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Spectral-angular characteristics of the radiation of a charged particle in the field of a frequency-modulated electromagnetic wave

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Based on the solution of the equation of motion of a charge in a frequency-modulated electromagnetic field, a classical theory of radiation of a relativistic charged particle linearly accelerated by a high-intensity chirped laser pulse is constructed. This solution was used to study the spectral-angular characteristics of the emission of a charged particle. According to the calculated formulas for the phase and phase-angular distributions of the intensity and power of the radiation of a particle in a modulated field, graphs of the dependence on the value of the modulation coefficient are plotted. Also obtained is the Fourier transform of the electric field strength of the radiation and the spectral density of the radiation of the particle in the case of linear polarization of the wave.

Keywords: chirped pulse, spectral-angular characteristics, charged particle, plane electromagnetic wave, high-power laser radiation.

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Introduction

Over the last 60 years, rapid developments in laser physics have enabled a range of innovations in non-linear optics, including frequency conversion, ultrafast optics and fully optical modulation. Record intensities at the laser beam focus were achieved — 10^{23} W/cm^2 [1,2]. The attainment of such powerful electric fields has been made possible, in part, by improvements in laser focusing technique [3,4], the advent and development of laser installations that allow optical pulses of a few femtosecond duration (10^{-15} s) [5], and the use of chirped pulse amplification (CPA) [6–8]. The technology is one of the most modern and advanced, and is planned for use in medicine, namely in the diagnosis and treatment of cancer and the manufacture of radioactive isotopes. However, there are still some problems that need to be solved before this technology can be applied to real cancer therapy, but thanks to advances in this and related fields, laser therapy can be expected in the future [9].

The theory of laser-assisted plasma acceleration of charged particles was put forward as early as 1979 [10] and is still being improved upon. The development of CPA-based lasers has been of great benefit in improving charged-particle acceleration circuits [11].

The essence of the chirped pulse amplification method is that the laser pulse is stretched in time and spectrum, amplified and then compressed again. In this way, the laser pulse is initially subjected to frequency modulation. The question of particle acceleration in the field of a frequency-modulated (FM) electromagnetic wave has been studied in [12,13].

In the present paper an investigation was undertaken on the spectral-angular characteristics of charged particle radiation in the field of FM electromagnetic wave based on results obtained in [12], in which energy characteristics of a charged particle without taking into account radiation friction were calculated based on classical equation of motion of a charge in electromagnetic field. It is known from paper [14] that electron energy loss due to hard radiation is reached at energy 1 GeV which corresponds to laser field intensity 10^{22} W/cm^2. In the present paper, all characteristics have been calculated at intensities 10^{19} W/cm^2. However, during the long interaction of the wave with the particle even a small radiation friction parameter can give a significant contribution to the particle dynamics, so it is assumed that the PM electromagnetic wave in this paper is represented as a chirped ultrashort laser pulse.

Of interest are intensity and power of charge radiation and their angular and phase-angle distributions, as well as the Fourier transform of the electric field strength of radiation and spectral density of particle radiation. These results can be useful in controlling the operation of laser accelerators in which the radiation is subject to frequency modulation.

The authors have undertaken a similar study in [15] in the case of a superposition of a plane monochromatic electromagnetic wave field with a co-directed external constant magnetic field (Redmond field).

1. Problem formulation

The high-frequency Lorentz force acts on a particle with charge q and mass m, then the equation of motion for the charge is

$$\frac{d\mathbf{p}}{dt} = q\mathbf{E} + \frac{q}{c}[\mathbf{v}\times\mathbf{H}], \quad (1)$$

where $\mathbf{p}$ — particle momentum; $\mathbf{E}$, $\mathbf{H}$ — the electric and magnetic field strength vectors of the electromagnetic wave.

Equation (1) is supplemented by initial conditions for the velocity and coordinates of the charged particle:

$$\mathbf{v}(0) = \mathbf{v}_0, \qquad \mathbf{r}(0) = \mathbf{r}_0.$$

The relativistic factor γ is related to the electromagnetic field strength by the following relation:

$$\gamma = \sqrt{1 + I/I_{\text{rel}}},$$

and is also $\gamma = mc(1 - \nu_{z0})/\sqrt{1 - \nu_0^2/c^2}$, where the relativistic intensity I_{rel} (W/cm^2) is:

$$I_{\text{rel}} = \frac{m^2c^3\omega^2}{8\pi q^2} = \frac{1.37\cdot 10^{18}}{\lambda^2}.$$

