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Influence of slow dynamics effects on the elastic relaxation properties of polycrystalline metal rods

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A theoretical model for the formation of ultrasonic signals in metallic microcrystalline rods taking into account the metastable behavior of their defective states is proposed. The influence of metastable states of the defective structure of samples on the features of changes in their resonant frequencies in ultrasonic experiments of fast dynamics is analyzed. The increase in the Young's modulus and the dynamics of changes in resonant vibrations of rods made of aluminum alloy D16T under conditions of free relaxation are explained. Based on the results obtained, the concentration of metastable defects was estimated..

Keywords: nonlinear resonant ultrasonic spectroscopy, defect structure, mechanical stresses, effects of fast and slow dynamics.

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Considerable attention is being paid at present to the examination of non-steady deformation processes in materials with a complex rheological structure. Metals and their alloys with a microcrystalline structure, rocks, and ceramics belong to this class of materials. Mesoscale structure nonuniformities in these materials exert a considerable influence on their elastic properties [1,2]. This translates into the emergence of acoustic and elastic effects that cannot be explained within the common theory of elasticity. Relaxation processes of various types [3–5] need to be taken into account in order to characterize these effects. We have demonstrated that such processes allow one to characterize correctly the experimental data obtained under excitation of acoustic vibrations in aluminum membranes by non-steady laser radiation [6,7]. This approach also provided an explanation for the specific features of behavior of laser ultrasonic signals in stressed D16T alloy samples [8,9] and fast dynamics effects in vibrations of plastically deformed rods made of this alloy [10].

Fast and slow dynamics effects were revealed in [11] in ultrasonic experiments with plastically deformed D16T aluminum rods. Fast dynamics effects were observed in the process of variation of the resonant frequency of rods with acoustic vibrations of a given amplitude and frequency applied to one of their ends (see the figure), while slow dynamics effects were detected after the vibrations ceased and the rods entered the free relaxation mode. In the fast dynamics mode, their resonant frequency decreased, reaching a certain steady-state value that depended on the amplitude of applied vibrations. In the slow dynamics mode, the resonant frequency was found to increase slightly in [11]. Fast dynamics effects were observed in D16T rods within the 9−10 kHz range of acoustic frequencies, while slow dynamics effects were confined to a significantly lower-frequency range and were characterized by a relaxation time of 534 s [11]. Since only the fast dynamics case was analyzed theoretically in our study [10], it appears reasonable to extend the proposed model to the slow dynamics case.

An approach based on the theory of acoustoplastic effect in solids [12–14] was used to furnish an explanation for the fast dynamics effects in [10]. The following equation is normally used in the theory of acoustoplastic effect to characterize the dynamics of temporal variation of stresses in a sample:

$$\frac{1}{E}\frac{\partial\sigma}{\partial t} = \dot{\varepsilon} - \dot{\varepsilon}_p, \qquad (1)$$

where E is the Youngs modulus of a material; $\dot{\varepsilon}$ is the rate of variation of object deformation, which is often specified by a certain external impact; and $\dot{\varepsilon}_p$ is the rate of variation of plastic deformation of a material. The value of $\dot{\varepsilon}_p$ is defined by relation

$$\dot{\varepsilon}_p = \dot{\varepsilon}_0 \exp\left(-\frac{U - \Omega(\sigma - \sigma_p)}{k_b T} - \frac{t}{\tau}\right),$$

where U is the activation energy of metastable defects, τ is the time of free stress relaxation, σ_p is the internal stress in a sample due to the presence of defects, $\dot{\varepsilon}_0$ is the pre-exponential factor, Ω is the activation volume of a defect, k_b is the Boltzmann constant, and T is the sample temperature.

It should be noted that parameter $\dot{\varepsilon}_0$ in Eq. (1) is normally assumed to be constant. At the same time, it may depend on the nature of external impact on a sample. Specifically, plastically deformed samples in fast-dynamics experiments in [11] were subjected to fairly strong vibrations, which facilitated a gradual reduction in the concentration of defects with their subsequent relaxation to a certain new quasi-equilibrium state with a corresponding stress decrease in samples. Therefore, the rate of deformation variation in such experiments is $\dot{\varepsilon}_0 > 0$. In experiments with slow relaxation, samples were subjected to vibratory inputs for a certain period of time. These vibrations then ceased, and the resonant properties of samples were examined in the free relaxation mode. A certain restoration of the defect structure occurred in this case in the samples after the abatement of vibrations, and $\dot{\varepsilon}_0 < 0$.

