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Analytical solution of the non-monochromatic directed radiation transfer in a resonantly absorbing medium

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In this work, a stationary problem analytical solution of the non-monochromatic directional radiation transfer through a medium filled with alkali metal atoms and a buffer gas is obtained. The three-level atom approximation was used to describe the level population kinetics. The processes of absorption and stimulated emission of photons, collisional mixing of the levels of the alkali metal atom fine structure, and spontaneous transitions were taken into account. The absorption of spontaneous radiation by the medium was not taken into account. Using the example of a medium consisting of potassium vapor and a helium buffer gas, the analytical and numerical solutions are compared.

Keywords: level population kinetics, alkali metal atom, transfer of nonmonochromatic radiation, resonantly absorbing medium.

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1. Introduction

To study problems associated with the operation of alkali metal vapor lasers, the three-level atom approximation is used. To transfer atoms from the ground state $n^2S_{1/2}$ to an excited state, for example, $n^2P_{3/2}$, radiation from laser diodes is usually used. If there is a buffer gas in the medium, then collisions of alkali metal atoms with atoms of the buffer gas lead to mixing of fine structure levels n^2P. If the medium is placed in a resonant cavity, then with sufficient diode radiation intensity between the $n^2P_{3/2}$ and $n^2S_{1/2}$ levels, inversion and lasing can be obtained. Due to spontaneous transitions of alkali atoms from excited states to the ground state, spontaneous radiation is formed in the medium, which can affect the processes occurring in the resonantly absorbing medium [1,2].

The width of the emission spectrum of laser diodes is either comparable or greater than the width of the absorption line of an alkali atom, and the spectrum is well approximated by a Lorentz or Gaussian function [3,4].

For the theoretical study of radiation propagation in a resonantly absorbing medium consisting of alkali metal atoms, the equations of population kinetics and radiation transfer equations are usually used. In the general case, the system of these equations is non-linear. Numerical approaches are usually used to find solutions to such problems. When creating calculation programs, there is a problem related to their verification. As model problems, it is proposed to select those that have analytical solutions [5–8].

The paper reviews the transfer process of directed non-monochromatic (the width of the emission spectrum is greater than the width of the absorption spectrum of the medium) radiation in a medium consisting of three-level atoms. The processes of nonlinear absorption and stimulated emission of radiation, spontaneous transitions of atoms, the dependence of the photon mean free path on the coordinate in which it is located, and the spectral dependence of the absorption and reemission coefficients are taken into account. The center of the emission line may not coincide with the maximum of the absorption line of the medium. The transfer of spontaneous radiation is not taken into account. The resulting analytical solution can be used to test computational programs.

Statement and solution of the problem

Let us review a layer of thickness b, consisting of atoms that resonantly absorb radiation at a wavelength of λ_{13} (transition $S_{1/2} \rightarrow P_{3/2}$), and buffer gas atoms. This layer is irradiated with non-monochromatic directional radiation with an integrated intensity J_{po}. We assume that the radiation does not scatter and propagates in the form of parallel rays. In this case, the problem of radiation transfer can be considered one-dimensional.

Fig. 1 schematically depicts the processes responsible for the population kinetics in a three-level atom. We designate the levels $S_{1/2}$, $P_{1/2}$ and $P_{3/2}$ as 1, 2 and 3, respectively, for brevity.