It is assumed that the modulated pulse propagates along axis z with duration τ, and if $t < \tau$, the components of the wave magnetic and electric field strengths will have the form (2), otherwise ($t > \tau$) the components of the strengths will be zero. The constant phase plane is perpendicular to one axis:

$$\begin{cases} E_x = H_y = b_x \sum_{n=0}^{N} J_n(\delta_{\text{FM}})\cos\Phi_n, \\ E_y = -H_x = f b_y \sum_{n=0}^{N} J_n(\delta_{\text{FM}})\sin\Phi_n, \\ E_z = H_z = 0, \end{cases} \quad (2)$$

where the axes x and y coincide with the direction of the half-axes of the polarization ellipse b_x and b_y, with $b_x \geq b_y \geq 0$; $\Phi_n = \Phi + \Phi'_n = (\omega + n\omega')\xi + \varphi_0 + n\alpha_0$; $\xi = t - z/c$; ω — carrier frequency; ω' — modulation frequency, $J_n(\delta_{\text{FM}})$ — Bessel function n-order, δ_{FM} — frequency modulation factor, $f = \pm 1$ — polarization parameter, with upper sign corresponding to right polarization E_y, and lower sign corresponding to left polarization.

For an example, take the parameters of a terawatt femtosecond titanium-sapphire laser system ($\lambda = 800$ nm; 40 fs), then $\Phi \in [0; 94.2]$.

At a peak intensity of 10^{19} W/cm ^{2}, the pulses of this laser system are generated in mode synchronization mode. The frequency spectrum of the emitted radiation is represented by „ridges“, consisting of a number of discrete, evenly spaced frequency lines. Such radiation is described by the task at hand [16].

2. Intensity of radiation of a charged particle in the field of an FM electromagnetic wave

Applying vector product to equation (1) by vector $\mathbf{H}$, we obtain the Umov–Poynting vector in the following form:

$$\mathbf{S} = \frac{c}{4\pi}[\mathbf{E}\times\mathbf{H}] = \frac{c}{4\pi q}[\mathbf{F}\times\mathbf{H}] - \frac{1}{4\pi}\big[[\mathbf{v}\times\mathbf{H}]\times\mathbf{H}\big], \quad (3)$$

The vector components (3) take the form

$$S_x(t) = \frac{1}{4\pi}H_y\left[(\nu_x H_y - \nu_y H_x) - \frac{c}{q}F_z\right],$$

$$S_y(t) = \frac{1}{4\pi}H_x\left[(\nu_x H_y - \nu_y H_x) - \frac{c}{q}F_z\right],$$

$$S_z(t) = \frac{1}{4\pi}\left[(\nu_z\left(H_x^2 + H_y^2\right) + \frac{c}{q}(E_xF_x + E_yF_y)\right]. \quad (4)$$

Substituting the system (2) into the vector components $\mathbf{S}$ (4), we get the following form:

$$S_x(t) = 0, \qquad S_y(t) = 0,$$

$$S_z(t) = \frac{c}{4\pi}\sum_{n=0}^{N} J_n^2(\delta_{\text{FM}})(b_x^2\cos^2\Phi_n + b_y^2\sin^2\Phi_n).$$

Circular polarization

In the case of circular polarization, the polarization semiaxis of the polarization ellipse $b_x = b_y = b/\sqrt{2}$;

$$|\mathbf{S}(t)| = \sqrt{S_x^2(t) + S_y^2(t) + S_z^2(t)} = I_{cir}\sum_{n=0}^{N} J_n^2(\delta_{\text{FM}}), \quad (5)$$

where $I_{\text{cir}} = cb^2/8\pi$.

Now let us estimate the intensity of radiation of a particle averaged over the period of its oscillation in the field of an FM wave:

$$I_{\text{rad}} = \frac{\omega}{2\pi(1+h)}\int\limits_{\Phi(t)}^{\Phi(t)+2\pi} |\mathbf{S}(t)|\frac{1+g}{\omega}d\Phi' = I_{cir}\sum_{n=0}^{N} J_n^2(\delta_{\text{FM}}). \quad (6)$$

Let us use the formula for the longitudinal component of the particle's momentum, corrected for the relativistic multiplier of [10]:

$$g = h - \frac{q^2(b_x^2 - b_y^2)}{4\gamma^2\omega^2}\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2}\cos 2\Phi_n$$

$$+ \frac{q}{\gamma^2\omega}\sum_{n=-N}^{N}\frac{J_n(\delta_{\text{FM}})}{1+n\eta}(\chi_x b_x\sin\Phi_n \mp \chi_y b_y\cos\Phi_n),$$

where $h = \frac{1}{2}\left[\frac{m^2c^2}{\gamma^2} - 1 + \frac{\chi_x^2+\chi_y^2}{\gamma^2} + \frac{q^2(b_x^2+b_y^2)}{2\gamma^2\omega^2}\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2}\right]$; $\eta = \omega'/\omega$.

