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Structure and self-modulation features of the superradiant states in asymmetric Fabry-Perot cavity

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The dependence of the structure and stability of strongly asymmetric stationary states of a superradiant laser with a slightly asymmetric low-Q Fabry-Perot cavity on its length, reflection factors of mirrors, and pumping level is studied. The states are related to a self-consistent inhomogeneous half-wavelength population inversion grating. The possibility of the existence of two dynamic phase transitions from a stationary (monochromatic) state of this type to a nonstationary one is established: 1) a dissipative superradiant transition to a regime with a quasi-continuous lasing spectrum (in a weakly asymmetric cavity) and 2) a self-modulation transition to a regime with a discrete lasing spectrum. It is shown that the latter can be caused by excitation of both polariton and electromagnetic laser modes due to resonant Rabi oscillations of active centers with a sufficiently long phase relaxation time.

Keywords: superradiant phase transition, population inversion grating, polariton modes, electromagnetic modes, self-modulation, discrete spectrum, continuous spectrum, low-Q cavity, Rabi oscillations.

1. Introduction

According to the results of recent studies [1–4], in superradiant lasers with a low-Q Fabry-Perot cavity and not very different reflection factor of mirrors, where the photon lifetime T_E is much shorter than the phase relaxation time of optical dipole oscillations of active centers T_2, the existence of strongly asymmetric stationary (monochromatic) or periodically oscillating states of the electromagnetic field and polarization of the active medium is possible. These states are associated with the formation of a self-consistent asymmetric half-wavelength population inversion grating of working energy levels of active centers, induced by counterpropagating waves. This phenomenon is expected to be observed in dense ensembles of active centers with a weak inhomogeneous broadening of the spectral line, which is much smaller than the spectral width of the resonant cavity modes $2/T_E$ and less than the homogeneous broadening of the spectral line $2/T_2$, which in turn should be much less than the so-called cooperative frequency (see formula (5) below).

This study is focused on the properties of stationary states of this type depending on the length of the resonant cavity, the reflection factors of its mirrors and the pumping level, as well as the nature of the instability of these states that arises when changing the parameters of the resonant cavity or pumping and determines the dynamic phase transition to one or another non-stationary operating mode of the laser. Such strongly asymmetric stationary and non-stationary lasing modes are achieved when the lasing threshold is exceeded many times over and can exist in various, including semiconductor, lasers with a fairly weak inhomogeneous broadening of the spectral line of active centers, which can be, for example, excitons, impurities or electrons and holes in highly magnetized quantum wells (see [5–11]). It should be noted that researchers of superradiant phase transitions have not yet discussed the possibility of existence of the strongly asymmetric coherent states we are considering, that arise in nonequilibrium ensembles of active centers (see, for example, [12–21]). The relevance of theoretical and experimental studies of such states is determined both by the fundamental nature of the physical phenomena associated with them in the emerging new field of laser physics, and by the possibilities of various applications of coherent radiation of active media in systems with low-Q cavities, which have their Q-factor and asymmetry of properties largely determined by the active medium itself through a half-wavelength grating of population inversion of active centers.

The study is structured as follows. Section 2 provides a brief description of the superradiant laser model used. Section 3 presents typical stationary asymmetric profiles of population inversion and counterpropagating waves of field and polarization of the active medium. Section 4 is devoted to the effect of the difference between the opposite mirrors of the resonant cavity on the above-mentioned profiles of the stationary state and the results of the analysis of its dynamic phase transitions to the non-stationary states. In Section 5, by an example of lasers with significantly different lengths, the structures of a strongly asymmetric superradiant state and the nature of its self-modulation depending on the pumping level are studied. Section 6 contains a comparative analysis of self-modulation due to the excitation of electromagnetic and polariton laser modes. General conclusions are given in Section 7.

2. Model of a superradiant laser

The initial relationships are well-known one-dimensional optical Maxwell–Bloch equations [1–3,22–24] that describe the (z, t)-dynamics of the superradiant state of an extended system of two-level active centers (qubits with transition frequency ω_0 and optical dipole moment d) under the action of continuous incoherent pumping based on (i) the introduction of counterpropagating waves of the field and polarization (density of optical dipole moments) with smoothly varying complex amplitudes

$$E = \mathrm{Re}\big[A_+(z, t)\exp(ik_0z - i\omega_0t) + A_-(z, t)\exp(-ik_0z - i\omega_0t)\big]/\sqrt{\varepsilon_0}, \quad (1)$$

$$P = \mathrm{Re}\big[P_+(z, t)\exp(ik_0z - i\omega_0t) + P_-(z, t)\exp(-ik_0z - i\omega_0t)\big]\sqrt{\varepsilon_0} \quad (2)$$

and (ii) explicit separation of the population inversion into a smoothly inhomogeneous component $n(z, t)$ (per active center) and a half-wavelength grating $n_z(z, t)$ (with a spatial period of $\lambda_0/2 = \pi/k_0$)

$$D(z, t) = n(z, t) + \mathrm{Im}\big[n_z(z, t)\exp(2ik_0z)\big], \quad (3)$$

where $k_0 = \omega_0c^{-1}\sqrt{\varepsilon_0}$ is wave number, c is speed of light in vacuum, ε_0 is permittivity of the active medium matrix. Equations the space-time function for n, n_z, $p_\pm = P_\pm/(dN_0)$,

