

# An experiment to improve the accuracy of a quantum level using hydrogen quantum clock with GLONASS/GPS phase measurements

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The article investigates the accuracy of measuring the difference of gravitational potentials and orthometric heights with a quantum level based on a hydrogen quantum clock. To improve the accuracy of time scale comparison, the authors propose to use a high-precision method of absolute navigation based on global navigation satellite systems phase measurements with integer ambiguity resolution. With a measurement accumulation interval of about 5 days and the use of mobile quantum clocks with a daily relative instability of the order of  $1 \cdot 10^{-15}$ , the proposed method allows to measure the difference in orthometric heights and gravitational potentials with an error of 7.7 m and  $75.3 \text{ m}^2/\text{s}^2$ , respectively. At the same time, it is still possible to reduce the error when using quantum clocks with higher stability and increasing the measurement accumulation interval.

**Keywords:** orthometric height difference, gravitational potential difference, phase ambiguity integer resolution.

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## Introduction

To date, in the Russian Federation, based on the gravitational effect of Einstein time dilation [1], several experiments were carried out to measure the difference in gravitational potentials and the corresponding difference in orthometric heights using stationary and mobile highly stable hydrogen quantum clocks (QCs) [2]. In this case, to compare the time scales of these clocks at the measuring points, either the relativistic synchronization method [3], or a fiber-optic communication line, or signals from global navigation satellite systems (GNSS) [4] were used. In a foreign ground-based gravity experiment, a duplex communication system via communication satellites was used for this purpose [5]. One of the proposals for increasing the accuracy of measurements in experiments of this kind is based on the use of quantum optical systems, or satellite laser rangefinders [6].

At the same time, the method using GNSS signals provides the simplest implementation of a quantum level, since, in addition to quantum clocks it requires the installation at measurement points of small-sized satellite navigation system signal receivers only. In well-known experiments using GNSS navigation receivers, code GNSS measurements were used to measure the accumulated effect of gravitational time dilation during the effect accumulation interval. This ensured an error of measurement of time scales differences of a pair of spatially separated quantum clocks equal to 0.3 ns and more, which did not allow us to obtain an acceptable accuracy in measuring the difference in gravitational potentials. To improve the accuracy of gravitational potential difference measurements, this paper

proposes the use of GNSS phase measurements and phase integer ambiguity resolution methods.

## 1. Experiment idea, basic relationships

The experiment is based on the use of stationary hydrogen quantum clocks located at point „Mendeleevo“, Moscow region, and transportable hydrogen quantum clocks, which, after mutual synchronization with the stationary clock, move to the point „Evpatoriya“.

The purpose of the experiment: to measure the difference in gravitational potentials and orthometric heights between the points „Mendeleevo“ and „Evpatoriya“ based on the use of continuous phase measurements of GNSS signals.

As stationary quantum clock KCh-0 with a basic intrinsic time scale  $\tau_0$  we used the primary standard of units of time and frequency of the national time scale RF GET 1-2022 [7] with relative instability  $5 \cdot 10^{-16}$ , located at the point „Mendeleevo“. As relocated clocks KCh-M we used transportable hydrogen quantum clocks (PKChV-N) produced by JSC „Vremya-Ch“ (Russia) with its own on-board scale of its own (measured) time  $\tau_m$ , as well as relative instability no worse than  $(\sigma f/f_0) = 1 \cdot 10^{-15}$  for 3600 s. To measure the difference between the time scales of quantum clocks located at the ends of the route, a consumer navigation equipment (CNA) GNSS Javad Sigma G3T with a data reading frequency of 1 Hz was used. The current temperature of KCh-M during the experiment was monitored using on-board weather station Combi-Sensor DTF 1MV with sensitivity  $0.01^\circ\text{C}$  and measurement error  $0.3^\circ\text{C}$ .