No externally induced deformations are present in rods in the free relaxation mode. However, a certain deformation due to relaxation of the defect subsystem and stresses may still be present; therefore, we assume that $\dot{\varepsilon} = \dot{\varepsilon}_r(t)$ in Eq. (1). If one assumes that the vibrational impact ceased at time point $t = 0$ and the rod then entered the free relaxation mode, the stress in it may be written, in accordance with Eq. (1), as

$$\sigma(t) = E\big(\varepsilon_r(t) - \varepsilon_r(0)\big) + \sigma_p^{(0)} + \Delta\sigma_p(t), \qquad (2)$$

where

$$\Delta\sigma_p(t) = -\frac{k_bT}{\Omega}\ln\Bigg[1 - \frac{\Omega E}{k_bT}\dot{\varepsilon}' \times \int_0^t \exp\bigg(-\frac{t'}{\tau} - \frac{\Omega\big(\sigma_p(t') - \sigma_p(0)\big)}{k_bT} - \frac{\Omega E\big(\varepsilon_r(t') - \varepsilon_r(0)\big)}{k_bT}\bigg)dt'\Bigg],$$

$$\varepsilon' = \dot{\varepsilon}_0 \exp\bigg(-\frac{U}{k_bT}\bigg).$$

It was demonstrated in [15] that, alongside with elastic deformations, defects in a sample affect the magnitude of stresses in it. In the simplest case of one-dimensional deformation, this relation takes the form

$$\sigma = E\varepsilon_e + \Omega E n, \qquad (3)$$

where ε_e is elastic deformation and n is the concentration of defects in a sample.

With the variation of the defect concentration taken into account, relation (3) in the one-dimensional model for longitudinal rod vibrations yields the following equation of motion:

$$\rho\frac{\partial^2\varepsilon_e}{\partial t^2} = E\frac{\partial^2\varepsilon_e}{\partial x^2} + E\Omega\frac{\partial^2\Delta n}{\partial x^2}, \qquad (4)$$

where ρ is the density of the rod material, Δn is the variation of the defect concentration upon excitation of elastic vibrations, and x is the axial coordinate.

In the general case, Eq. (4) is nonlinear, since the defect concentration may depend on stresses. One needs to know the dependence of concentration Δn on stresses to transform Eq. (4) further. If we assume that the diffusion of defects is negligible at room temperature, the concentration of defects involved in the process may be determined using the following equation:

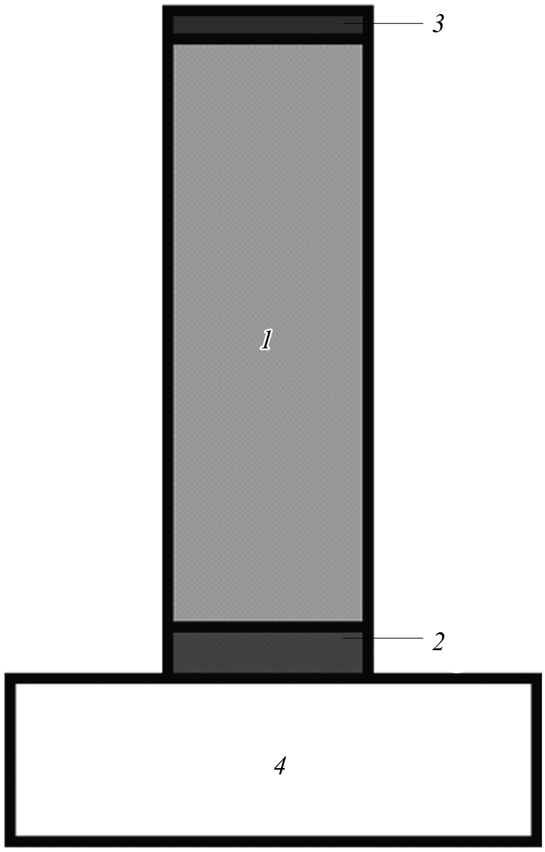

Diagram of excitation and measurement of elastic vibrations of the rod [7]. *1* — Sample, *2* — exciting piezoceramic transducer, *3* — piezoceramic vibration detection sensor, and *4* — massive base.

$$\frac{\partial\Delta n}{\partial t} + \frac{\Delta n}{\tau} = J, \qquad (5)$$

where J is a volume defect source in a sample defined by the Arrhenius law

$$J = \frac{N}{\tau}\Bigg[\exp\bigg(-\frac{U - \Omega(\sigma_p^{(0)} + \Delta\sigma_p + \sigma_e)}{k_bT}\bigg) - \exp\bigg(-\frac{U - \Omega(\sigma_p^{(0)} + \sigma_e)}{k_bT}\bigg)\Bigg],$$

N is a quantity on the order of the concentration of atoms in the sample material, $\sigma_p^{(0)}$ is the internal stress in the sample at the onset of free relaxation, and $\Delta\sigma_p(t)$ is the stress variation in the sample in the process of relaxation of defects.