Fig. 1 shows the following processes: 1) absorption of radiation (P_{13}), 2) stimulated emission of a photon (P_{31}), 3) spontaneous emission of a photon from state 3 (S_{31}), 4) nonradiative transition from state 3 to state 2 due to collisions of an alkali metal atom with buffer gas atoms (M_{32}), 5) transition without absorption of a photon

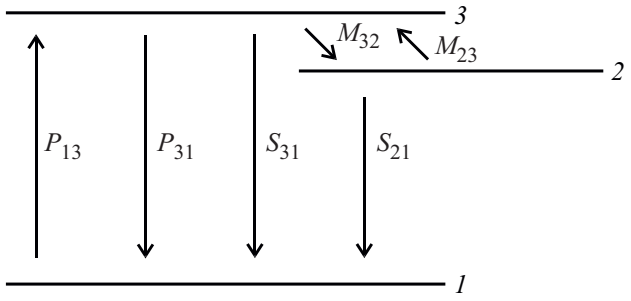

Figure 1. Diagram of population kinetics in a three-level atom.

from state 2 to state 3 due to collisions of a resonantly absorbing metal atom with atoms of a buffer inert gas (M_{23}) (mixing [9]), 6) spontaneous emission of a photon from state 2 (S_{21}).

It is assumed that due to the bleaching effect (in this paper due to the absorption of pump radiation) [2] and the redistribution of photons over frequencies, spontaneous photons are practically not absorbed by the medium [10]. Consequently, the effect of spontaneous emission on the level population kinetics can be neglected.

Let us write down the equations of level population kinetics and the equations of radiation transfer for a three-level atom. The medium interacts with continuous radiation, i. e., the equations of population kinetics and radiation transfer can be considered stationary:

$$\begin{cases} \frac{n_1}{E_{13}}\int\limits_0^\infty J_\nu\sigma_{13,\nu}d\nu - \frac{g_1}{g_3}\frac{n_3}{E_{13}}\int\limits_0^\infty J_\nu\sigma_{13,\nu}d\nu \\ \quad +n_2R_{23} - n_3R_{32} - n_3A_{31} = 0, \\ n_3R_{32} - n_2R_{23} - n_2A_{21} = 0, \\ n_1 + n_2 + n_3 = N_0, \\ \frac{dJ_\nu}{dz} = -(n_1 - g_1/g_3 \cdot n_3)\sigma_{13,\nu}J_\nu, \\ \sigma_{13,\nu} = \frac{g_3}{g_1}\cdot\frac{d^2A_{31}}{4\nu^2}\frac{\Delta\nu_{13}}{2\pi[(\nu-\nu_{13})^2+(\Delta\nu_{13}/2)^2]}. \end{cases} \quad (1)$$

Here n_1, n_2 and n_3 are the concentrations of atoms in states 1, 2 and 3, respectively; g_1, g_2 and g_3 — multiplicities of degeneration of states 1, 2 and 3; E_{13} — photon energy corresponding to the transition $3 \to 1$; J_ν — spectral intensity of radiation (energy of photons passing through a single area per unit time and being in the range from ν to $\nu + \Delta\nu$) at a depth z; $\sigma_{13,\nu}$ — absorption cross section of the $1 \to 3$ transition, which has a Lorentz shape due to the impact broadening mechanism [11]; A_{31}, A_{21} — Einstein coefficients for transitions $3 \to 1$, $2 \to 1$; N_0 — metal vapor concentration; $\Delta\nu_{13}$ — absorption line width; c — speed of light;$\nu_{13} = c/\lambda_{13}$ — photon frequency corresponding to the transition $3 \to 1$; $R_{32} = n_{\text{bg}}V\sigma_{\text{mix}}$ — rate of transitions of atoms from state 3 to state 2; $R_{23} = (g_3/g_2)R_{32}e^{-\Delta E/(kT)}$ — rate of transitions of atoms from state 2 to state 3; $n_{bg} = P_{bg}/(kT)$ — concentration of buffer gas atoms; V — relative velocity of metal atom and buffer gas atom; σ_{mix} — cross section for mixing levels 2 and 3; k — Boltzmann constant; T — medium temperature; ΔE — energy difference between levels 2 and 3.

In the first equation of system (1), the first term — the processP_{13}, shown in Fig. 1, the second — P_{31}, the third — M_{23}, the fourth — M_{32} and the fifth — S_{31}. In the second equation of the system (1) the third term — process S_{21}.