The frequency difference between the relocated quantum clock KCh-M, arrived to the point „Evpatoriya“ and installed there without movement, as well as the stationary clock KCh-0, located at the point „Mendeleevo“, is determined by the known relation [8]:

$$f_E - f_0 = \Delta f_{GR} + \Delta f_{\Omega} + \Delta f_K + \Delta f_T + \Delta f_R, \quad (1)$$

where the indices 0, E refer to the points „Mendeleevo“ and „Evpatoriya“ respectively;  $f_{GR}/f_0 = \frac{\varphi_0 - \varphi_E}{c^2}$  — gravitational displacement in the Earth’s field;  $\varphi_0, \varphi_E$  — gravitational potentials, respectively, at the points „Mendeleevo“ and „Evpatoriya“;

$$\Delta f_{\Omega}/f_0 = \frac{\Omega^2}{2c^2} [(x_0^2 + y_0^2) - (x_E^2 + y_E^2)]$$

— frequency difference due to the centrifugal potentials of the Earth;  $\Omega$  — angular speed of the Earth rotation;  $(x_0, y_0), (x_E, y_E)$  — geocentric coordinates of the points „Mendeleevo“ and „Evpatoriya“;  $\Delta f_K$  — initial calibration frequency difference of the relocated standard in relation to the reference frequency of the master oscillator KCh-0  $f_0$ ;  $\Delta f_T = K_T^f \Delta T$  — frequency deviation due to temperature KCh-M, which is calculated through the temperature coefficient of the frequency of its master oscillator  $K_T^f$  and temperature deviation  $\Delta T$  of clock KCh-M;  $\Delta f_R$  — unpredictable random change in the frequency of clock KCh-M during time of movement from the point „Mendeleevo“ to the point „Evpatoriya“ due to the own instability of the standard. The Sagnac effect, accumulated during the time of movement of clock KCh-M and unchanged when the clock KCh-M is stationary at the point „E“, is of no interest for solving the problem of measuring the gravitational potentials difference, so here and further it is not taken into account.

In relation to this experiment, when the difference in measured orthometric heights does not exceed 250 m, we express the gravitational potentials  $\varphi_0, \varphi_E$  through the geoid potential  $\varphi_G$ , the same for all points on the Earth surface:

$$\begin{aligned} \varphi_0 &= \varphi_G + \int g_0(H) dH_{ort}^0 \approx \varphi_G - g_0 H_{ort}^0, \\ \varphi_E &= \varphi_G + \int g_E(H) dH_{ort}^E \approx \varphi_G - g_E H_{ort}^E, \end{aligned} \quad (2)$$

where  $H_{ort}^0, H_{ort}^E$  — orthometric heights of the quantum clock placement points „Mendeleevo“ and „Evpatoriya“ relative to the geoid surface, respectively;  $g_0, g_E$  — gravitational acceleration at these points, respectively.

As a result, neglecting the minor influences of the heterogeneity of the Earth’s gravitational field, and taking into account that for a small difference in orthometric heights (not exceeding one kilometer)  $g_0 = g_m = g$  [9], the desired gravitational displacement (1) is determined by the following approximate formula:

$$f_{GR} = f_0 \frac{\varphi_0 - \varphi_E}{c^2} = f_0 \frac{\Delta\varphi_{0E}}{c^2} \approx f_0 \frac{g \Delta H_{ort}}{c^2}, \quad (3)$$

where  $\Delta\varphi_{0E}; \Delta H_{ort} = H_{ort}^E - H_{ort}^0$  — the required difference in potentials and orthometric heights, respectively.

## 2. Determination of frequency difference of remote frequency standards using GNSS phase measurements

To compare the time scales and frequencies of KCh-M and KCh-0, this paper proposes the use of high-precision absolute positioning method with ambiguity integer resolution, known as Integer Precise Point Positioning (IPPP or PPP-AR) [10]. The input data for solution making using this method are code and phase measurements of the GNSS receiver in two frequency ranges. Information on high-precision orbits and corrections to the on-board time scales of navigation satellites is required also. High-precision corrections are calculated by independent GNSS measurement analysis centers, and are available a posteriori with a delay of 1 to 14 days. In this paper, high-precision products from the CODE analysis center (Bern, Switzerland) were used. For the highly accurate comparison of time scales it is necessary to accumulate code and phase GNSS measurements over several days with a period of 30 s maximum. An important condition is the continuity of phase measurements over the entire time interval of solving the problem.