$$\alpha_\pm = a_\pm\frac{\nu_c}{\omega_0} = \frac{A_\pm}{2\pi dN_0} \equiv \frac{dA_\pm}{\hbar\nu_c\varepsilon_0} \quad (4)$$

are given, for example, in [3], as well as in [1], where $a_\pm$ are introduced instead of $\alpha_\pm$ (N_0 is concentration of active centers, $\hbar$ is Planck's constant). Important parameters of the problem under consideration are the cooperative frequency

$$\nu_c = \sqrt{\frac{2\pi d^2\omega_0N_0}{\varepsilon_0\hbar}} \quad (5)$$

and the cooperative length $B_c = c/\nu_c\sqrt{\varepsilon_0}$, which are further used to normalize frequency detuning $\Delta = (\omega - \omega_0)/\nu_c$, the rates of field relaxation, population inversion and polarization $\Gamma_{E,1,2} = 1/(\nu_cT_{E,1,2})$, laser length $L = B/B_c$, time $\tau = \nu_ct$ and coordinate $\xi = z/B_c$. For definiteness, we assume that the resonant cavity contains an integer number of half-wavelengths of the radiation, i.e. its length B is a multiple of $\lambda_0/2$.

Bloch equations for complex amplitudes of counterpropagating polarization waves (see (2))

$$\left[\frac{\partial}{\partial\tau} + \Gamma_2\right]p_+ = -in\alpha_+ - \frac{n_z}{2}\alpha_-,$$

$$\left[\frac{\partial}{\partial\tau} + \Gamma_2\right]p_- = -in\alpha_- + \frac{n_z^*}{2}\alpha_+ \quad (6)$$

include nonlinear interaction of waves and describe enrichment of their spectrum due to the known effect of Rabi oscillations of population inversion (3) of working energy levels of active centers with frequency of [3,24,25] $\omega_R = \nu_c\sqrt{\varepsilon_0}|\alpha(\xi, \tau)|$ when exposed to the resonant field (1). Here $\alpha = \alpha_+\exp(ik_0z) + \alpha_-\exp(-ik_0z)$ is the normalized complex amplitude of the field, the real part of which $\mathrm{Re}\big[\alpha\exp(-i\omega_0t)\big]$ characterizes the ratio between the Rabi frequency and the cooperative frequency.

To study properties of the superradiant state of a laser with a low-Q asymmetric Fabry-Perot cavity, the Maxwell–Bloch equations were solved numerically for various levels of homogeneous continuous-wave pumping $D_p = n_p$ using the modified 4th order Runge–Kutta method and grid method. The difference in the reflection factors of the mirrors was taken into account by different boundary conditions at opposite ends of the cavity: $\alpha_+(-L/2) = R_1\alpha_-(-L/2)$ and $\alpha_-(L/2) = R_2\alpha_+(L/2)$. For definiteness, at the initial moment of time, only small polarization noises of the active medium $|p_\pm| \sim 10^{-4}$ were specified, and the electric field and population inversion grating were considered absent: $\alpha_\pm = 0$, $n_z = 0$.

The following rates of polarization relaxation and population inversion were chosen as characteristic values in the study: $\Gamma_2 = 2\Gamma_1 = 0.02$. Calculations have shown that sufficiently small values of these relaxation parameters and a sufficiently large excess of the lasing threshold are needed in order that counter-propagating superradiance pulses, which appear usually not quite simultaneously at the initial transition stage of the process, can spontaneously form a highly inhomogeneous half-wavelength population inversion grating near one of the laser mirrors, usually the one with a higher reflection factor. We further assume that this is the left mirror, i.e. $R_1 \geq R_2$, and the ratio of these reflection factors $\rho = R_2/R_1$ can be either close to unit or noticeably less than it. In the steady state, the resulting strongly asymmetric grating turns out to be stationary or oscillates, without moving to the other half of the laser and providing its strongly asymmetric radiation only for certain intervals of laser lengths, mirror reflection factors, and pumping levels. Outside these parameter intervals, numerical solutions show that the population inversion grating is weak or strongly fluctuating and quasi-stationary single-mode lasing or various modes of quasi-periodic or quasi-chaotic lasing of superradiance pulses are usually realized.

3. Stationary profiles of fields, polarization and population inversion

According to numerous calculations, the formation of strongly asymmetric superradiant states caused by a self-consistent half-wavelength grating of population inversion is possible in wide ranges of cavity lengths $L \sim 1-10$ and reflection factors of its mirrors $R_{1,2} \gtrsim 0.1$, $1 - R_{1,2} \gtrsim 0.1$ in a suitable range of pumping levels, significantly exceeding the lasing threshold. Moreover, in all the cases considered, the condition of $\Gamma_2 \ll \Gamma_E \lesssim n_p^{1/2}$ was met.

