho l}{\pi r^2 - \pi(r-\delta_s)^2} \text{ for } \delta_s \le r. \tag{4}$$

Samples of aluminum and copper wires of round cross-section with different diameters were used for studies (Table 1).

Measurements of active resistance $Rs$ of the studied straight pieces of metal wires were carried out using a four-contact circuit (in Kelvin bridge mode [8]) using LCR E4980A full conductivity meter (Agilent Technologies Inc., USA) in frequency range of 20 Hz to 2 MHz. Pairs of contacts $L_{\mathrm{curr}}$, $L_{\mathrm{pot}}$ and $H_{\mathrm{curr}}$, $H_{\mathrm{pot}}$ are connected to the opposite ends of the wire under test using Kelvin clamps [8]

**Table 1.** Parameters of studied aluminum and copper wires of 62 cm long

| Name | Material | Diameter, mm | Resistivity $\rho$, $\Omega{\cdot}$m [7] |
|---|---|---|---|
| $Al_1$ | Aluminium | 1.77 | $2.650 \cdot 10^{-8}$ |
| $Al_2$ | | 2.22 | |
| $Al_3$ | | 2.67 | |
| $Cu_1$ | Copper | 0.77 | $1.724 \cdot 10^{-8}$ |
| $Cu_2$ | | 1.34 | |
| $Cu_3$ | | 1.75 | |

(Fig. 1, *a*). Since the metal wire samples under study had low impedance ($< 100\,\Omega$), measurements of the frequency dependence of the total electrical resistance were carried out in the mode of a series equivalent electrical circuit $Rs-Ls$ (active resistance —inductance). In this mode, alternating current is supplied to the sample through a pair of connections $L_{curr}$ and $H_{curr}$ (Fig. 1, *a*). To measure the AC voltage drop $V$ on the wire sample under measurements a pair of measuring connections $L_{pot}$ and $H_{pot}$ is used (Fig. 1, *a*). It was experimentally established that the best signal-to-noise levels were achieved at alternating current amplitude equal to 30ṁA.

## 2. Results

Fig. 1, *b* shows the measurements of the frequency dependence of the inductance $Ls$ and active resistance $Rs$ of the sample $Cu_1$ (Table 1) in the form of a straight piece of copper wire with a diameter of 0.77 mm and 62 cm long. In the low-frequency region (below 20 kHz), in accordance with expression (2), the thickness of the skin layer $\delta_s$ is significantly greater than the radius of the conductor $r$, therefore active resistance $Rs$ is practically independent of the frequency of the measuring signal and is determined by expression (3). When the frequency of the measuring signal increases above 20 kHz, the thickness of the skin layer $\delta_s$ becomes smaller than the radius of the conductor $r$, which leads to a sharp increase in active resistance $Rs$ (Fig. 1, *b*). Inductance $Ls$ of the sample $Cu_1$ weakly depends on the frequency of the measuring signal in the frequency range under study from 20 Hz to 2 MHz and is about $1\,\mu$H (Fig. 1, *b*).

Fig. 2 shows the results of measurements of active resistance $Rs$ of aluminum ($Al_1$, $Al_2$, $Al_3$) and copper wires ($Cu_1$, $Cu_2$, $Cu_3$) with different diameters, but the same length 62 cm (Table. 1). Comparison of experimental data with theoretical calculations of the frequency dependence of the active resistance of ideal conductors of the same length and diameter using expressions (3) and (4) shows very good agreement in the frequency range below 200 kHz. It was assumed that the electrical resistivity $\rho$ of copper and aluminum samples is uniform over the cross-section of the