At the first stage, independent processing of base and mobile station data was carried out using the IPPP method. Integer ambiguity resolution was carried out using a method developed at CNES, France [11]. As a result of this processing, the differences in time scales (TS) of KCh-0 and KCh-M were independently calculated relative to some reference highly stable time scale, which is used in the applied high-precision ephemerides. Since the desired value is the difference between TS of KCh-0 and KCh-M, the third scale is of no interest, since it is excluded when forming the difference. Thus, we obtain a highly accurate estimate of the difference between the time scales of the relocated and stationary quantum standards.

The advantage of IPPP technology is to obtain a single solution for the entire array of processed measurements, subject to continuity of phase measurements. This solution is not subjected to day-boundary discontinuities and other factors that impair accuracy. As a result, the time scale difference between any time points within the processing interval can be determined with very high accuracy. In this case, for each processing session there will be a constant error in estimating TS, which depends on the noise of code measurements and a number of other factors, however, to solve the problem of measuring the potential difference, the absolute value of the TS difference is not important, you only need to know the TS phase increment between the start and end points of the processing interval. Thus, the constant error for each of the independently processed stations does not affect the result, and the error in estimating the potential difference is determined only by the random component

of the phase measurements and the models used in the solution.

The random error in estimating TS using the IPPP method is evaluated as not exceeding 50 ps, which corresponds to the error in frequency comparison for about  $n \cdot 10^{-16}$  one day. Based on the results of the experiment conducted by VNIIFTRI specialists, an estimate of the Allan deviation was obtained equal to  $3 \cdot 10^{-16}$  on a daily averaging interval for short baselines. The systematic error in estimating the relative frequency difference using the IPPP method does not exceed  $\pm 1 \cdot 10^{-16}$ . This result was obtained at VNIIFTRI based on the comparison of the processing results with measurement data obtained using a phase comparator [12]. Foreign experts also conducted experiments during which they compared the estimates obtained by Integer PPP and via the optical frequency comparison channel. Experiments shown [13] that the random error of the Integer PPP method does not exceed  $1 \cdot 10^{-16}$  over an averaging interval of 3 days for baselines of the order of 1000 km. Comparisons on longer baselines are difficult due to the lack of an alternative comparison channel with the required accuracy to validate the method.

### 3. Experiment execution

Experiment was performed in stages.

#### 3.1. Stage I. Initial frequency calibration

At this stage, the frequency difference  $\Delta f_K = f_m - f_0$  of the base and mobile quantum clocks was determined with an error of maximum  $1 \cdot 10^{-16}$ . The measurements were carried out using VCH-314 frequency comparator when the clocks were placed in close proximity to each other in a temperature-stabilized room for three days. The relative initial calibration mismatch, taking into account the linear frequency drift of the master oscillator of KCh-M, was:  $\Delta f_{\text{init}}/f_0 = (66.32 - 1.99 \cdot T \pm 4.40) \cdot 10^{-16}$ , where  $T$  — observation time in days.

The temperature coefficient of frequency (TCF) was determined by comparing the differences in the frequencies of the master oscillators of clock KCh-M and KCh-0 at different temperatures of the internal volume of an automobile thermostabilized laboratory, which was located near the standard GET1-2022, which contains KCh-M. The temperature fluctuations in the mobile laboratory during calibration were monitored using the on-board thermohygrometer IVA-6A-KP-D with sensitivity  $0.1^\circ\text{C}$  and measurement error  $\pm 0.3^\circ\text{C}$ . With the laboratory temperature difference  $7.194^\circ\text{C}$  and the observation interval of 2.51 days, the TCF in relative terms was:  $K_T^f = 2.18 \cdot 10^{-16} \text{ }^\circ\text{C}^{-1}$ .

#### 3.2. Stage II. Moving the clock KCh-M to the point „Evpatoriya“ and measuring the difference in frequencies between the clocks KCh-M and KCh-0 at an interval of 5 days using CNA GNSS

Using the phase method, continuous measurements of the time scales difference of the base clock KCh-0, located in „Mendeleevo“, and KCh-M, located in „Evpatoriya“, are carried out on CNA GNSS:

$$\Delta\tau(\tau_0) = [\tau_E(\tau_0) - \tau_0] + \delta\tau_R + \delta\tau_S, \quad (4)$$

where  $\delta\tau_R$ ,  $\delta\tau_S$  — random and systematic error in comparison of time scales of KCh-M and KCh-0.