In papers [3,4] it is proposed to use the Gaussian or Lorentz form to approximate the emission spectrum of laser diodes:

$$J_{\nu o} = \begin{bmatrix} \frac{J_{po}\Delta\nu_p}{2\pi}\frac{1}{[(\nu-\nu_p)^2+(\Delta\nu_p/2)^2]}, \\ \frac{2J_{po}}{\Delta\nu_p}\sqrt{\frac{\ln(2)}{\pi}}\left[-4\ln(2)\frac{(\nu-\nu_p)^2}{\Delta\nu_p^2}\right]. \end{bmatrix} \quad (2)$$

Here $J_{\nu o}$, J_{po} — spectral and integral (summed over all frequencies) radiation intensity at the entrance to the medium;$\Delta\nu_p$ — emission spectrum width; Δ_p — central frequency of radiation at which its spectral intensity is maximum.

From system (1) we obtain expressions for the concentrations of atoms in different states:

$$D = \frac{S}{E_{13}}(R_{23} + A_{21} + R_{32}) + A_{21}\left(A_{31} + R_{32} + \frac{g_1}{g_3}\frac{S}{E_{13}}\right) + R_{23}\left(A_{31} + \frac{g_1}{g_3}\frac{S}{E_{13}}\right),$$

$$n_1 = \left[A_{21}\left(A_{31}+R_{32}+\frac{g_1}{g_3}\frac{S}{E_{13}}\right)+R_{23}\left(A_{31}+\frac{g_1}{g_3}\frac{S}{E_{13}}\right)\right]\frac{N_0}{D},$$

$$n_2 = R_{32}\frac{S}{E_{13}}\frac{N_0}{D}, \quad n_3 = (R_{23} + A_{21})\frac{S}{E_{13}}\frac{N_0}{D},$$

$$S = \int\limits_0^\infty J_\nu\sigma_{13,\nu}d\nu. \quad (3)$$

To obtain an analytical solution, we introduce the following substitutions:

$$d\mu = \left(n_1 - \frac{g_1}{g_3}n_3\right)\sigma_m dz, \qquad J_\nu = J_{\nu o}[a(z)]^{\frac{\sigma_{13,\nu}}{\sigma_m}},$$

$$\frac{\sigma_{13,\nu}}{\sigma_m} = \frac{\left(\frac{\Delta\nu_{13}}{2}\right)^2}{(\nu-\nu_{13})^2 + \left(\frac{\Delta\nu_{13}}{2}\right)^2}. \quad (4)$$

Here σ_m — is the value of the absorption cross section at the center of the line ($\nu = \nu_{13}$), μ — optical layer thickness, a — unknown function depending on z.

Then the transfer equation for nonmonochromatic radiation reduces to the equation

$$\frac{da}{a} = -d\mu \to a = e^{-\mu}, \qquad J_\nu = J_{\nu o}e^{-\mu\frac{\sigma_{13,\nu}}{\sigma_m}}. \quad (5)$$

Let us calculate the integral S from the system (3). We consider that $\Delta\nu_p \gg \Delta\nu_{13}$. By replacing

$\xi = 2(\nu - \nu_{13})/\Delta\nu_{13}$, we get the following:

$$S = J_{vo}(\nu = \nu_p)\frac{g_3}{g_1}\frac{c^2 A_{31}}{4\pi\nu_{13}^2}\int\limits_{-\frac{2\nu_{13}}{\Delta\nu_{13}}}^{\infty}\frac{e^{-\frac{\mu}{1+\xi^2}}}{[1+\xi^2]}d\xi$$

$$= J_{vo}(\nu = \nu_p)\frac{g_3}{g_1}\frac{c^2 A_{31}}{4\pi\nu_{13}^2}\int\limits_{-\infty}^{\infty}\frac{e^{-\frac{\mu}{1+\xi^2}}}{[1+\xi^2]}d\xi. \qquad (6)$$

