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Interaction of H-wave with the thin metal layer with generalized boundary conditions

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The incidence of electromagnetic H-wave on the thin metal layer at the angle of theta is considered, taking into account the dependence of the specularity coefficients on the surfaces of the thin layer q_1 and q_2 on the angle of incidence of electrons. The behavior of reflection, transmission and absorption coefficients depending on the thickness of the thin layer is analyzed. The obtained results are compared with experimental data.

Keywords: electromagnetic wave, thin metal layer, conductivity, reflection coefficient.

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Introduction

The electromagnetic properties of thin metal films can differ significantly from the properties of bulk samples [1–9]. In the case under consideration, the thickness of the thin metal layer a does not exceed the thickness of the skin layer δ and is comparable to the mean free path of electrons in the metal Λ. It is known that the thickness of the skin layer strongly depends on the frequency of the external field ω: as the frequency increases δ decreases [10]. In our case ω does not exceed $7 \cdot 10^{11}$ Hz and the inequality $a < \delta$ is satisfied, so the skin effect is not taken into account. Since the thickness of the thin layer is much greater than the de Broglie wavelength, which in typical metals is about the interatomic distance (~ 0.3 nm), the quantum effects can be neglect ed.

1. Electromagnetic H-wave and thin metal layer

Let us consider the thin metal layer with thickness a, specularity coefficients q_1 and q_2 when electrons are reflected respectively from the upper and lower surfaces of this layer, in the case of electromagnetic H-wave incident on it at an angle θ. Since the vector of the electric field of an electromagnetic wave is parallel to the surface of the thin layer, then such wave is called H-wave. The electric field of the electromagnetic wave is parallel to the coordinate axis Y, the coordinate axis X is directed deep into the thin metal layer. Let us clarify that the electric field $\mathbf{E}$ is uniform and periodic in time:

$$\mathbf{E} = \mathbf{E}_0 \exp(-i\omega t).$$

Here ω — frequency of electric field.

The behavior of the electromagnetic field inside the thin metal layer can be described by the following system of equations [11]:

$$\begin{cases} rot\,\mathbf{H} = \mathbf{j} + \varepsilon\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \\ rot\,\mathbf{E} = -\mu\mu_0 \frac{\partial \mathbf{H}}{\partial t}. \end{cases} \tag{1}$$

Here $\mathbf{j}$ — electric current density,μ and μ_0 — permeability of the medium and magnetic constant; ε and ε_0 — permittivity of the medium and electrical constant.

Write according to definition the curl of the vectors $\mathbf{E}$ and $\mathbf{H}$, leaving only the projections E_y and H_z in accordance with the choice of coordinate axes for the thin metal layer:

$$rot\,\mathbf{E} = -\frac{\partial E_y}{\partial z}\mathbf{i} + \frac{\partial E_y}{\partial x}\mathbf{k},$$

$$rot\,\mathbf{H} = -\frac{\partial H_z}{\partial y}\mathbf{i} + \frac{\partial H_z}{\partial x}\mathbf{j}. \tag{2}$$

Since in our case $\frac{\partial E_y}{\partial z} = 0$, the first equation of system (2), taking into account the second equation of system (1), will take the form

$$\frac{\partial E_y}{\partial x} = -\mu\mu_0 \frac{\partial}{\partial t} H_z$$

$$= -\mu\mu_0 \frac{\partial}{\partial t} H_0 \exp(-i\omega t) = \mu\mu_0 i\omega H_z.$$

Thus, the first equation of the system (1) in SI system will take the final form

$$\frac{\partial E_y}{\partial x} - \mu\mu_0 i\omega H_z = 0.$$

The second equation of the system (2) taking into account that $\frac{\partial H_z}{\partial y} = 0$ can be rewritten as follows:

$$\frac{\partial H_z}{\partial x} = -j_y - \varepsilon\varepsilon_0 \frac{\partial}{\partial t} E_y \exp(-i\omega t).$$

Here j_y — projection of electric current density.

As a result of similar transformations, we can obtain the second equation of the system (1) in SI system:

$$\frac{\partial H_z}{\partial x} + i\varepsilon\varepsilon_0\omega(\sin^2\theta - 1)E_y = -j_y.$$

Here θ — angle of incidence of H-wave on thin metal layer.