Then, based on the TS estimates the average difference in frequencies of two standards  $f_s - f_0$  is calculated over an interval of 5 days. To do this, using the least squares method  $\Delta\tau(\tau_0)$  is approximately represented by a first-order polynomial. The coefficient of the linear term of the polynomial represents the desired frequencies difference of the master oscillators of the compared quantum clocks. As noted above, the systematic error does not affect the estimate of this frequency difference; therefore, the estimate error depends only on the random error  $\delta\tau_R$ . With an increase in the measurement accumulation interval  $\tau_H$ , under the condition of a normal distribution law of the random variable  $\delta\tau_R$ , the error in estimating the frequency difference decreases linearly.

The result of measuring the frequency difference of KCh-M and KCh-0, according to formulas (1) and (3), is presented in the form

$$(f_E - f_0)^{\text{meas}} = f_0 \frac{g\Delta H_{\text{ort}}}{c^2} + \Delta f_K + \Delta f_\Omega + \Delta f_T + \Delta f_C + \delta f^{\text{meas}}, \quad (5)$$

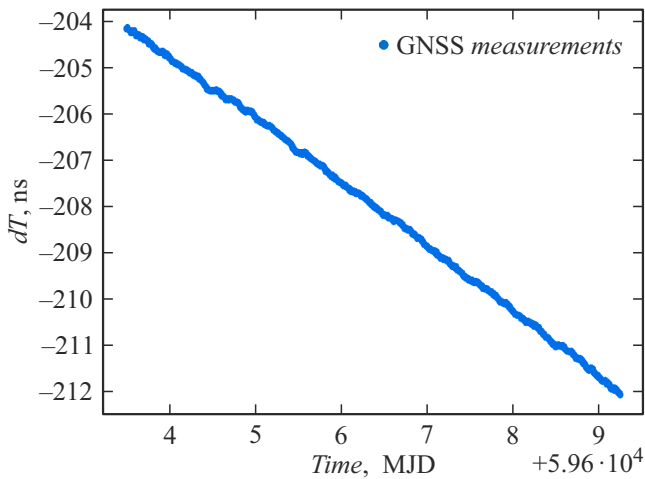
where  $\delta f^{\text{meas}}$  — error in measuring the frequency difference.

Measurements accumulation interval  $\tau_{\text{Acc}}$  in the experiment was 5 days. The resulting current difference of the time scales of the base clock KCh-0, located in the point „Mendeleevo“, and the mobile clock KCh-M, located in the point „Evpatoriya“, are presented in Fig. 1.

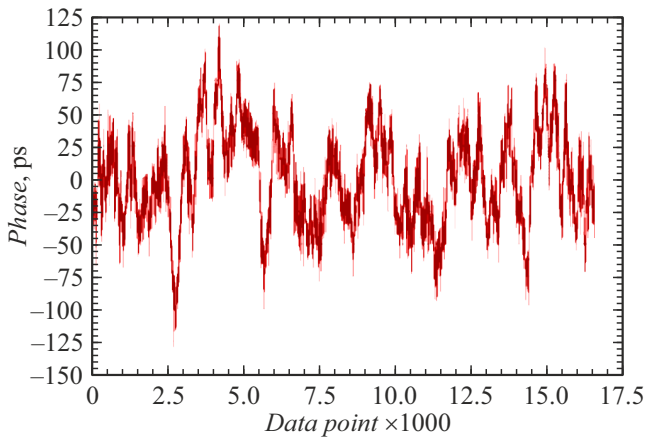
Measurement processing was carried out in a posteriori mode after the mobile clock was returned back to VNIIFTRI.

#### 3.3. Stage III. Moving KCh-M back to Mendeleevo

Upon returning to VNIIFTRI, measurements of the frequencies difference of the base and mobile quantum clocks were carried out using the VCH-314 phase comparator (similar to stage I). The return occurred approximately 13.7 days later. Substituting this value into the initial calibration mismatch equation, we obtain the predicted value of the relative frequency difference equal to  $\tilde{f}_{\text{final}}/f_0 = 39.06 \cdot 10^{-16}$  taking into account linear drift.