In the integral (6), the lower limit of integration is replaced by $-\infty$, since $\nu_{13} \sim 10^{14}$ Hz, and $\Delta\nu_{13} \sim 10^{10}$ Hz, therefore, for $\xi = 2\nu_{13}/\Delta\nu_{13} \sim 10^4$ the value of the integrand can be considered the same as for $\xi \to \infty$ (compared to the maximum of the integrand). Next, a trigonometric replacement is made, which will reduce the integral to a modified Bessel function of the 1st kind of zero order (I_0) [12]:

$$S = J_{vo}(\nu = \nu_p)\frac{g_3}{g_1}\frac{c^2 A_{31}}{4\nu_{13}^2}e^{-\frac{\mu}{2}}I_0\left(\frac{\mu}{2}\right). \qquad (7)$$

Find the relationship between the z coordinate and the μ parameter using the left equation from (4):

$$\mu + F\,\mu e^{-\frac{\mu}{2}}\left[I_0\left(\frac{\mu}{2}\right) + I_1\left(\frac{\mu}{2}\right)\right] = N_o\sigma_m z$$

$$F = \frac{g_3}{g_1}\frac{\left(R_{23} + A_{21} + R_{32} + \frac{g_1}{g_3}A_{21} + \frac{g_1}{g_3}R_{23}\right)\lambda_{13}^2 A_{31}}{4(A_{21}A_{31} + R_{32}A_{21} + R_{23}A_{31})E_{13}}$$

$$\times J_{vo}(\nu = \nu_p). \qquad (8)$$

Here I_1 — modified Bessel function of the 1st kind of the 1st order [12].

It remains to obtain the dependence of the radiation intensity integrated over the spectrum on the parameter μ. To do this, we transform the transfer equation from the system (1) to the following form:

$$dJ_\nu = -d\mu\,\frac{\sigma_{13,\nu}}{\sigma_m}J_\nu. \qquad (9)$$

We integrate equation (9) over the frequencies ($d\nu$), and then over the optical thickness ($d\mu$):

$$J = J_{po} - J_{vo}(\nu = \nu_p)\frac{g_3}{g_1}\frac{\lambda_{13}^2 A_{31}}{4\sigma_m}\mu e^{-\frac{\mu}{2}}\left[J_0\left(\frac{\mu}{2}\right) + I_1\left(\frac{\mu}{2}\right)\right]. \qquad (10)$$

Equations (2), (3), (5), (7) and (8) combined into a system give an analytical solution of system (1).

The analytical solution contains a special case of a two-level atom. If the pressure of the buffer gas is 0, then there is no mixing between the levels of the fine structure, and there will be no atoms in the medium that are in the $P_{1/2}$ state.

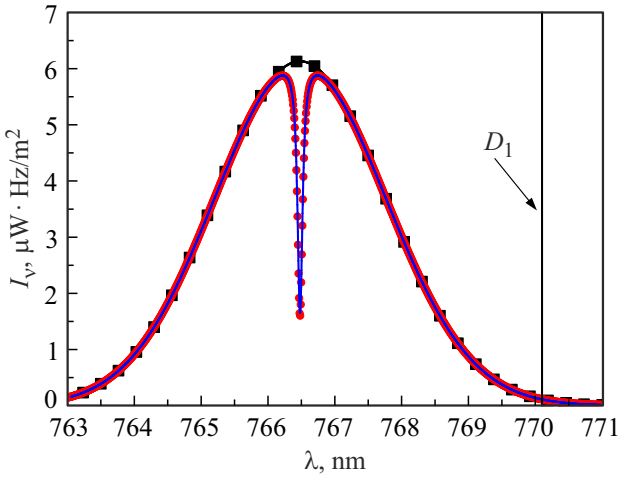

Figure 2. Spectrum of radiation at the input and output. $\Delta\nu_p = 3000$ pm, $\nu_p = \nu_{13}$. Solid curve with squares — radiation spectrum at the input to the medium; solid curve — spectrum of transmitted radiation obtained by numerical methods; curve with circles — spectrum of transmitted radiation obtained by analytical solution.