So, the system of equations (1) in SI system will obtain the final form:

$$\begin{cases} \frac{\partial E_y}{\partial x} - \mu\mu_0 i\omega H_z = 0, \\ \frac{\partial H_z}{\partial x} + i\varepsilon\varepsilon_0\omega(\sin^2\theta - 1)E_y = -j_y. \end{cases} \tag{3}$$

Interaction of electromagnetic$H$- wave With thin metal layer is characterized by coefficients: reflection R, transmission T and absorption A[12]:

$$T = \frac{1}{4}\left|P^k - P^l\right|^2,$$

$$R = \frac{1}{4}\left|P^k + P^l\right|^2,$$

$$A = 1 - T - R. \tag{4}$$

P^k and P^l are related to the surface impedances Z^k and Z^l on the lower surface of the thin metal layer and the wave incidence angle θ. They are defined as follows [12]:

$$P^k = \frac{Z^k\cos\theta - 1}{Z^k\cos\theta + 1},$$

$$P^l = \frac{Z^l\cos\theta - 1}{Z^l\cos\theta + 1}. \tag{5}$$

Z^k corresponds to the antisymmetric electric field configuration of the external field: $E_y(0) = -Ey(a)$, $H_z(0) = H_z(a)$, and Z^l — symmetrical external field configuration: $E_y(0) = E_y(a)$, $H_z(0) = -H_z(a)$. The impedance in both cases has the form

$$Z^k = Z^l = \frac{E_y(0)}{H_z(0)}. \tag{6}$$

The electric and magnetic fields will change very little at distances smaller than the skin-layer depth ($a < \delta$). In the case of an antisymmetric configuration of the external field with respect to the electric field, when $H_z(0) = H_z(a)$, we can take the value H_z as constant, and the change in the electric field with thickness can be determined from the first equation of the system (3):

$$E_y(a) - E_y(0) = \mu\mu_0 i\omega a H_z. \tag{7}$$

Accounting for the antisymmetric nature of the electric field reduces expression (7) to the form

$$E_y(0) = -\frac{\mu\mu_0 i\omega a H_z}{2}.$$

Taking into account the expression (6) for the impedance, in the case of the antisymmetric external field configuration with respect to the electric field, we obtain

$$Z^k = -i\mu\mu_0\omega a/2. \tag{8}$$

In the case of the symmetric configuration of the external field with respect to the electric field, when $E_y(0) = E_y(a)$, we can take the value E_y as constant, and the change in the magnetic field with thickness can be determined from the second expression of the system of equations (3):

$$H_z(a) - H_z(0) = -i\varepsilon\varepsilon_0(\sin^2(\theta) - 1)E_y - \int_0^a j_y dx. \tag{9}$$

Let us introduce the electrical conductivity, which will be averaged over the thickness of the thin metal layer a:

$$\sigma_a = \frac{1}{E_y a}\int_0^a j_y(x)dx = \frac{1}{a}\int_0^a \sigma(x)dx. \tag{10}$$

For the electric current density, we have

$$j_y = \sigma(x)E_y.$$

Here $\sigma(x)$ is the local electrical conductivity of the layer.

Taking into account the expression (10) and the symmetry of the magnetic field, the expression (9) will take the form

$$H_z(0) = -\frac{1}{2}i\varepsilon\varepsilon_0\omega a(\sin^2(\theta) - 1)E_y + a\sigma_a E_y.$$

Taking into account (6) in the case of the external field configuration symmetric with respect to the electric field, we obtain

$$Z^l = \frac{2}{-i\varepsilon\varepsilon_0\omega a(\sin^2(\theta) - 1) + 2a\sigma_a}. \tag{11}$$

Now suppose that the wavelength of the incident radiation is much greater than the thickness of the layer a. In this case $\mu\mu_0\omega a \ll 1$, $\varepsilon\varepsilon_0\omega a \ll 1$ or $\mu\mu_0 2\pi c a/\lambda \ll 1$, $\varepsilon\varepsilon_0 2\pi c a/\lambda \ll 1$ will be met. Estimates of the wavelength λ and frequency of the electromagnetic field ω for the considered thin metal layer are as follows: $\lambda \sim 2.7 \cdot 10^{-3}$ m, $\omega \sim 7 \cdot 10^{11}$ Hz.