Typical population inversion profiles and amplitudes of counterpropagating polarization and field waves in a Fabry-Perot resonant cavity with mirror reflection factors of $R_1 = 0.5$, $R_2 = 0.4$ for lengths of $L = 2$ and $L = 10$ at a pumping level of $n_p = 0.5$ are shown in Figure 1. In this specific case and the general case, according to the stationary Maxwell–Bloch equations, for the above-specified functions the positions of the maxima of grating and smoothly inhomogeneous component of the population inversion coincide, and at this point the amplitudes of counterpropagating waves of both field and polarization are equal. On the right slope of the indicated peaks for a wave traveling to the left, at the same point, the field amplitude reaches a minimum and the polarization amplitude becomes zero, while the amplitude of the counterpropagating polarization wave in the region of this point has a smooth maximum. On the contrary, on the left slope of these peaks for a wave traveling to the right, at the same point the field amplitude becomes minimal and the polarization amplitude reached zero value, and in the region of the same point there is a smooth maximum of the polarization amplitude of the counterpropagating wave.

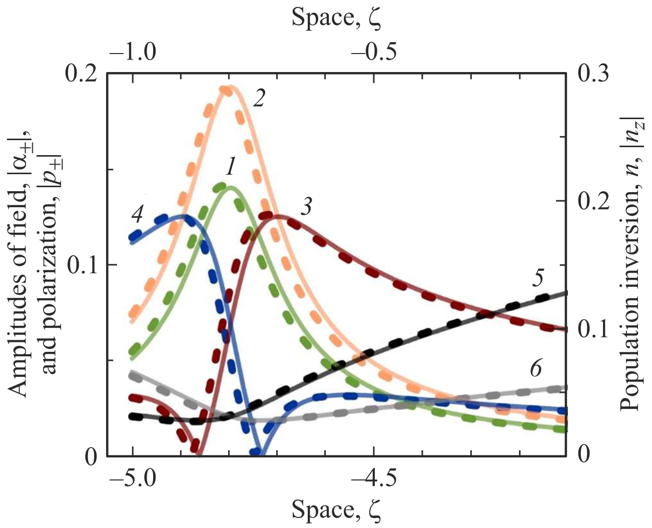

Figure 1. Typical asymmetric profiles of the smoothly inhomogeneous component n (*1*) and the amplitude of the population inversion grating $\left|n_z\right|$ (*2*), the amplitudes of counterpropagating waves of polarization $|p_\pm|$ (*3, 4*) and field $|\alpha_\pm|$ (*5, 6*) in steady-state stationary lasing of a superradiant laser with mirror reflection factors $R_1 = 0.5$, $R_2 = 0.4$. Solid lines show structures in the left part of the laser with dimensionless length $L = 10$ (lower coordinate axis), dashed lines show that of the laser with $L = 2$ (upper coordinate axis). Pumping level is $n_p = 0.5$. (The colored version of the figure is available on-line.)

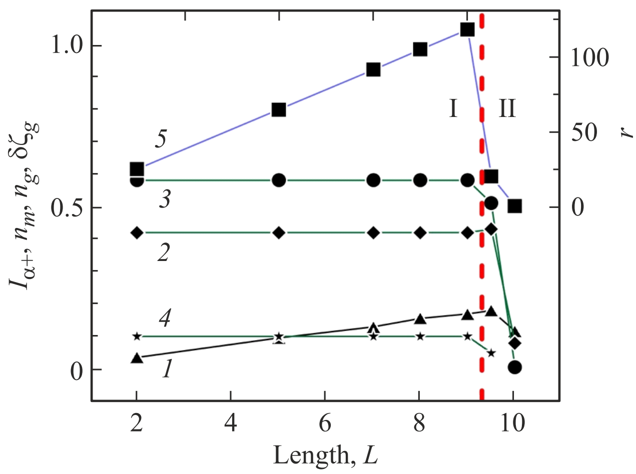

Figure 2. Characteristics of steady-state strongly asymmetric lasing of a superradiant laser with mirror reflection factors of $R_1 = 0.5$, $R_2 = 0.4$ and a pumping level of $n_p = 1$ as functions of length L: average intensity of radiation $\left\langle I_{\alpha_+}\right\rangle_\tau$ from the right end with coordinate $\xi = L/2$ (line *1*), average value of the maximum of the smoothly inhomogeneous component of population inversion n_m (line *2*), average value of the maximum amplitude of the half-wavelength population inversion grating n_g (line *3*), distance from it, i.e. from the grating peak at point $\xi_g < 0$, to the left end, $\delta\xi_g = L/2 + \xi_g$ (line *4*), radiation asymmetry factor r (line *5*). The dashed vertical line separates the zones of stationary (I) and non-stationary (II) lasing (self-modulation or superradiance).