2. Comparison of numerical and analytical solutions in the problem of transfer of non-monochromatic radiation

Using the example of a medium consisting of potassium vapor and helium buffer gas, let us compare the results of numerical and analytical solutions to the problem of non-monochromatic radiation transfer [13,14]. The numerical values of the physical quantities used in the calculation are presented in the table. The values of fundamental values, such as Einstein coefficients, transition wavelengths, and others, are presented in papers [15,16].

The value γ_{13}, the numerical value of which is determined from the experiment and depends on the temperature of the medium T_0, is required to calculate the width of the absorption line using the following formula [15]:

$$\Delta\nu_{13} = \gamma_{13}P_{bg}\sqrt{\frac{T}{T_o}} \approx 67\ pm. \qquad (11)$$

As can be seen from formula (11) and the table, the width of the emission spectrum is much greater than the width of the absorption line.

In this case, the width of the emission spectrum cannot be arbitrarily large. Its value is limited by the physical approximations that were made in the problem. It was assumed that the radiation is not absorbed at the $1 \to 2$ transition. Consequently, the width of the emission spectrum should be much smaller than twice the width between the fine structure levels ($\Delta\nu_p < 2\Delta E/h = 3.5$ THz $\to 6.8$ nm for potassium). The best fit for this condition is the cesium atom, for which $\Delta E = 42$ nm.

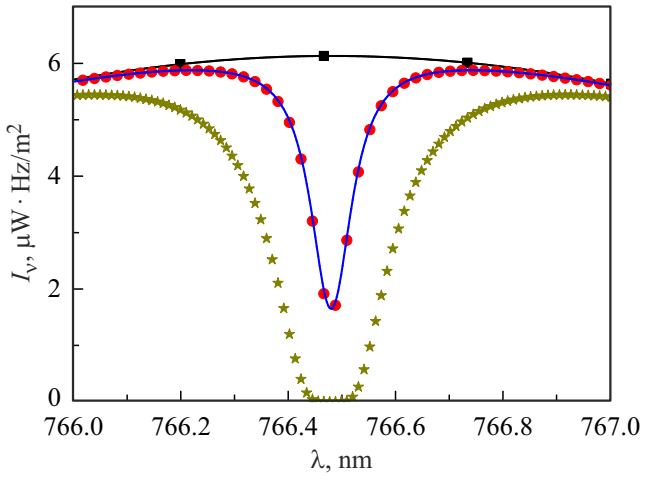

Figure 3. Spectrum of radiation at the input and output. $\Delta\nu_p = 3000$ pm, $\nu_p = \nu_{13}$. Solid curve with squares — radiation spectrum at the entrance to the medium; solid curve — spectrum of transmitted radiation obtained by numerical methods; curve with circles — spectrum of transmitted radiation, obtained using an analytical solution; the curve with stars — the spectrum of the transmitted radiation, obtained using the BLB formula (12).

To obtain a numerical solution of system (1), we will use the implicit Euler scheme and the spectral-group approximation. Let the radiation spectrum at the input be Gaussian.

Fig. 2 and 3 show the radiation spectrum at the entrance to the medium and the transmitted radiation spectrum obtained by numerical and analytical methods. In order to demonstrate the bleaching effect, Fig. 3 shows the spectrum of the transmitted radiation, which is described by the Bouguer–Lambert–Baer (BLB) formula [17]:

$$J_\nu = J_{\nu 0} e^{-N_o \sigma_{13,\nu} b}. \qquad (12)$$

Such a formula can be obtained when the problem of propagation through a medium of radiation with an intensity much lower than the saturation intensity $I_s = E_{13}A_{31}/\sigma_{13,\nu}$ is solved. In this case, we can assume that almost all atoms are in the ground state ($n_1 \approx N_0$, $n_2 \approx 0$ and $n_3 \approx 0$). Then the equations become linear and integrable.