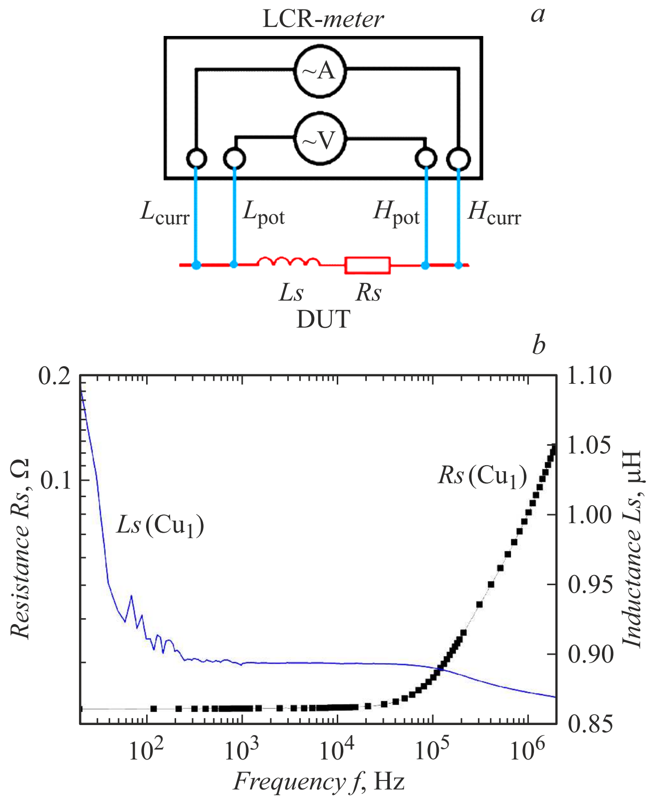


**Figure 1.** *a* — circuit for measuring active resistance using the four-contact method in Kelvin bridge mode using full conductivity meter LCR, where $L_{curr}$, $L_{pot}$ and $H_{curr}$, $H_{pot}$ — points of measurement of electric current and voltage, DUT — measured sample; *b* — graph of the experimental dependence of active resistance $Rs$ and inductance $Ls$ of copper wire on current frequency.

wires and coincides with the values known from the literature (Table 1) [7]. For both types of wire materials, model calculations of active resistance $Rs$ quite well describe the shift to the high frequency region taking into account the contribution of the skin effect with a decrease in the diameter of the wires under study (Fig. 2). Note that in the frequency range above 200 kHz for all wire samples under study, a stronger increase in the experimentally measured active resistance $Rs$ is observed in comparison with what is obtained from the calculations. This behavior may be due to the contribution of the impedance of the measuring leads $L_{curr}$, $L_{pot}$ and $H_{curr}$, $H_{pot}$, used for implementation four-contact method in Kelvin bridge mode (Fig. 1, *a*). To evaluate their influence, measurements were made of the frequency dependence of the active resistance $Rs$ for copper wires of type $Cu_1$ (Table 1) of different lengths from 62 to 8 cm (Fig. 3, *a*). Modeling the frequency dependence of the active resistance $Rs$ using expressions (3) and (4) for all samples showed fairly good agreement in the range from 20 Hz to 200 kHz. However, in the range above 200 kHz the experimental values of the active resistance $Rs$ are higher than the calculated ones (Fig. 3, *a*), while the magnitude