**Figure 1.** Current difference between the values of the time scales of KCh-0 and KCh-M during the stay in Evpatoriya.



**Figure 2.** Noise in estimating the TS difference after removing the linear trend.

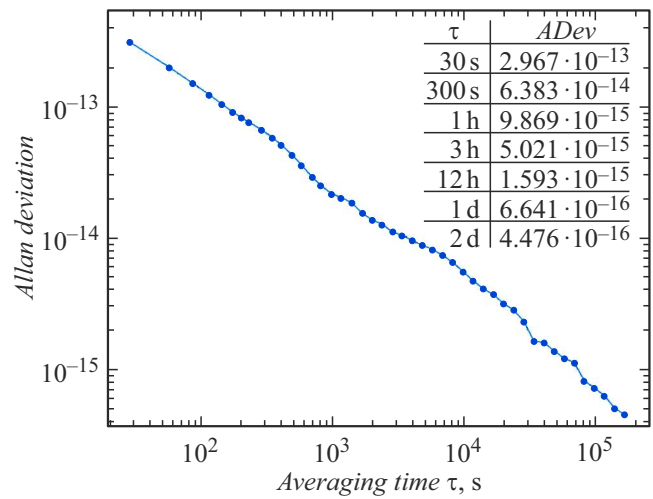
The measured actual value of the relative frequency difference was  $\Delta f_{\text{final}}/f_0 = 37.658 \cdot 10^{-16}$ , which coincides with the predicted value within the intrinsic instability of the clock.

The standard deviation of TS difference of KCh-0 and KCh-M over an observation interval of 5 days is 36 ps (after linear trend removal to estimate the frequency difference). Fig. 2 shows the phase noise of the measurements, and Fig. 3 shows the Allan deviation of the resulting TS difference.

#### 4. Discussion of results

The nature of the obtained Allan deviation allows us to conclude that the distribution of the random error of TS comparison is close to the normal law. The Allan deviation decreases linearly with increasing the averaging time.

Note that the presented Allan deviation, in addition to the actual error of the method, also contains components of the own instability of relocated and stationary quantum clocks.



**Figure 3.** Allan deviation of the time scale difference estimate.

Based on this, the relative error of frequency comparison using the proposed method is estimated by the authors as not exceeding  $5 \cdot 10^{-16}$  over the observation time interval of 5 days. Also, using the obtained estimate of the Allan deviation, it is possible to estimate the actual total instability of the relocated and stationary frequency standards during the experiment. It can be seen that the Allan deviation in a two-day averaging interval is about  $5 \cdot 10^{-16}$ . This value can be taken as a pessimistic estimate of the total instability of standards, since it is a component of the resulting Allan deviation, as noted above.

Next, from formula (5) taking into account formula (3), the required potential difference is determined:

$$\Delta\varphi_{0e} \approx g\Delta H_{ort} = \frac{c^2}{f_0} [(f_E - f_0)^{meas} - \Delta f^{calc} - \Delta f_R - \delta f^{meas}], \quad (6)$$

where  $(f_E - f_0)^{meas}$  — measured with error  $\delta f^{meas}$ , the frequency difference of the master oscillators of the quantum clocks in the accumulation interval of the time scale divergence  $\tau_{Acc}$ ;  $\Delta f^{calc} = (\Delta f_{\Omega} + \Delta f_K + \Delta f_T)^{calc}$  — the calculated value of the sum of the disturbing frequency differences presented in formula (1);  $\Delta f_R$  — the total value of unpredictable random frequency deviations of the master oscillators of both quantum clocks from their nominal values, caused by their own instability.

In this case, it is calculated using formula (1) based on the available input data:

$$\begin{aligned} \Omega &= 7.29 \cdot 10^{-5} \text{ 1/s}, & x_0 &= 2845476.75 \text{ m}, \\ y_0 &= 2160917.71 \text{ m}, & z_0 &= 5265974.39 \text{ m}, \\ x_E &= 3760896.45 \text{ m}, & y_E &= 2473953.78 \text{ m}, \\ & & z_E &= 4503304.79 \text{ m}. \end{aligned}$$

The height of the point „Mendeleev“ above the ellipsoid is 238.35 m, above the geoid — 222.25 m. The heights of the point „Evpatoriya“ are 45.95 and 20.88 m, respectively.

The orthometric heights used differ from the given normal heights by a few centimeters, which is negligible and does not change the results of the experiment.