In the resulting analytical solution, the value of the radiation intensity can be any.

In Fig. 2 and 3, the solid curve is the numerically obtained spectrum of the transmitted radiation, the curve with circles — is the analytically obtained spectrum of the transmitted radiation, they coincide with each other.

In Fig. 2, the wavelength at which the maximum spectral intensity of radiation falls corresponds to the maximum absorption. The vertical line marks the position of the D_1 line (770.1 nm). Using formula (2), we calculate how many times the spectral intensity of radiation on the D_2 (766.5 nm) line is greater than the spectral intensity of radiation on the D_1 line using the following formula:

$$\frac{J_{\nu o}(\nu_{D_2})}{J_{\nu o}(\nu_{D_1})} = \exp\left[4\ln(2)\frac{(\nu_{D_2} - \nu_{D_1})^2}{\Delta\nu_p^2}\right] \approx 114. \qquad (13)$$

From formula (13) and Fig. 3 we can conclude that in the kinetic equations we can neglect the absorption of radiation by the D_1 line.

Fig. 3 shows an enlarged image of Fig. 2 in the area of the D_2-line. It demonstrates a noticeable difference between the results of the analytical solution and the result obtained by the BLB (12) formula. This difference is due to a non-linear effect — bleaching [18]. To obtain the formula BLB (12), using the obtained analytical solution in the formula (8), let us direct the intensity of the incident radiation $J_{\nu o}$ to 0. In this case, as can be seen from formula (8), $F \to 0$, and hence $\mu = N_0\sigma_m b$. Substituting the value of the optical thickness into formula (5), we obtain that the radiation spectrum at the outlet of the medium in the case of low intensity is determined by formula (12).

Environment bleaching effect

To describe in more detail the essence of the medium bleaching effect, we assume that the radiation is monochromatic, i.e., the width of its spectrum is much less than the width of the absorption spectrum, and $\nu_p = \nu_{D2}$. Then, in expressions (3), the value S can be represented as follows (for $\Delta\nu_p \ll \Delta\nu_{13}$, the function is $J_\nu = J\delta(\nu - \nu_{13})$, where δ is — the Dirac delta function):

$$S = \int_0^\infty J \cdot \delta(\nu - \nu_{13})\sigma_{13} d\nu = J\sigma_{13}(\nu = \nu_{13}). \qquad (14)$$

Fig. 4 presents dependence of $S(z = 0)$ on the width of the emission spectrum λ_p. This graph is obtained by numerical calculation of $S(z = 0)$ from formula (3) for

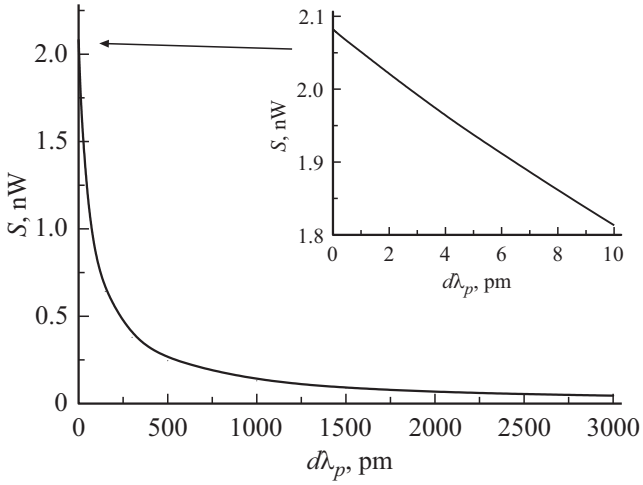

Figure 4. Dependence of $S(z = 0)$ on the width of the emission spectrum $\Delta\lambda_p$.