A similar symmetrical structure of the superradiant state with a strongly pronounced population inversion grating is formed in a very limited region of the laser near the mirror with the highest reflection factor (left) and occupies a relatively small region with a length less than or of the order of the cooperative length B_c. This structure is almost independent of the laser length (Figure 2) and, with increase in distance from it toward the remaining part of the laser to the right, a monotonous increase takes place in the amplitudes of both counterpropagating field waves, which is much greater for the wave traveling to the right, and a monotonic decrease takes place in the amplitudes of both counterpropagating polarization waves. In the stationary case, the latter waves are proportional to the derivatives of the corresponding field waves, and therefore the polarization wave traveling to the right prevails over the polarization wave traveling to the left. With increasing laser length, the predominance of the wave traveling to the right over the wave traveling to the left increases, because it is the traveling to the right wave that takes almost all the pumping energy, so that a significant increase takes place in the radiation asymmetry factor (line *5* in

Figure 2):

$$r = \max\left[\frac{(1-R_2^2)|\alpha_+(\xi=L/2)|^2}{(1-R_1^2)|\alpha_-(\xi=-L/2)|^2}\right]. \qquad (7)$$

This factor depends almost linearly on the laser length at a given pumping level and is directly proportional to this level at a given laser length, because, as numerical calculations show, the intensity of the counterpropagating wave $I_{\alpha_-} = (1-R_1^2)|\alpha_-|^2$ emitted from the laser to the left weakly depends on these values, if the asymmetry factor r significantly exceeds the asymmetry of the cavity mode in the absence of grating $r_0 = R_1(1-R_2^2)/(R_2(1-R_1^2))$. In this case, the radiation intensity from the right end $I_{\alpha_+} = (1-R_2^2)|\alpha_+|^2$ is directly proportional to the laser length (see line *1* in Figure 2 in the non-stationary lasing zone II where the radiation intensities of counterpropagating waves are equalized to a significant extent, in particular, due to the weakening of the influence of the population inversion grating on them because of its oscillations and a decrease in amplitude).

4. The influence of the difference between the opposite mirrors of the cavity on the superradiant state and dynamic phase transitions

All calculations performed (for the selected ratios of rates of the polarization and population inversion relaxation to the cooperative frequency $\Gamma_2 = 2\Gamma_1 = 0.02$) indicate that in the specified ranges of length of 1–10 and mirror reflection factors of 0.1–0.9 strongly asymmetric superradiant states turn out to be stationary (in a certain range of pumping levels) only if the difference in mirror reflection factors is sufficiently large: $R_1 - R_2 \gtrsim 0.1R_1$, i.e. $1-\rho \gtrsim 0.1$. In this case, in contrast to the laser length L, the reflection factors R_1, R_2, as well as the pumping level n_p (see Sections 5 and 6) significantly affect the profiles of both the grating and smoothly inhomogeneous component of the population inversion peak, and the amplitudes of the counterpropagating field and polarization waves associated with them.

For very close (usually within one or a few percent, $1-\rho < 0.1$) values of the mirror reflection factors, self-modulation of a strongly asymmetric superradiant state is observed in all the cases considered. In the range of pumping levels where this self-modulation is small, the time-averaged structure of the superradiant state depends relatively weakly, and the average intensity $\langle I_{\alpha^+}\rangle_\tau$ of a strong wave traveling to the right is almost independent on the difference in the reflection factors of the mirrors ρ (see zone II in Figure 3). The rather significant change in the laser radiation asymmetry factor (7) r with a change in the value of $\rho = R_2/R_1$ shown in the figure is mainly due to a change in the average intensity $\langle I_{\alpha_-}\rangle_\tau$ of the weak counterpropagating wave traveling to the left.

In the case of moderately close (usually within $\sim 10\%$, $1-\rho \sim 0.1$) reflection factors of the mirrors, according to the numerical calculations analysis performed, the loss of stability of the stationary strongly asymmetric superradiant state at all laser lengths is possible for two reasons. Firstly, when the reflection factors of the mirrors are equalized ($\rho \to 1$), either with an increase in the laser length or an increase in the pumping level (see below) an instability arises due to the excitation of discrete polariton modes by resonant Rabi oscillations of active centers located in the region of peak of population inversion under the effect of a matched electromagnetic field of counterpropagating waves; this mechanism of dynamic phase transition to self-modulation with a discrete spectrum is considered in [3,4]. Secondly, when approaching the lasing threshold, namely, when the laser length or pumping level decreases, the instability of the strongly asymmetric stationary state is caused by a dynamic phase transition to pulsed superradiant lasing with a quasi-continuous spectrum and almost symmetric time-averaged profiles of counterpropagating waves (which is typical for superradiant lasers with a low-Q cavity, where individual discrete modes do not play a leading role (see [1,22,26])).

In the case of a sufficiently large difference in the reflection factors of the mirrors $R_1 - R_2 \gtrsim 0.2R_1$, i.e. $1-\rho \gtrsim 0.2$, the dynamic phase transition to superradiant lasing with a continuous spectrum turns out to be impossible, and the instability of a strongly asymmetric state with increase in the pumping level or laser length