Then, the expressions for the impedances (8) and (11) will take the form

$$Z^k = 0,$$

$$Z^l = \frac{1}{a\sigma_a}. \tag{12}$$

In accordance with (12), the expression (5) will take the form

$$P^k = -1,$$

$$P^l = \frac{\cos\theta - a\sigma_a}{\cos\theta + a\sigma_a}. \tag{13}$$

Taking into account (13) and (5), the expressions for the coefficientsIFx70x E 4) will take the form

$$T = \left| \frac{\cos\theta}{\cos\theta + a\sigma_a} \right|^2,$$

$$R = \left| \frac{a\sigma_a}{\cos\theta + a\sigma_a} \right|^2,$$

$$A = 1 - T - R, \quad (14)$$

where σ_a has the form [13]:

$$\sigma_a = \frac{3}{4}\frac{\sigma_0}{\tau}\frac{1}{(1/\tau - i\omega)}\int_0^{\frac{\pi}{2}} \sin^3\alpha \Bigg[2a + \left[-\frac{a\cos(\alpha)(\exp(-\Omega/\cos\alpha) - 1)}{\Omega}\right] \times \left[\frac{q_1\left[1 - \exp(-\Omega/\cos\alpha) + q_2\exp(-\Omega/\cos\alpha)\right]}{1 - q_1 q_2 \exp(-2\Omega/\cos\alpha)} + \frac{q_1\left[1 - \exp(-\Omega/\cos\alpha) + q_1\exp(-\Omega/\cos\alpha)\right]}{1 - q_1 q_2 \exp(-2\Omega/\cos\alpha)}\right]\Bigg] d\alpha. \quad (15)$$

Here $\sigma_0 = ne^2\tau/m$, n — concentration of conduction electrons, e — electric charge of electron, τ — electron relaxation time, m — electron mass, $\Omega = \frac{a}{v_F}\left(\frac{1}{\tau} - i\omega\right)$, q_1 and q_2 — specularity coefficients of the surfaces of the thin layer, when electrons are reflected from its surfaces.

The relationship between the electric field strength **E** and the current density **j** in the case when the layer thickness a is comparable to the mean free path of electrons in the metal λ turns out to be mainly nonlocal. To describe the relation between **E** and **j**, the paper [13] used the Boltzmann kinetic equation [14] (in the relaxation time approximation) for a degenerate Fermi gas of conduction electrons located in the thin layer:

$$\frac{\partial f}{\partial t} + v_z\frac{\partial f}{\partial z} + \frac{eE}{m}\frac{\partial f_0}{\partial v_x} = -\frac{f - f_0}{\tau}.$$

Here f_0, f, e, m, v_z, v_x, τ — respectively the FermiIFx87x Dirac function, electron distribution function in the presence of external electric field, electron charge, its mass, velocity projections of conduction electrons, electron relaxation time.

This equation can be linearized with respect to the field **E** and small deviations f_1 from the Fermi–Dirac equilibrium electron distribution function f_0 $(f = f_0 + f_1)$), and as a result the following equation is obtained:

$$v_z\frac{\partial f_1}{\partial z} - \frac{eE}{m}\frac{\partial f_0}{\partial v_x} - i\omega f_1 = -\frac{f_1}{\tau}. \quad (16)$$

Solving equation (16) and taking into account Fuchs specular-diffuse boundary conditions [13,15], the expression (15) was obtained for the electrical conductivity of the thin metal layer, averaged over the thickness IFx6x E of this layer.

However, it was not taken into account that, in the general case, the specularity coefficients q_1 and q_2 may depend on the angle of incidence of electrons from the surface of the thin layer. The model of specular-diffuse boundary conditions for electrons on the surface of the thin metal layer, taking into account the dependence of the specularity coefficient on the angle of incidence of electrons α, was first proposed in the paper [16], where the expression for the specularity coefficient q was obtained

$$q(\alpha) = q_0 + (1 - q_0)\exp(-b_1\cos\alpha - b_2\cos^2\alpha). \quad (17)$$

Here b_1 and b_2 — are some positive coefficients, when $b_1 \gg 1$ and $b_2 \gg 1$, then $q(\alpha) = q_0$ — we get the Fuchs model, and if $q_0 = 0$ and IFx10 0xE — we get the Soffer model.