Following [6–10], we assume that the concentration of defects in a sample in the free relaxation mode varies in a quasi-equilibrium fashion. The variation of concentration of defect centers involved in the process of rod relaxation may then be estimated as

$$\Delta n \cong N\left[\exp\left(-\frac{U-\Omega(\sigma_p^{(0)}+\Delta\sigma_p+\sigma_e)}{k_bT}\right)\right.$$
$$\left.-\exp\left(-\frac{U-\Omega(\sigma_p^{(0)}+\sigma_e)}{k_bT}\right)\right]. \quad (6)$$

The activation volume of a defect is usually commensurate with the lattice cell volume of a material [16]. Therefore, we set it to $10^{-28}\,\mathrm{m}^3$ for the aluminum alloy. At stresses on the order of 10 MPa and deformations below 10^{-3}, the last two exponents under the integral sign in expression (2) are then near-unity and may be neglected. If the indicated conditions are satisfied, it may also be assumed that $\Omega E\dot{\varepsilon}\tau < k_bT$. Using expression (2), we then write the relaxation law for stress $\Delta\sigma_p(t)$ in the form

$$\Delta\sigma_p(t) \cong E\dot{\varepsilon}'\tau\left(1-e^{-\frac{t}{\tau}}\right), \quad (7)$$

while the concentration of excited defects is given by

$$\Delta n \cong N'\frac{\Omega E}{k_bT}\dot{\varepsilon}'\tau\left[1-\exp\left(-\frac{t}{\tau}\right)\right]\exp\left(\frac{\Omega\sigma_e}{k_bT}\right), \quad (8)$$

where $N' = N\exp\left(-(U-\Omega\sigma_p^{(0)})/k_bT\right)$.

The nature of variation of the resonant frequency of a rod probed by a weak acoustic signal was examined in [11]. If $\Omega\sigma_e \leqslant k_bT$ is assumed in relation (8), equation of motion (4) may be transformed to

$$\rho\frac{\partial^2\varepsilon_e}{\partial t^2} = E_{eff}(t)\frac{\partial^2\varepsilon_e}{\partial x^2}, \quad (9)$$

where

$$E_{eff}(t) \cong E\left[1+\Omega N'\dot{\varepsilon}'\tau\left(\frac{\Omega E}{k_bT}\right)^2\left(1-\exp\left(-\frac{t}{\tau}\right)\right)\right].$$

The samples in [11] were excited in the quarter-wave elastic resonator mode. Its first resonant frequency is given by $f_0 = \sqrt{E/\rho}/(4L)$, where ρ is density and L is the sample length. If we assume that during the experiments the density and length of the samples did not change, the influence of the effects of fast dynamics on the resonant frequency of sample vibrations may be estimated using this equality at $E = E_{eff}(t)$. In accordance with the results reported in [11], the relative variation of the rod elasticity modulus due to the presence of defects is small. Therefore, the resonant frequency of the rod varies in the process of free relaxation in the following way:

$$f_0(t) \cong \frac{1}{4L}\sqrt{\frac{E}{\rho}}\left[1+\frac{\Omega N'\dot{\varepsilon}'\tau}{2}\left(\frac{\Omega E}{k_bT}\right)^2\left(1-\exp\left(-\frac{t}{\tau}\right)\right)\right]. \quad (10)$$

Thus, owing to a partial restoration of the defect subsystem, the resonant frequency increases somewhat in the mode of free relaxation of the deformed rod after the removal of load. If we present this frequency in the form used in [11]:

$$f_0(t) = f_0(t\to\infty) - C\exp\left(-\frac{t}{\tau}\right), \quad (11)$$

the coefficient is, in accordance with (10), written as

$$C = \frac{1}{8L}\sqrt{\frac{E}{\rho}}\Omega N'\dot{\varepsilon}'\tau\left(\frac{\Omega E}{k_bT}\right)^2.$$

The obtained theoretical result verifies the conclusions made in [11] regarding the nature of temporal variation of the resonant frequency of aluminum-alloy rods with residual deformations in the free relaxation mode. It was demonstrated in [11] that, if expression (11) is used, $C = 96.4\,\mathrm{Hz}$ provides the closest agreement between the experimental and theoretical data for a rod made of aluminum alloy D16T. Knowing the value of C, one may estimate the concentration of defects involved in the relaxation process.

Aluminum alloy D16T has $\rho = 2700\,\mathrm{kg/m}^3$ and $E = 71\,\mathrm{GPa}$. The length of rods examined in [11] was 0.14 m. With the activation volume of a defect being commensurate with the lattice cell volume of a material, we assume that $\Omega \approx 10^{-28}\,\mathrm{m}^3$ and $\dot{\varepsilon}'\tau \approx 10^{-5}$. If the value of coefficient C is known, we may then estimate the concentration of defects involved in the relaxation process. An estimate of N' corresponding to the presented data is $N' \cong 10^{25}\,\mathrm{m}^{-3}$. This value matches the concentration of defects in stressed D16 alloy samples that was determined in our experiments on laser generation of ultrasound [8–10].

The proposed theoretical model provides an explanation for the effects of slow dynamics in metal rods with defects. It relates the dynamics of temporal variation of the Young's modulus of a material with such characteristics of its defect subsystem as the density of defects, their relaxation time, and activation volume.

Conflict of interest

The authors declare that they have no conflict of interest.

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