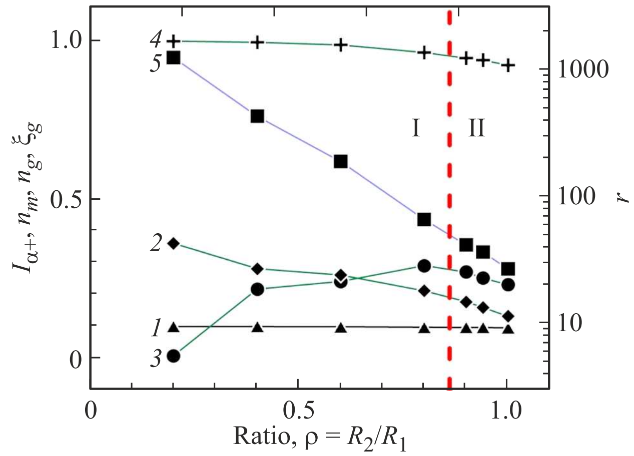

Figure 3. Characteristics of steady-state strongly asymmetric lasing of a superradiant laser with a length of $L = 10$, and a pumping level of $n_p = 0.5$ as functions of asymmetry $\rho = R_2/R_1$ of Fabry-Perot cavity with a left mirror reflection factor of $R_1 = 0.5$: average intensity of radiation $\langle I_{\alpha_+}\rangle_\tau$ from the right end with coordinate $\xi = L/2$ (line *1*), average value of the maximum of the smoothly inhomogeneous component of population inversion n_m (line *2*), average value of the maximum amplitude of the half-wave population inversion grating n_g (line *3*), its relative position inside the laser $\xi_g = |\xi_g|/(L/2)$ (line *4*), radiation asymmetry factor r (line *5*). The dashed vertical line separates the zones of stationary (I) and non-stationary (II) lasing (self-modulation).

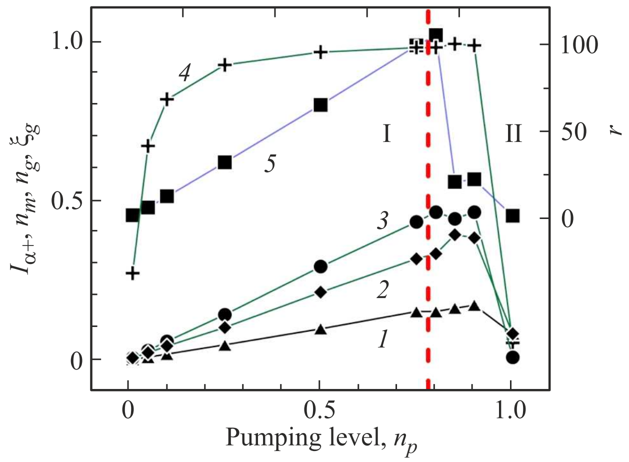

Figure 4. Characteristics of steady-state strongly asymmetric lasing of a superradiant laser with the reflection factors of the mirrors of $R_1 = 0.5$, $R_2 = 0.4$ and a length of $L = 10$ as functions of the pumping level n_p: average radiation intensity $\langle I_{\alpha+} \rangle_\tau$ from the right end with coordinate $\xi = L/2$ (line *1*), average value of the maximum of the smoothly inhomogeneous component of population inversion n_m (line *2*), average value of the maximum amplitude of the half-wavelength population inversion grating n_g (line *3*), its relative position inside the laser $\xi_g = |\xi_g|/(L/2)$ (line *4*), radiation asymmetry factor r (line *5*). The dashed vertical line separates the zones of stationary (I) and non-stationary (II) lasing (self-modulation or superradiance).

can only be associated with a phase transition to self-modulation with a discrete spectrum. As shown below, for sufficiently long lasers, the latter phase transition can also be caused by the excitation of electromagnetic modes of the cavity, and not only by the above-mentioned polariton modes. In both cases, as the lasing threshold is further exceeded (for example, because of increase in the laser length, as in Figure 2, or the pumping level, as in Figure 4), the self-modulation is replaced by a quasi-chaotic mode of superradiance pulses lasing with a quasi-continuous spectrum. This mode rules out a strong radiation asymmetry, which time-averaged value usually does not exceed the mode asymmetry of the Fabry-Perot cavity r_0.

As the difference in the reflection factors of the mirrors increases to tens of percent or by several times, the region of stability of the stationary superradiant state with respect to changes in the laser length and pumping level, as a rule, expands, and for a given length and pumping, the radiation intensity of a strong wave traveling to the right and exiting through the weakly reflecting mirror in a stationary state remains almost unchanged (see line *1* in Figure 3). At the same time, the reflected counterpropagating wave turns out to be increasingly weaker, its amplification in the active medium is small and in fact occurs only near the left mirror with stronger reflection, where a high (line *2* in Figure 3) but narrow peak of population inversion is kept. In this case, the peak of the half-wavelength grating generated by counterpropagating waves also turns out to be weaker (line *3* in zone I), and both peaks, having maxima at the same point, are pressed against the left mirror (line *4*). As a result, with a decrease in the ρ parameter in the region of $\rho \lesssim 0.5$, the asymmetry factor of the laser radiation becomes very large (line *5* in zone I).

Under these conditions, with increasing laser length or pumping level (at a fixed value of ρ), the structure of the superradiant state, either stationary or with moderate self-modulation, does not change qualitatively, however, its asymmetry increases and corresponds to increasingly strong localization of a layer of high population inversion near the mirror with a higher reflection factor. Moreover, the peaks of the grating and the smoothly inhomogeneous components of the population inversion with coinciding positions can be so close to the left mirror that their left slopes are largely or completely lost, and, consequently, there are not (cf. Figure 1) the zero of polarization amplitude, and the minimum of field amplitude of the wave traveling to the right, and the maximum of the polarization amplitude of the wave traveling to the left, that all could be located in the same region and have coinciding positions as well. This is the situation, in particular, for lasers with $L = 10$, $R_1 = 0.5$, $n_p = 0.5$ at $\rho < 0.5$ parameters, which characteristics are shown in Figure 3.