In our case, taking into account that the specularity coefficients of the surfaces of the thin layer can be different, expression (17) will take the form

$$q_1 = q_{01} + (1 - q_{01})\exp(-b_1\cos\alpha - b_2\cos^2\alpha),$$

$$q_2 = q_{02} + (1 - q_{02})\exp(-b_1\cos\alpha - b_2\cos^2\alpha). \quad (18)$$

Here q_{01} and q_{02} is the value of the reflection coefficients of the surfaces of the thin layer.

Thus, in further analysis of the behavior of the reflection coefficients R, transmission coefficients T, and absorption coefficients A, it is necessary to take into account in the expression (15) for the conductivity of the thin layer the dependence of the specularity coefficients on the angle of incidence of electrons α (expression (18)).

Consider the case of a thick metal layer, when its thickness a is much greater than the mean free path of electrons in the metal Λ. Let us also assume that $b_1 \gg 1$ and $b_2 \gg 1$, then the expression for the electrical conductivity of the thin metal layer σ_a will take the form

$$\sigma_a = a\frac{\sigma_0}{1 - i\omega\tau}.$$

We obtained a classical result for the conductivity of the thick layer — the Drude formula.

2. Discussion of results

Consider the behavior of the reflection coefficients R, transmission coefficients T and absorption coefficients A in the case of a copper layer of thickness a. For further calculations we will use the parameters for copper from [17]. In particular, the electron relaxation time is $\tau = 1.9 \cdot 10^{-14}$ s.

Let us compare the obtained results with the experimental data of the paper [18]. In this paper, the resistivity of thin copper films was studied. Taking the experimental values of the specific conductivity of the thin copper film for the specified thickness, it is possible to calculate the experimental values of the reflection coefficients R, transmission coefficients T and absorption coefficients A.

The obtained values for the coefficients R, T and A are comparable with the theoretical results of this paper. Let us specify that for the Fuchs model the values of the specularity coefficients q_{01} and q_{02} were taken from [19].

It can be seen in Fig. 1 that the variation of the specularity coefficients of the thin metal layer has a significant effect on the behavior of the reflection coefficient R. With the increase in the thickness of the thin metal layer, the value of the reflection coefficient increases R.

It can be observed in Fig. 2 that the angle of incidence of H-wave θ on thin metal layer also makes a significant contribution to the behavior of the reflection coefficient R.

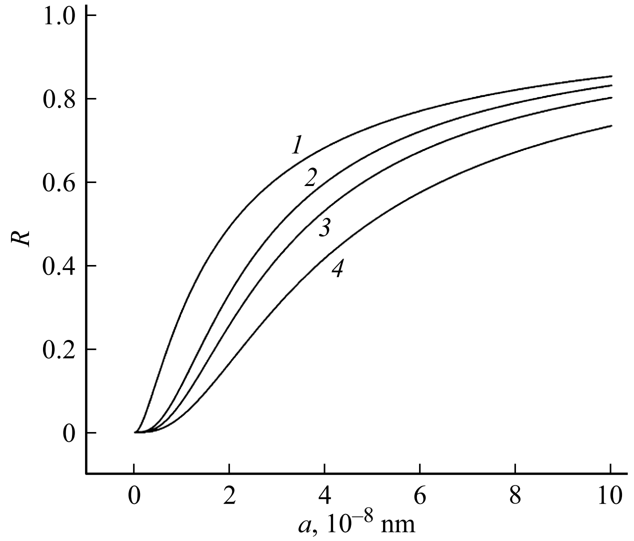

Figure 1. Reflection coefficient R vs. thickness of thin metal layer a. For all curves $\omega = 0$, $\theta = 60°$, $b_1 = b_2 = 10$. Curve *1* — $q_{01} = 1$, $q_{02} = 1$; curve *2* — $q_{01} = 0.5$, $q_{02} = 0.8$; curve *3* — $q_{01} = 0.3$, $q_{02} = 0.5$; curve *4* — $q_{01} = 0$, $q_{02} = 0$.