The relative value of the initial frequency deviation of the quantum clock  $\Delta f_K/f_0 = \Delta f_{init}/f_0$  was determined by us at stage I during the initial frequency calibration, where the random component of the calibration is equal to  $\sigma_{\Delta f_K} \approx 4.4 \cdot 10^{-16}$ .

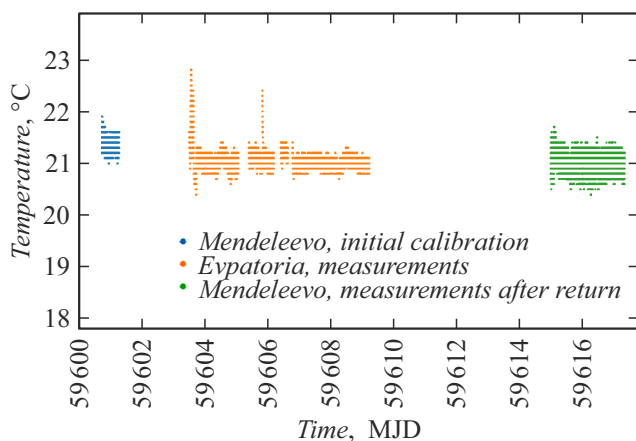
To calculate the temperature deviation of frequency  $\Delta f_T^{calc}$ , data on the change in average temperature at the initial calibration stage and at the measurement stage in Evpatoriya are used. Fig. 4 shows temperature measurements inside the mobile laboratory during the entire experiment, obtained with an on-board thermohygrometer. At the initial calibration stage, the average temperature value was  $+21.371 \pm 0.173^\circ\text{C}$ , at the measurement stage in Evpatoriya —  $+21.065 \pm 0.221^\circ\text{C}$ , when carrying out measurements on the original standard upon return of KCh-M, the average temperature was  $+21.010 \pm 0.238^\circ\text{C}$ . So, the known TCF allows calculation of the average frequency deviation due to temperature change  $\Delta f_T = K_T \Delta T = 2.18 \cdot 10^{-16}^\circ\text{C} \cdot 0.306^\circ\text{C} = 6.67 \cdot 10^{-17}$  in relative units of frequency. Since the standard deviation of the temperature deviation from its average value during the experiment did not exceed  $0.3^\circ\text{C}$ , the residual influence of these deviations on the standard frequency will also not exceed  $7 \cdot 10^{-17}$ , which is approximately by 15 times less, than the standard's own instability.

Next, moving in formula (6) to random errors in determining the components of this formula, we find the error in determining the potential difference and the difference in orthometric heights:

$$\sigma_{\Delta\varphi} = g \Delta H_{ort} = c^2 \sqrt{\sigma_{meas}^2 + \sigma_g^2 + \sigma_R^2 + \sigma_{calc}^2},$$

$$\sigma_{\Delta H} = \frac{\sigma_{\Delta\varphi}}{g}. \quad (7)$$

Here the following designations are used:



**Figure 4.** Graph of changes in on-board temperature of KCh-M during the experiment.

1)  $\sigma_{meas} \approx 5 \cdot 10^{-16}$  — error in determining the frequency difference between the master oscillators of the quantum clocks KCh-0 and KCh-M based on measuring the divergence of their time scales  $\Delta\tau(\tau_0)$  (4) at accumulation interval of 5 days;

2)  $\sigma_R$  — total relative standard error of frequency deviation from nominal values due to instability of KCh-0 frequency and KCh-M frequency ( $\sigma_m$ ). Since the movement of the mobile standard took about two days, it is proposed to take the previously obtained Allan deviation at this interval as a measure of error. Thus  $\sigma_R \approx 5 \cdot 10^{-16}$ ;

3)  $\sigma_E$  — error due to discarding the term  $\Delta f \Delta H$  in formula (3), since measurements of the acceleration of free fall at the end points of the route were not carried out. According to the model of the Earth's gravitational field EGM2008,  $\Delta g$  between the points „Mendeleevo“ and „Evpatoriya“ is about  $0.01 \text{ m/s}^2$ , which at the height difference under study of about 200 m leads to a change in the frequency of the standard of about  $\sigma_g \approx 2 \cdot 10^{-17}$ ;