5. The effect of pumping on the superradiant state and its instability

Coming back to the case of very close mirror reflection factors, let us make an indicative comparison of the asymmetric states evolution for the laser as its pumping level increases at two significantly different cavity lengths of $L = 2$ and $L = 10$ for the case of a weakly asymmetric cavity with $R_1 = 0.5$, $R_2 = 0.47$, i.e. the case of a 6-percent difference in mirror reflection factors ($\rho = 0.94$).

In a short laser with $L = 2$ not far from the lasing threshold, for example, for $n_p = 0.05$, a stationary (monochromatic) mode of weakly asymmetric radiation (with $r \approx r_0 = 1.1$) of one resonant polariton mode is realized with an extremely weak population inversion grating. For a significant excess of the lasing threshold, for example, for $n_p = 0.1$, the mode of quasiperiodic radiation of superradiance pulses with a quasi-continuous spectrum takes place, which is specified in the previous Section and also weakly time-averaged asymmetric. However, for an even greater, many times exceeding of the lasing threshold, for example, for $n_p = 0.25$, a highly inhomogeneous population inversion grating is spontaneously formed in the laser together with the strongly asymmetric stationary (again monochromatic) superradiant state caused by this grating, with a radiation asymmetry factor of $r = 2$. Finally, for the range of extremely high pumping levels of $n_p \sim 0.5 - 1$, this superradiant state acquires the self-modulation with a discrete spectrum due to the excitation of „side“ polariton modes of the laser, detuned from the frequency of the two-level transition, maintaining a strong time-averaged asymmetry radiation with a factor of $r > 5$.

In a long laser with $L = 10$, not far from the lasing threshold for $n_p \sim 0.003$, a single-mode stationary mode of weakly asymmetric radiation is also realized with the same $r \approx r_0 = 1.1$ and an extremely weak inversion population grating. At a significantly higher level of the lasing threshold, when $n_p \sim 0.05$, superradiance again occurs with a quasi-continuous spectrum and a weak asymmetry. However, if the lasing threshold is exeeded even more (tens of times), in the range of pumping levels of $n_p \sim 0.1-0.5$, not only a highly inhomogeneous population inversion grating is spontaneously formed in the laser, which is matched with counterpropagating waves of the resonant polariton mode and maintaining a strongly asymmetric superradiant state, but at the same time, „side“ polariton modes are excited, which are detuned from the transition frequency in the active medium. They introduce a self-modulation with a discrete spectrum and rule out the existence of a stationary state, however, do not disturb the strong time-averaged asymmetry of radiation with a factor of $r \sim 20$. Finally, for the limiting pumping level of $n_p \sim 1$, a more complex self-modulation of a strongly asymmetric superradiant state with a quasi-discrete spectrum becomes possible. The latter consists of a strong fundamental harmonic, which is broadened by polariton modes with a frequency spread of $|\Delta| \lesssim 0.03$ (discussed in the previous Section), and two symmetrical weak, also slightly broadened, „side“ harmonics, which are detuned in frequency from the fundamental harmonic by the amount of $\Delta \approx \pm 0.3$ and are caused by the excitation of electromagnetic modes of the cavity under the effect of Rabi oscillations of active centers in the resonant field of this harmonic (for more details, see Section 6).

Except for the above-specified last step for a long laser with $L = 10$, in the case of a perfectly symmetric cavity with $R_1 = R_2 = 0.5$ for a short laser with a length of $L = 2$, in general, qualitatively the same sequence of lasing modes change takes place. In particular, a strongly asymmetric superradiant state with $r \approx 10$ with a weak self-modulation due to a pair of excited polariton modes is realized at the limiting pumping levels of $n_p \sim 0.5-1$ and can no longer be stationary.

On the contrary, in the case of a more asymmetric cavity with $R_1 = 0.5$, $R_2 = 0.4$, i. e. with a 20 percent difference in mirror reflection factors ($\rho = 0.8$, see Section 3), not only for short ($L = 2$) lasers but also for long ($L = 10$) lasers a wide range of pumping levels takes place where a stationary, strongly asymmetric superradiant state is realized. A similar state with weak self-modulation can exist in an even larger range of pumping levels. This fact is illustrated in Figure 4 for a laser with a length of $L = 10$, where a stationary strongly asymmetric state is realized at $0.05 \lesssim n_p < 0.8$ (zone I). Its radiation asymmetry factor increases (up to $r \sim 100$, line *5*) with increase in pumping level simultaneously with the growth of peaks of the population inversion grating and the smoothly inhomogeneous component of the population inversion (lines *2* and *3*) and their displacement to the left mirror (line *4*). At higher pumping levels n_p, approximately in the interval of $0.8-0.9$ (zone II), quasiperiodic self-modulation of the asymmetric state takes place, in which the above-mentioned, now oscillating, peaks of the population inversion remain pressed to the left mirror, but the radiation asymmetry factor turns out to be much smaller and of the order of magnitude $r \sim 20$, as well as at low pumping levels of $n_p \sim 0.1$ (in zone I).

