Criteria for electron runaway in a gas diode with a needle cathode

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The conditions of electron runaway in a gas diode with a cathode in the form of a needle are studied theoretically. It is shown that the runaway conditions are qualitatively different for needles with relatively large and small tip radii, i.e., in fact, for different degrees of electric field inhomogeneity. In a weakly inhomogeneous field, the transition of electrons to the runaway regime is determined by the local distribution of the field near the place of their start — the tip of the needle. In a strongly inhomogeneous field, the runaway condition has a nonlocal character: it is determined by the behavior of electrons in the near-anode region. This difference leads to a nonmonotonic dependence of the threshold runaway voltage on the tip radius.

Keywords: Runaway electrons, needle cathode, gas diode, pulsed breakdown.

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Free electrons in gas in an electric field may undergo continuous acceleration (i.e., a transition to the runaway regime) if the field is sufficiently strong [1–4]. Runaway electrons (RAEs) produced in a centimeter-scale gas gap under the influence of a voltage pulse with an amplitude of several tens or hundreds of kilovolts cross this gap with velocities comparable to the speed of light. RAEs induce preionization of gas, governing the dynamics of its subnanosecond breakdown (see [2–7] and references therein). In a homogeneous or weakly inhomogeneous field, electrons enter the runaway regime if the field intensity at their origin site exceeds a certain critical value (E_c) that depends on the type of gas and pressure. In a strongly inhomogeneous field produced with the use of pointed cathodes (see, e.g., [8–11]), this condition is not sufficient for continuous acceleration of electrons within the entire gap [12]. The field decays rapidly with distance from the tip, and an electron accelerated in the near-cathode region may start decelerating in the periphery.

In the present study, the dependence of conditions for electron runaway in a gas diode with a needle cathode on the needle tip radius is analyzed. It is demonstrated that a variation of the tip radius (i.e., the degree of inhomogeneity of the electric field distribution in a gap) induces a qualitative change in these conditions and, consequently, a non-monotonic radius dependence of the threshold runaway voltage.

Let us examine a needle cathode with tip rounding radius R. Its shape is approximated by a paraboloid of revolution $r^2 = -2Rz$ (cylindrical coordinates with their origin at the needle tip are used; see the inset in Fig. 1). The following solution of the Laplace equation for the electric field potential is used to describe the field distribution in a gap (D is the interelectrode distance along axis z and U is the applied voltage):

$$\varphi(r, z) = \frac{U}{\ln(1 + 2D/R)} \times \ln\left(\frac{z + R/2 + \sqrt{r^2 + (z + R/2)^2}}{R}\right). \quad (1)$$

Let us determine the minimum voltage U at which electrons originating from the cathode undergo continuous acceleration within the entire interelectrode gap (i.e., enter the runaway regime). It is evident that direction z is the preferred one for RAEs. Therefore, it is sufficient to consider one-dimensional motion of electrons in gas (let us choose atmospheric air for definiteness). The absolute value of field intensity E on axis z is written as

$$E(z) \equiv \left|\frac{\partial\varphi(0, z)}{\partial z}\right| = \frac{U}{(z + R/2)\ln(1 + 2D/R)}. \quad (2)$$

The field distribution corresponding to (2) with $D = 10$ mm, $R = 50\,\mu$m, and $U = 50$ kV is presented in Fig. 1 (the electrode geometry is shown in the inset). It is evident that the field is strongly inhomogeneous. It decays by a factor of 400 with distance from the cathode.

The motion of an electron with kinetic energy ε is described by equation [2–4]

$$d\varepsilon/dz = eE(z) - F(\varepsilon), \quad (3)$$

where e is the elementary charge and F if the friction force for a free electron in gas. This force is known [1,3,4] to have a non-monotonic dependence on the electron energy. It has maximum $F_{\max}$ corresponding to certain energy ε_c on the order of 100 eV within the non-relativistic energy range ($\varepsilon < 510$ keV). In the case of a homogeneous field, an electron is accelerated at any ε if its intensity exceeds critical value $E_c \equiv F_{\max}/e$. The values of $\varepsilon_c = 110\,\mathrm{eV}$ [13] and $E_c = 270\,\mathrm{kV/cm}$ may be set for atmospheric air [3,4]. The following approximations (e is the base of the natural logarithm) are used in subsequent calculations:

$$F(\varepsilon) = eE_c\sqrt{4\mathrm{e}\varepsilon/(9\varepsilon_c)}, \quad \varepsilon < \varepsilon_0 \equiv \mathrm{e}^{-1/3}\varepsilon_c,$$

$$F(\varepsilon) = eE_c\,\frac{\varepsilon_c}{\varepsilon}\,\ln\left(\frac{\mathrm{e}\varepsilon}{\varepsilon_c}\right), \quad \varepsilon > \varepsilon_0 \qquad (4)$$

These expressions combine power dependence $F \propto \varepsilon^{1/2}$ (the force of friction is proportional to velocity at low energies) with non-relativistic Bethe formula $F(\varepsilon) = 2\pi Ze^4 n\varepsilon^{-1}\ln(2\varepsilon/I)$, where Z is the number of electrons in a neutral molecule, n is the concentration of molecules, and I is the mean inelastic loss energy [1–4], at point $\varepsilon = \varepsilon_0$.