Values of physical parameters

Parameter	Value	Parameter	Value
T_0	328 K	T	433 K
γ_{13}	19.8 MHz/Torr	N_0	$5 \cdot 10^{12}$ cm^{-3}
I_{po}	1 kW/cm^2	$\Delta\nu_p$, $\Delta\lambda_p$	1.53 THz, 3000 pm
b	1 cm	P_{bg}	1 atm

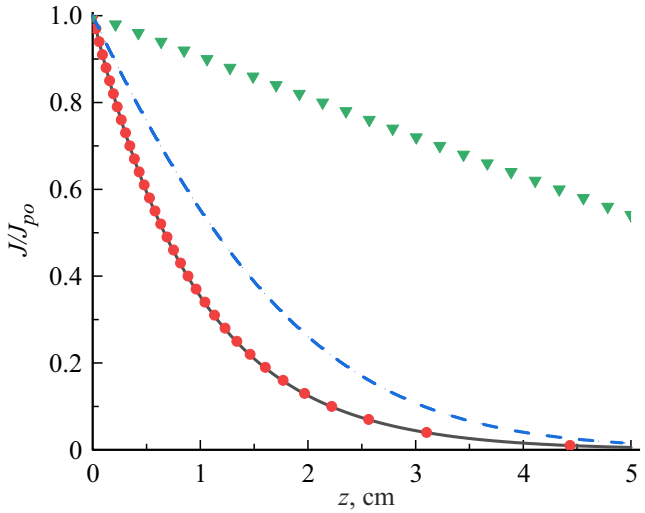

Figure 5. Dependence of the radiation intensity on the coordinate z at different ratios J_{po}/J_s. Solid curve — BLB formula; curve with circles — $J_{po}/J_s = 0.01$; dashed curve — $J_{po}/J_s = 1$; curve with triangles $J_{po}/J_s = 10$.

various values of $\Delta\lambda_p$. The calculation was performed using the parameters taken from the table.

From the calculation presented in Fig. 4, it follows that at $\Delta\lambda_p = 0.5$ pm the value of $S(z = 0) = 2.067$ nW. The value of $S(z = 0)$ calculated by formula (14) is 2.082 nW. As the radiation spectrum width decreases, the difference between S calculated by formulas (3) and (14) will decrease.

From expressions (3) and (14) we obtain the formula for the free path of a photon:

$$l(z) = \frac{1}{\left(n_1 - \frac{g_1}{g_3} n_3\right)\sigma_m} = \frac{1}{N_0\sigma_m}\left(1 + \frac{J}{J_s}\right) = l_o\left(1 + \frac{J}{J_s}\right),$$

$$J_s = \frac{E_{13}}{\sigma_{13}} \frac{(A_{21}A_{31} + A_{21}R_{32} + R_{23}A_{31})}{\left(R_{23} + A_{21} + R_{32} + \frac{g_1}{g_3}A_{21} + \frac{g_1}{g_3}R_{23}\right)} = \frac{E_{13}A_{\text{eff}}}{\sigma_{13}}. \quad (15)$$

Here J_s — saturation intensity, which depends only on the parameters of the medium. For the parameters presented in the table, the saturation intensity is $J_s = 3.6$ W/cm^2.