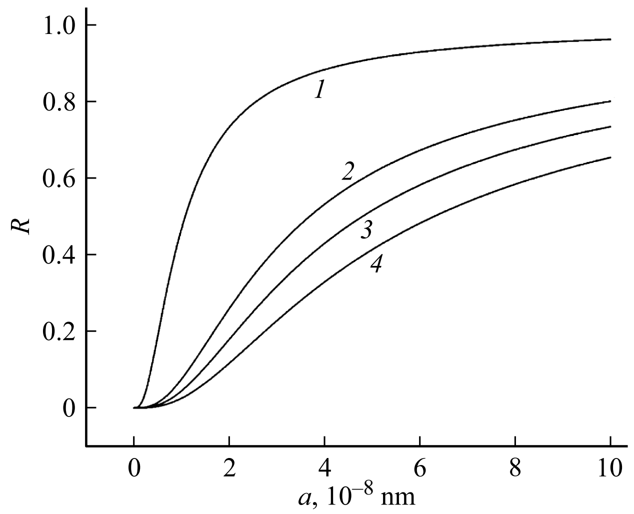

Figure 2. Reflection coefficient R vs. thickness of thin metal layer a. For all curves $\omega = 0$, $b_1 = b_2 = 10$, $q_{01} = 0.3$, $q_{02} = 0.5$. Curve *1* — $\theta = 85°$; curve *2* — $\theta = 60°$; curve *3* — $\theta = 45°$; curve *4* — $\theta = 0°$.

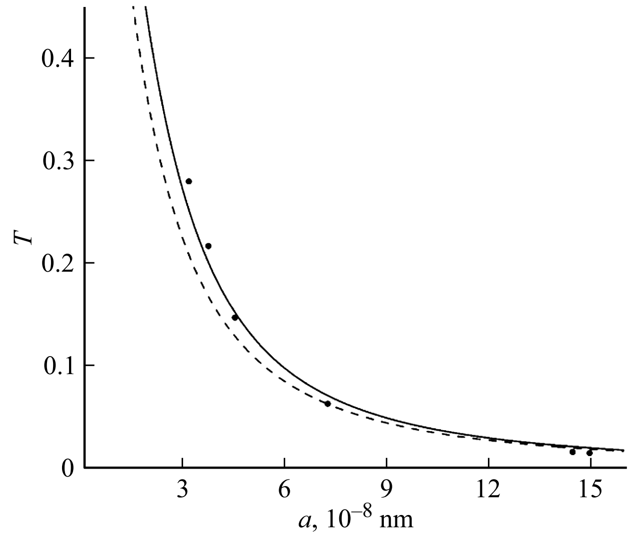

Figure 3. Transmission coefficient T vs. thickness of thin metal layer a. Solid curve — model of generalized boundary conditions. Dashed curve — Fuchs model. Dots on the graph — experimental. For solid curve: $\omega = 0$, $\theta = 0°$, $b_1 = 100$, $b_2 = 0, 1$, $q_{01} = 0$, $q_{02} = 0$. For dashed curve: $\omega = 0$, $\theta = 0°$, $q_{01} = q_{02} = 0.5$.

Relative error of the results presented in Fig. 3.

Thickness of thin of copper layer a, nm	Relative error Δ, % model of generalized boundary conditions	Relative error Δ, % Fuchs model
31.7	12.1	35.4
37.5	9.6	33.1
45.3	2.7	15.2
72.7	9.4	0.5
145	4.2	4.1
150	4.1	4

In particular, the smaller the angle of incidence of H-wave θ is, the greater the rate of increase in the reflection coefficient R is with thickness increasing of the thin layer.

Fig. 3 compares the theoretical model of the present paper with the experimental data of the paper [18]. In the paper [18], the electromagnetic properties of thin copper films were studied in the static case, when$\omega = 0$, and when the angle of incidence of the electromagnetic wave is $\theta = 0°$. Thus, the results obtained using the generalized boundary conditions are in better agreement with the experimental data than the results obtained using the Fuchs boundary conditions (see Table).