Equation (3) was solved numerically with initial condition $\varepsilon(0) = 0$ (i.e., electrons were initially positioned at the needle tip and had zero velocity). The interelectrode distance was set to $D = 10\,\mathrm{mm}$. Tip rounding radius R was varied within a wide range. Runaway voltage U_c was determined for each R. It is evident that a change in R at constant D may be interpreted as a change in the degree of inhomogeneity of the electric field distribution in a gap. In the $R \gg D$ limit, the field becomes homogeneous. The field distribution at $R \ll D$ is strongly inhomogeneous. Specifically, it follows from Fig. 1 that the field exceeds critical value E_c only in the immediate vicinity of the cathode (in the region with $z < 280\,\mu\mathrm{m}$). In the rest of the gap, $E < E_c$ and runaway is made possible by the fact that the force of friction does, in accordance with (4), decay rapidly with increasing ε. The right-hand part of Eq. (3) is positive, and an electron is accelerated if it manages to acquire a sufficiently high energy in the near-cathode region.

The calculation results are represented by the solid curve in Fig. 2. It is evident that the dependence of U_c on R is non-monotonic with a minimum at $R_c \approx 475\,\mu\mathrm{m}$.

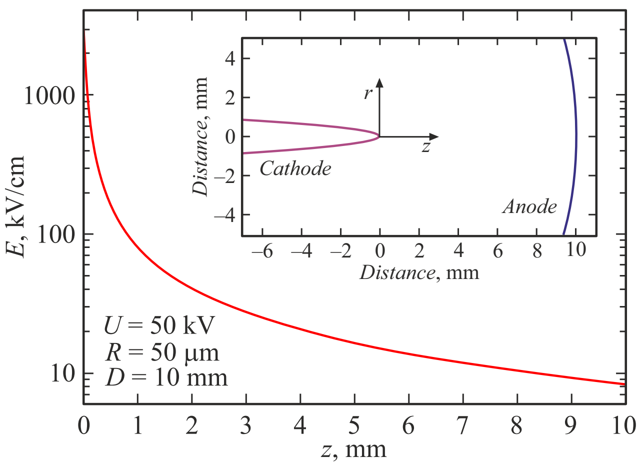

Figure 1. Distribution of electric field intensity E along axis z ($D = 10\,\mathrm{mm}$, $R = 50\,\mu\mathrm{m}$, and $U = 50\,\mathrm{kV}$). The model geometry of electrodes is shown in the inset.

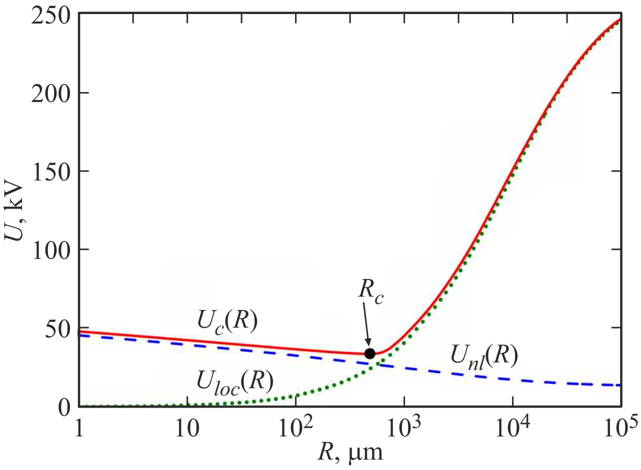

Figure 2. Results of numerical calculation of the dependence of threshold runaway voltage U_c on needle tip radius R ($D = 10\,\mathrm{mm}$ and the gas is atmospheric air) (solid curve). Dotted and dashed curves represent the dependences of U_{loc} and U_{nl} on R (formulae (5) and (6)) corresponding to local and nonlocal runaway criteria, respectively.