6. The self-modulation with involvement of polariton and (or) electromagnetic modes

The issue of the self-modulation nature of a strongly asymmetric superradiant state is very complex and little studied (even less studied is the problem of the dynamic phase transition of such a state into the mode of pulsed superradiant lasing with a quasi-continuous spectrum, which is almost time-averaged symmetric). Therefore, using the considered examples of numerical calculations, this Section will be limited to qualitative comments on the operation of the mechanism of this self-modulation with the involvement of not only polariton modes (discussed earlier [3,4]) but also electromagnetic modes of the resonant cavity. The latter modes, for the inverted active medium of a superradiant laser of interest to us, have significantly larger frequency detunings from the fundamental mode (strictly resonant to the active centers) than polariton modes (for the spectrum of modes, see, for example [1,3,22,26,27]). Hence, according to the Bloch equation for the polarization of a medium (6), the excitation of certain „side“ modes due to the resonant to them oscillations of dipole moments of the active centers (playing the role of resonant current sources) is caused by nonlinear processes in different parts of the laser. It is in such parts where the field amplitude of the fundamental mode corresponds to the Rabi frequency $\Omega_R = dE/\hbar$ of the oscillations of the population inversion of active centers, which is equal to the detuning of the frequency of the excited mode from the frequency of the fundamental mode, i. e. from the frequency of a two-level laser transition.[1]

The fulfillment of this condition, supplemented by the requirement to maintain undamped Rabi oscillations of active centers due to the nonlinear interaction and nonadiabatic dynamics of inhomogeneous counterpropagating waves in a

[1] For cases of sufficiently long lasers with simultaneous excitation of both polariton and electromagnetic modes, which have significantly different frequency detunings from the fundamental, „central“ mode, in numerical calculations not only oscillating rather sharp peaks of grating and smoothly inhomogeneous component of the population inversion are observed being pressed against the left mirror (cf. Figure 1), but also similar oscillating, although not so sharp, peaks closer to the center of the laser. In these parts, the field of the fundamental mode is stronger and the Rabi frequency is higher, and therefore, the conditions for resonant nonlinear excitation of electromagnetic modes are realized, despite the fact that they are low-Q.

low-Q cavity, depends on many factors, including the laser length L and the reflection factors of the mirrors R_1, R_2, level n_p and rate Γ_1 of pumping, ratio of relaxation times T_2 and T_E.

So, for the option considered at the end of Section 5 (Figure 4) $L = 10$, $R_1 = 0.5$, $R_2 = 0.4$, $n_p = 0.85$, $\Gamma_2 = 2\Gamma_1 = \Gamma_E/4$, as shown in Figure 5, two spectrally symmetric electromagnetic modes are excited with minimal (for this type of mode) frequency detunings from the fundamental central mode $\Delta \approx \pm 0.3$ and there is no excitation of polariton modes with smaller frequency detunings. The spectrum pattern remains qualitatively the same at pumping levels of $n_p = 0.8$ and $n_p = 0.9$. However, the excitation of electromagnetic modes, and therefore self-modulation, stops at lower pumping levels. At the maximum pumping level of $n_p = 1$, the strongly asymmetric state is completely destroyed due to the appearance of superradiance pulses with a continuous spectrum (see Figure 5 and the right edge of zone II in Figs. 2 and 4).

For the option considered in the middle of Section 5 with $L = 10$, $R_1 = 0.5$, $R_2 = 0.47$, $n_p = 1$, $\Gamma_2 = 2\Gamma_1 = \Gamma_E/3.6$, as shown in Figure 6, not only approximately the same spectrally symmetric electromagnetic modes are excited but also two polariton modes with maximum (for this type of mode) frequency detunings from the fundamental central mode $\Delta \simeq \pm 0.03$. Moreover, in this option and the previous option of the laser, the waves of fundamental mode traveling