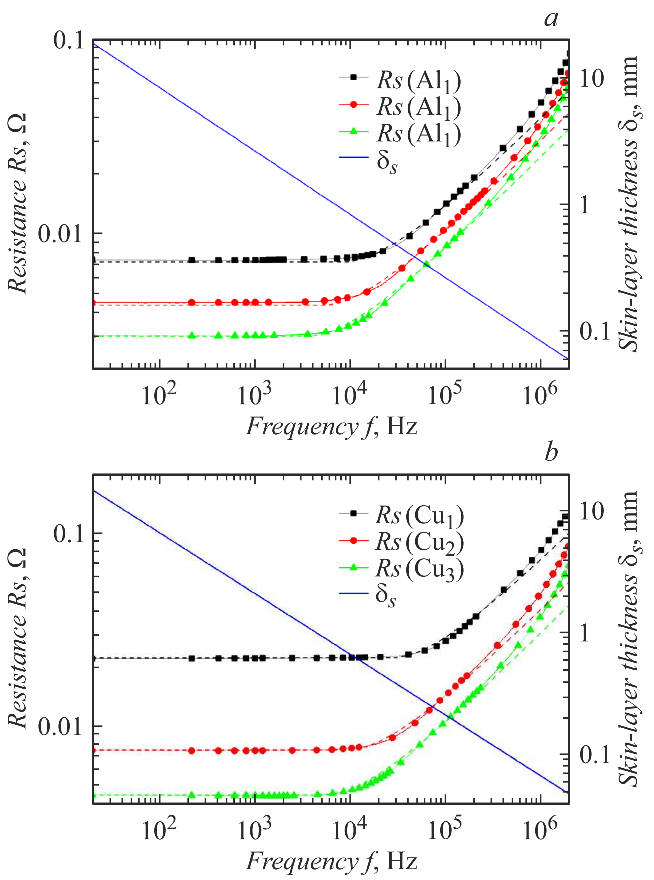


**Figure 2.** Frequency dependence of active resistance $Rs$, aluminum ($a$) and copper ($b$) wires of various diameters (calculation and measurements). The skin layer thickness $\delta_s$ is calculated using expression (2). The dashed line shows the simulated frequency dependences of the electrical resistance of ideal conductors of the same length and diameter using the parameters from Table 1.

of the deviation is scaled proportionally to the length of the wire. This indicates that there is no contribution to the measured active resistance $Rs$ from $L_{\text{curr}}$, $L_{\text{pot}}$ and $H_{\text{curr}}$, $H_{\text{pot}}$ leads, used to implement the Kelvin bridge. At frequencies above 200 kHz the thickness of the skin layer $\delta_s$, calculated using expression (2), for copper wire is less than 200 $\mu$m (Fig. 2, $b$), therefore, increased values of active resistance $Rs$ may be due to the fact that the resistivity of the surface layers is higher than in the bulk of the conductor.

Metal wire is produced by drawing [9]. The operation consists of pulling a metal workpiece through a die, which has a hole with a diameter smaller than that of the original wire material. As a result of passing through the die, the initial material is compressed, and the wire becomes thinner and longer. To obtain wire of the required diameter, it is possible to use several dies with different successively decreasing diameters. Wire production is completed by annealing. Compression of the wire and rubbing it against the surface of the die can lead to the formation of defects in its near-surface layer, which, in turn, causes increase in resistivity compared to the bulk value characteristic for defect-free material.

To simulate the active resistance of a wire with a surface defective layer of thickness $T_d$, a parallel connection of an ideal conductor in the form of a solid cylinder with resistivity $\rho$ and radius $(r-T_d)$ and a conductor in the form of a hollow cylinder with an increased resistivity of the near-surface defective layer $\rho_d$ and wall thickness $T_d$ was considered. Thus, the total value of the wire radius $r$ remained unchanged.

The active resistance of the ideal conductor $R_i$ in the form of solid cylinder of length $l$ is calculated using the following expressions:

$$R_i(f) = \frac{\rho l}{\pi(r - T_d)^2} \text{ for } \delta_s > r, \tag{5}$$

$$R_i(f) = \frac{\rho l}{\pi((r - T_d)^2 - (r - \delta_s)^2)} \text{ for } \delta_s \leq r. \tag{6}$$