4)  $\sigma_{calc}^2 = \sigma_{\Delta f_\Omega}^2 + \sigma_{\Delta f_K}^2 + \sigma_{\Delta f_T}^2$  — the total relative error of calculations of the components of the sum  $\Delta f_{calc}$ , determined by formula (6). The components of this formula have the following meanings:

A) random error in calculating the relative frequency shift caused by the difference in centrifugal potentials of the extreme points of the route  $\sigma_{\Delta f_\Omega}$ . It is determined by the error in coordinate estimates at the starting point (Mendeleevo) and the end point of the route (Evpatoriya). Given approximately equal errors of the used CNA GNSS Javad Sigma G3T  $\sigma_x = \sigma_y \approx 1 \text{ m}$  for both points and taking into account the small value of the centrifugal potentials (at the equator the maximum centrifugal frequency shift is about  $10^{-12}$ ), we find that  $\sigma_{\Delta f_\Omega} \leq 10^{-18}$ ;

B) the error in determining the initial frequency mismatch, according to calibration measurements at stage I, is  $\sigma_{\Delta f_x} \sim 4.40 \cdot 10^{-16}$ ;

C) as already noted, the random error in calculating the temperature frequency drift  $\sigma_{\Delta f_T} \approx 7 \cdot 10^{-17}$ .

With these input data, as well as at the average value of the acceleration of gravity  $g \approx 9.81 \text{ m/s}^2$ , we obtain the required measurement errors:  $\sigma_{\Delta\varphi} \approx 75.3 \text{ m}^2/\text{s}^2$ ;  $\sigma_{\Delta H} \approx 7.7 \text{ m}$ . This is significantly less than in the experiment using code measurements of GNSS signals [4].

The measured value  $(f_E - f_0)^{meas}/f_0$  was  $-150.34 \cdot 10^{-16}$ ,  $\Delta f^{calc}/f_0$  taking into account linear drift and the moment of the beginning of the second stage of the experiment ( $T = 2.941$  days) amounted to  $60.46 \cdot 10^{-16}$ . By substituting all the obtained values into formula (6), we can determine the desired value  $\Delta H_{ort}$ . The estimate of  $\Delta H_{ort}$  is  $-193.127 \text{ m}$ , which is within the maximum uncertainty value ( $3\sigma_{\Delta H}$ ) relative to the actual height difference (201.37 m).

In the present experiment, the dominant factor limiting the accuracy of the method is the instability of the mobile quantum clock used ( $\sim 1 \cdot 10^{-15}$ ). Therefore, the accuracy of measurements increasing is possible through the use of microwave hydrogen quantum clocks created in Russia

with double sorting of atoms, which have stability up to  $0.7 \cdot 10^{-16}$  [14]. If we use a pair of such clocks with average relative instability  $0.9 \cdot 10^{-16}$  at the ends of the route, then with a commensurate random component of the initial calibration error  $\sigma_{\Delta f_x} \approx 0.9 \cdot 10^{-16}$  and constant other input data we obtain  $\sigma_{\Delta\varphi} \sim 15.3 \text{ m}^2/\text{s}^2$ ;  $\sigma_{\Delta H} \approx 1.6 \text{ m}$ .

As follows from formula (7) one of the ways to further improve accuracy is to reduce the measurement error of the frequency difference of the master oscillators of clock  $\delta f^{meas}$  (see formula (5)), which is possible by increasing the observation interval of the time scales difference (4).

## Conclusion

For the first time, the use of phase GNSS measurements of the current divergence of time scales and frequencies of stationary and mobile quantum clocks was proposed in the interests of measuring the difference in gravitational potentials and the difference in orthometric heights of points on the Earth's surface.

With a height difference between the extreme points of the route maximum 250 m, and also when using relocatable quantum hydrogen clocks with relative instability  $1 \cdot 10^{-15}$  and a measurement accumulation time of 5 days, the error in measuring the potential difference and orthometric heights was approximately  $75.3 \text{ m}^2/\text{s}^2$  and 7.7 m, respectively.

Ways to increase the accuracy of geodetic measurements are to use new domestic hydrogen quantum clocks with increased stability, as well as to reduce the measurement error from GNSS signals, which is possible by increasing the measurement accumulation interval.

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## Conflict of interest

The authors declare that they have no conflict of interest.

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