The transport equation and its solution have the form

$$\frac{dJ}{dz} = -\frac{J}{l} = -\frac{J}{l_o\left(1 + \frac{J}{J_s}\right)},$$

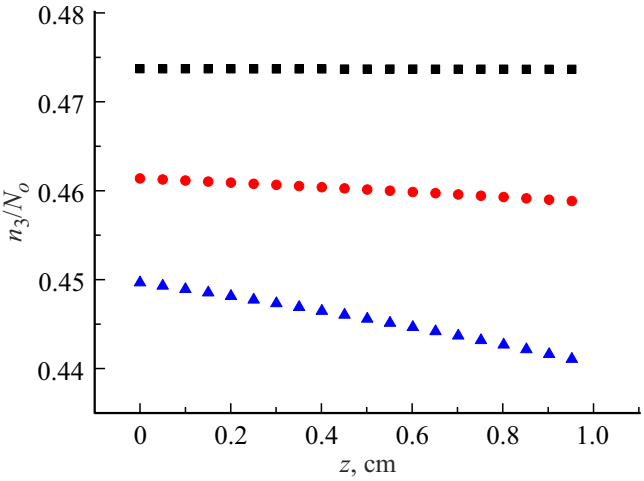

Figure 6. The dependence of the concentration of atoms in the excited state $4P_{3/2}$, at different values of the width of the emission spectrum $\Delta\lambda_p$. Curve with squares — $\Delta\lambda_p = 0.5$ pm; curve with circles — $\Delta\lambda_p = 500$ pm; curve with triangles — $\Delta\lambda_p = 1000$ pm.

$$J(z = 0) = \int_0^\infty J_{\nu o} d\nu = J_{po},$$

$$\ln\left(\frac{J_{po}}{J}\right) + \frac{J_{po} - J}{J_s} = \frac{z}{l_o}. \quad (16)$$

At $J_{po}/J_s \ll 1$, it follows from formula (15) that the mean free path does not depend on the radiation intensity. Then the solution of the transport equation (16) is the Bouguer– Lambert–Baer formula.

For $J_{po}/J_s > 1$, the dependence of the mean free path on the radiation intensity cannot be neglected. In this case, the transport equation (15) will be non-linear, and the dependence of the radiation intensity on the coordinate — will be different from the BLB formula.

Using the parameters taken from the table, calculations were carried out using the formula (15) for 3 different ratios J_{po}/J_s. The results are shown in Fig. 4.

As can be seen from Fig. 5, the curve obtained for the $J_{po}/J_s \ll 1$ ratio is in excellent agreement with the curve obtained using the BLB formula ($\exp(-N_0\sigma_m z)$). From the curves obtained for the ratios $J_{po}/J_s = 1$ and 10, it can be seen that the radiation for these cases is absorbed noticeably weaker, since the free path of the photon increases.

In addition, the dependence of the concentration of atoms in the excited state $4P_{3/2}$ on the width of the emission spectrum $\Delta\lambda_p$ was studied. For this, system (1) was solved numerically. The calculation was performed using the parameters taken from the table. The form of the emission spectrum of laser diodes has a Gaussian form. The calculation results are presented in Fig. 6.

It can be seen from Fig. 6 that as the width of the emission spectrum decreases, the number of atoms in the excited state $4P_{3/2}$ increases. Such a dependence is explained by the following way. A decrease in the spectrum

width (with the integral radiation intensity unchanged) leads to an increase in the probability of absorption of photons located outside the center of the $1 \rightarrow 3$ transition line. An increase in the number of absorbed photons leads to an increase in the number of atomic transitions from the $4S_{1/2}$ state to the $4P_{3/2}$ state, i. e. n_3 should increase.

Conclusion

An analytical solution to the problem of transfer of non-monochromatic directional radiation through a resonantly absorbing medium is obtained for the case when the width of the emission line spectrum is greater than the width of the absorption line spectrum of the medium, but less than the distance between the levels of the fine structure.

As the pressure of the buffer gas tends to 0, the resulting solution reduces to solving the problem of a two-level atom.

Using the example of a medium consisting of potassium and helium atoms, a comparison of numerical and analytical solutions is demonstrated, which are consistent with each other with a relative error $\sim 0.1\%$.

The result of this paper can be used to test various methods of numerical simulation of the problems of nonmonochromatic radiation transfer through a resonantly absorbing medium.

Additionally, the effect of radiation bleaching was studied. It is shown that the effect occurs when the radiation intensity is greater than the saturation radiation intensity. In this case, when describing the propagation of radiation through a medium, one cannot use the Bouguer– Lambert –Baer formula.

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