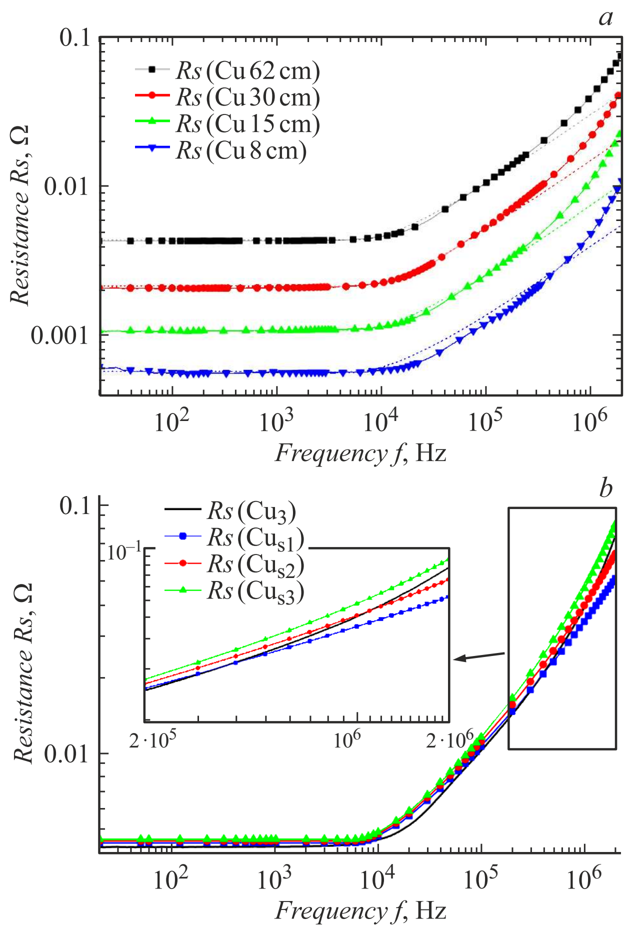


**Figure 3.** $a$ — frequency dependence of active resistance $Rs$ copper wires with diameter of 1.75 mm of various lengths; $b$ — simulated frequency dependences of active resistance $R$ of copper wire in the presence of a near-surface defective layer with an increased resistivity value using the parameters from Table 2. The insert shows an enlarged region of active resistance $Rs$ in the frequency range from 200 kHz to 2 MHz.

**Table 2.** Parameters of simulated copper wires with resistivity $\rho = 1.724 \cdot 10^{-8}\,\Omega\cdot$m and near-surface defective layer of thickness $Td = 200\,\mu$m

| Name | Modification coefficient, $Cf$ | Defective layer resistivity $\rho_d$, $\Omega\cdot$m |
|---|---|---|
| $Cu_{S1}$ | 1.05 | $1.810 \cdot 10^{-8}$ |
| $Cu_{S2}$ | 1.10 | $1.896 \cdot 10^{-8}$ |
| $Cu_{S3}$ | 1.15 | $1.983 \cdot 10^{-8}$ |

The active resistance of the surface defective layer $R_d$ in the form of hollow cylinder with wall thickness $T_d$ is calculated as

$$R_d(f) = \frac{\rho_d l}{\pi(r^2 - (r - T_d)^2)} \text{ for } \delta_d > T_d, \tag{7}$$

$$R_d(f) = \frac{\rho_d l}{\pi(r^2 - (r - \delta_d)^2)} \text{ for } \delta_d \le T_d, \tag{8}$$

where $\delta_d$ — thickness of the skin layer in the near-surface defective layer.

The final active resistance of the wire $R$ for a parallel connection of ideal conductor $R_i$ and near-surface defective layer $R_d$ is calculated as follows:

$$R(f) = \frac{R_i(f)R_d(f)}{R_i(f) + R_d(f)}. \tag{9}$$

The frequency dependences of the active resistance $R$ of copper wire with diameter $r$ equal to 1.75 mm, simulated using expression (9) taking into account the parameters of the near-surface defective layer given in Table 2, are represented in Fig. 3, *b*. It can be seen that the best agreement between calculations and experimental data is observed for the thickness of the defective layer $T_d$ equal to $200\,\mu$m, when the resistivity of the near-surface defective layer $\rho_d$ increases by 10% to $1.896 \cdot 10^{-8}\,\Omega\cdot$m ($\rho_d = Cf\,\rho$, modification factor $Cf = 1.10$).

The model proposed above assumes a sharp boundary between the defect-free central part of the wire and the defective near-surface layer. The most realistic assumption is that the defective area is not separated by a clear boundary from the main wire, but rather has an increasing resistivity. Thus, in the paper [3] it was shown that the density of defects in the near-surface layers of the wire changes smoothly with depth. Therefore, it is necessary to develop a model that takes into account the non-uniform distribution of resistivity across the thickness of the defective layer.

## Conclusions

So, this paper presents the frequency dependence of the active resistance of aluminum and copper conductors with round cross-section of various diameters in wide frequency range from 20 Hz to 2 MHz. Using skin effect modeling, it is shown that for all types of wires there is a significant increase in active resistance compared to the calculated one at frequencies above 200 kHz, where the thickness of the skin layer becomes less $200\,\mu$m. To describe this phenomenon, the model of the frequency dependence of the active resistance of cylindrical metal wire with near-surface defective layer characterized by increased resistivity was proposed. This may be due to the fact that during the metal wire manufacture by drawing, defects are formed in the near-surface layer, leading to increase in its resistivity.

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### Conflict of interest

The authors declare that they have no conflict of interest.

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