The dependence of threshold voltage on the degree of inhomogeneity of the electric field distribution is altered profoundly at this point. Let us analyze the reasons why it behaves in such a counterintuitive fashion (it would be logical to assume that an increase in the degree of field inhomogeneity (i.e., a reduction in R) makes it easier for electrons to enter the runaway regime and, consequently, induces a monotonic U_c reduction).

Let us examine the runaway conditions for electrons analytically. Naturally, an electron starting from a cathode needs to enter a supercritical field to undergo a transition to the runaway regime. This yields the following simple condition at the cathode: $E(0) > E_c$. It follows readily from (3) that the same condition in voltage terms is

$$U > U_{loc}(R) \equiv E_c R\ln(1 + 2D/R)/2. \qquad (5)$$

The corresponding runaway criterion is local in nature, since it is specified by the field at the electron origin site.

Condition (5) may be insufficient for an electron to undergo continuous acceleration within the entire gap if the field within the bulk of this gap is weaker than E_c. Let us examine the behavior of an electron in formal limit $z \to \infty$. The potential and intensity of the electric field are $\varphi \propto \ln z$ and $E \propto 1/z$. Let us assume that an electron is accelerated continuously (this occurs if electric force eE dominates over friction force F at large z). The electron energy is then defined approximately by the potential difference over which it has traveled: $\varepsilon \approx e\varphi \propto \ln z$. Friction force (4) is estimated in this case as $F \propto \varepsilon^{-1} \propto 1/\ln z$. At the same time, the force accelerating an electron is estimated as $eE \propto 1/z$. According to these estimates, $F \gg eE$ at $z \to \infty$ and, consequently, our assumption that the electron runs away at the periphery is incorrect. At a sufficient distance from the cathode, an electron decelerates and thermalizes. The process of deceleration should start „beyond the anode" for the runaway regime to be established in a finite-size gap. The runaway threshold then corresponds to condition $d\varepsilon/dz|_{z=D} = 0$; i.e., the electron energy reaches its maximum at the anode or, equivalently, the force acting on it vanishes: $eE(D) = F(\varepsilon(D))$. The entire past history of electron motion in a gap ($0 < z < D$) needs to be taken into account to determine the electron energy at the anode. The discussed runaway criterion is nonlocal in this respect.

Estimate U_{nl} for the threshold voltage under the nonlocal criterion may be obtained by setting the RAE energy at the anode to eU (i.e., using the $eE(D) = F(eU)$ condition). With (2) and (4) taken into account, this yields the following transcendental equation for U_{nl}:

$$\frac{eU_{nl}^2}{(D+R/2)E_c\varepsilon_c} = \ln\left(1+\frac{2D}{R}\right)\ln\left(\frac{\mathrm{e}eU_{nl}}{\varepsilon_c}\right). \quad (6)$$

Dotted and dashed curves in Fig. 2 represent the dependences of U_{loc} and U_{nl} on R (formulae (5) and (6)) corresponding to local and nonlocal runaway criteria, respectively. It can be seen that they approximate closely the dependence of U_c on R calculated numerically (solid curve). At $R > R_c \approx 475\,\mu$m, the local runaway criterion is more rigorous than the nonlocal one. If the near-cathode field exceeds critical value E_c, an electron is accelerated continuously within the entire gap. In contrast, the nonlocal criterion, which is concerned with the behavior of an electron in the region of a weak field (at the anode), is more stringent at $R < R_c$. It specifies the observed anomalous shape of the dependence of threshold voltage on the degree of field inhomogeneity.

Threshold R_c effectively separates the regions of strongly inhomogeneous ($R < R_c$) and weakly inhomogeneous ($R > R_c$) fields where the transition of electrons to the runaway regime is governed by two different criteria (nonlocal and local, respectively). The boundary between these regions is specified by condition $U_{nl}(R_c) = U_{loc}(R_c)$. Assuming that $D \gg R_c$ and neglecting weak logarithmic dependences in (5) and (6), we arrive at a simple relation for the problem parameters:

$$eE_cR_c^2 \propto \varepsilon_c D.$$

The following expression for calculation of the threshold may be used in practice:

$$R_c \approx 2.4\sqrt{\varepsilon_c D/(eE_c)}.$$

Thus, it was demonstrated that the dependence of U_c on R changes radically at threshold value R_c of the needle tip radius. This change is attributable to the fact that one runaway criterion is replaced by another. At $R > R_c$ (weakly inhomogeneous field distribution in a gap), the runaway threshold is set by the local field distribution near the cathode tip. At $R < R_c$ (strongly inhomogeneous distribution), the runaway condition is nonlocal in character: the outcome is defined by the behavior of electrons in the periphery (i.e., in the region of a weak field).

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Conflict of interest

The authors declare that they have no conflict of interest.

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