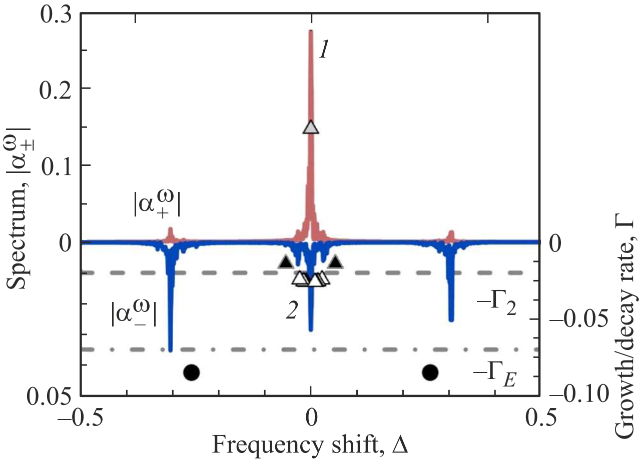

Figure 6. Typical discrete amplitude spectra $|\alpha_\pm^\omega|$ (shown on the scales on the left, going up and down from level 0) of counterpropagating waves (*1, 2*) at the $\xi = \pm L/2$ ends of superradiant laser for steady-state asymmetric quasi-monochromatic lasing at a pumping level of $n_p = 1$ as functions of the normalized frequency detuning $\Delta = (\omega - \omega_0)/v_c$. Triangles show frequencies and growth/decay rates Γ of polariton modes, circles show electromagnetic modes, calculated within the linear theory with a homogeneous population inversion equal to its average steady-state value in the nonlinear calculation $\langle \overline{n} \rangle_\tau = 0.015$. The dashed line shows the level of $-\Gamma_2 = -0.02$, the dash-dot line shows $-\Gamma_E = -0.07$. The laser has the mirror reflection factors of $R_1 = 0.5$, $R_2 = 0.47$ and a length of $L = 10$.

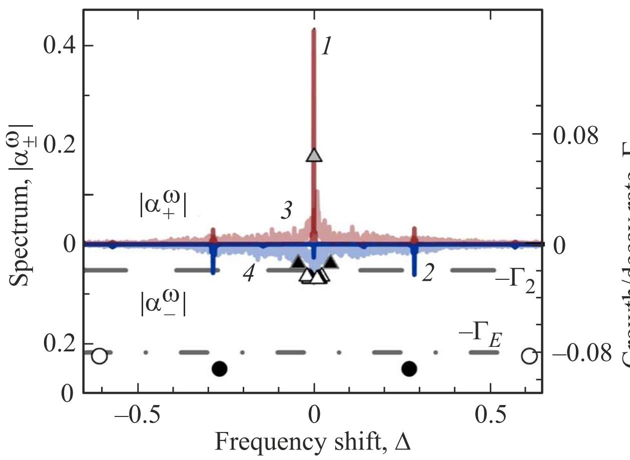

Figure 5. Typical amplitude spectra $|\alpha_\pm^\omega|$ (shown on the scales on the left, going up and down from level 0) of counterpropagating waves at the $\xi = \pm L/2$ ends of a superradiant laser: discrete (*1, 2*) for steady-state asymmetric quasi-monochromatic lasing at a pumping level of $n_p = 0.85$ and continuous (*3, 4*) for chaotic lasing of superradiance pulses at $n_p = 1$ as functions of the normalized frequency detuning $\Delta = (\omega - \omega_0)/v_c$. Triangles show frequencies and growth/decay rates Γ of polariton modes, circles show electromagnetic modes, calculated within the linear theory with a homogeneous population inversion equal to its average steady-state value in the nonlinear calculation $\langle \overline{n} \rangle_\tau = 0.013$. The dashed line shows the level of $-\Gamma_2 = -0.02$, the dash-dot line shows $-\Gamma_E = -0.08$. The laser has the mirror reflection factors of $R_1 = 0.5$, $R_2 = 0.4$ and a length of $L = 10$.

to the left (the weaker of its two counterpropagating waves) turns out to be even weaker than the waves co-directed with it in two „side“ electromagnetic modes, which beats with the fundamental mode determine the main component of the self-modulation.

Finally, for the option considered at the beginning of Section 5 with $L = 2$, $R_1 = 0.5$, $R_2 = 0.47$, $n_p \sim 0.5{-}1$, $\Gamma_2 = 2\Gamma_1 = \Gamma_E/18$, only two polariton modes are excited with maximum (for this type of mode) frequency detunings from the fundamental central mode $\Delta \approx \pm 0.03$. In this case, wave of fundamental mode traveling to the left turns out to be stronger than the waves co-directed with it in two „side“ polariton modes, which beats with the fundamental mode also determine the main component of the self-modulation. In this case, the pumping level and the field amplitude of the fundamental mode are insufficient for the operation of the Rabi mechanism for excitation of electromagnetic modes, which have their decay rates and frequency deturnings from the fundamental mode much greater than those for polariton modes.

7. Conclusion

In this study, a number of fundamental properties of strongly asymmetric stationary and nonstationary states of a superradiant laser in a slightly asymmetric low-Q Fabry-Perot resonant cavity have been identified. It is shown that these properties are largely due to the self-consistent inhomogeneous half-wavelength population inversion grating created by self-consistent counterpropagating waves, and depend on the length, reflection factors of the mirrors, and the pumping level of the laser. The features of two dynamic phase transitions, i. e. the pulsed superradiant transition and the quasiperiodic self-modulation transition, between similar stationary and non-stationary states are indicated. It has been established that for these states, the self-modulation with a discrete spectrum can be associated with both polariton and electromagnetic laser modes, the nonlinear excitation of which occurs due to resonant Rabi oscillations of active centers in the field of the fundamental laser mode. Further study of the discovered superradiant states for a system of many particles with radiative interaction in a low-Q cavity is of interest from the point of view of both the fundamental physics of dissipative phase transitions and the applied physics of self-organizing laser systems, in particular, allowing a spontaneous transition between modes of almost symmetric and almost unidirectional radiation.

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Conflict of interest

The authors declare that they have no conflict of interest.

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