Conclusion

Thus, expressions for the reflection coefficients R, transmission coefficients T, and absorption coefficients A are obtained for the first time, when the specularity coefficients q_{01} and q_{02} depend on the angle of incidence of electrons. The behavior of the reflection coefficient R is studied in the case of its dependence on the thickness of thin metal layer a. The variation of the specularity coefficients q_{01} and q_{02} with their dependence on the electron reflection angle makes a significant contribution to the conductivity of the thin layer and, consequently, to the behavior of the reflection coefficients R, transmission coefficients T and absorption coefficients A. As a result of the calculations it was found that the obtained theoretical model of generalized boundary conditions more accurately describes the behavior of the reflection coefficients R, transmission coefficients T and absorption coefficients A, in comparison with the Fuchs mirror-diffuse model, if we take into account that the specularity coefficients q_{01} and q_{02} depend on the electron incidence angle. In particular, the results for the transmission coefficient T are consistent with the experimental data within the error of the calculation method.

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] T.A. Vartanyan, N.B. Leonov, S.G. Przhibel'skii. J. Opt. Technol., **80** (2), 3 (2013). DOI: 10.1364/JOT.80.000088
[2] E.N. Kotlikov, V.N. Prokashev, V.A. Ivanov. Opt. Spectr., **108** (6), 1003 (2010). DOI: 10.1134/S0030400X10060196
[3] V.V. Kaminskii, N.N. Stepano, M.M. Kazanin, A.A. Molodykh, S.M. Solov'ev, V.A. Sidorov. Physics of the Solid State, **55** (5), 991 (2013). DOI: 10.1134/S106378341302011X
[4] E.N. Kotlikov, A.N. Tropin. J. Opt. Technol., **87** (1), 56 (2020). DOI: 10.1364/JOT.87.000045
[5] E.V. Vashchenko, I.A. Gladskikh, S.G. Przhibel'skiĭ, V.V. Khromov, T.A. Vartanyan. J. Opt. Technol., **80** (5), 24 (2013). DOI: 10.1364/JOT.80.000263
[6] W.E. Jones, K.L. Kliewer, R. Fuchs. Phys. Rev., **178**, 1201 (1969). DOI:10.1103/PhysRev.178.1201
[7] F.A. Korolev, V.I. Gridnev. Radiotekhnika i elektronika. 1718 (1965). (in Russian).
[8] S.G. Bezhanov, A.A. Ionin, A.P. Kanavin. J. Experiment. Theor. Phys., **120** (6), 937 (2015). DOI: 10.1134/S1063776115050106
[9] A. Paredes-Juarez, F. Dias-Monge, N.M. Makarov, F. Perez-Rodriguez. JETP Lett., **90** (9), 687 (2009). DOI: 10.1134/S0021364009210073
[10] E.M. Lifshitz, L.P. Pitaevskiy. *Teoreticheskaya fizika. T.X. Fizicheskaya kinetika* (Fizmatlit, M., 2002) (in Russian)
[11] D.I. Penner, V.A. Ugarov. *Elektrodinamika i spetsialnaya teoriya otnositelnosti* (Prosveschenie, M., 1980) (in Russian)
[12] A.V. Latyshev, A.A. Yushkanov. Russian Microelectronics, **41** (1), 25 (2012). DOI: 10.1134/S1063739712010076
[13] A.I. Utkin, A.A. Yushkanov, E.V. Zavitaev. Journal of Surface Investigation: X-ray, Synchrotron and Neutron Techniques, 10, 5 (2016). DOI: 10.1134/S1027451016050153
[14] A.A. Abrikosov. *Osnovy teirii metallov: uchebnoe rukovodstvo* (Nauka, M., 1987) (in Russian)
[15] A.V. Latyshev, A.A. Yushkanov. *Entsiklopediya nizkotemperaturnoi plazmy*. (Yanus-K, M, 2008), t. VII-1, gl. 10. (in Russian)
[16] F.A. Karimov, A.A. Yushkanov. Vestnik MGOU. Seriya: Fizika-matematika **1**, 50 (2020)(in Russian). DOI: 10.18384/2310-7251-2020-1-50-56
[17] N.A.Ashkroft, N.D.Mermin. *Fizika tverdogo tela* (Mir, M., 1979), t. 1. (in Russian)
[18] Sun Tik, Yao Bo, P. Warren Andrew, Kumar Vineet, Roberts Scott, Barmak Katayun, R. Coffey Kevin. J. Vac. Sci. Technol. A, **26**, 605 (2008). DOI: 10.1116/1.2938395
[19] A. Bid, A. Bora, A.K. Raychaudhuri. Phys. Rev. B, **74**, 035426-1-9 (2006).

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