Linear polarization

For the case of linearly polarized electromagnetic waves, one of the semi-axes of the polarization ellipse is zero and the other is equal to b $(b_x = b;\ b_y = 0)$

$$|\mathbf{S}(t)| = \sqrt{S_x^2(t) + S_y^2(t) + S_z^2(t)} = I_{\text{lin}} \sum_{n=0}^{N} J_n^2(\delta_{\text{FM}}) \cos^2 \Phi_n, \tag{7}$$

where $I_{\text{lin}} = cb^2/4\pi$.

Now let us estimate the intensity of radiation of a particle averaged over the period of its oscillation in the field of an FM wave:

$$I_{\text{rad}} = \frac{\omega}{2\pi(1+h)} \int_{\Phi(t)}^{\Phi(t)+2\pi} |\mathbf{S}(t)| \frac{1+g}{\omega} d\Phi'$$

$$= \frac{I_{\text{lin}}}{2} \sum_{n=0}^{N} J_n^2(\delta_{\text{FM}}) \left(1 - \frac{\mu}{8(1+h)} \sum_{n=0}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2}\right), \tag{8}$$

where $\mu = q^2 b^2/\gamma^2 \omega^2$.

The minimum will correspond to the phase $\Phi_0 = 0,\ \pi$, and is defined by the formula:

$$I_{\min} = I_{\text{lin}} \sum_{n=0}^{N} J_n^2(\delta_{\text{FM}}) \left(1 - \frac{\mu \sum\limits_{n=0}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2}}{8 + 2\mu \sum_{n=0}^{N} \frac{H_n^2(\delta_{\text{FM}})}{(1+n\eta)^2}}\right).$$

The maximum will correspond to phase $\Phi_0 = \pi/2,\ 3\pi/2$ and is defined by the formula:

$$I_{\max} = I_{\text{lin}} \sum_{n=0}^{N} J_n^2(\delta_{\text{FM}}) \left(1 - \frac{\mu \sum\limits_{n=0}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2}}{8 + 6\mu \sum_{n=0}^{N} \frac{H_n^2(\delta_{\text{FM}})}{(1+n\eta)^2}}\right).$$

The average radiation intensity of a particle can be obtained using the following formula [15]:

$$\langle f \rangle = \sup \lim_{\Phi_{0n} \to \infty} \frac{1}{\Phi_{0n}} \int_{0}^{\Phi_{0n}} f(\Phi_{0n}) dt,$$

where $f(\Phi)_{0n}$ shall be replaced with I_{rad}.

Thus, the radiation intensity of a particle, the field of a plane PM wave, averaged over the initial phase Φ_{0n}, is:

$$\langle I_{\text{rad}} \rangle = \frac{I_{\text{lin}}}{2} \sum_{n=0}^{N} J_n^2(\delta_{\text{FM}})$$

$$\times \left(2 - \frac{\mu \sum\limits_{n=0}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2}}{\sqrt{\sum\limits_{n=0}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2}\mu + 4}\ \sqrt{3 \sum\limits_{n=0}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2}\mu + 4}}\right). \tag{9}$$

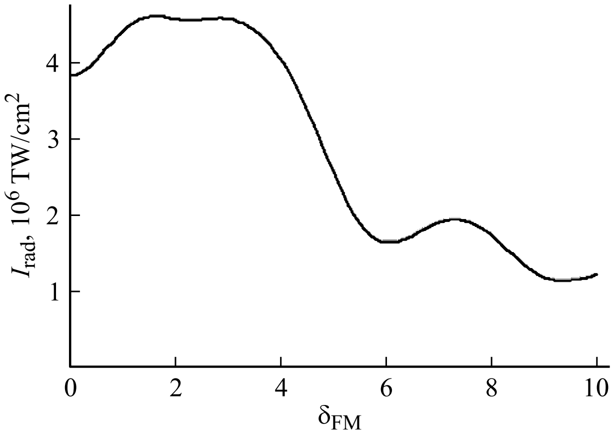

Figure 1. Dependence of average particle radiation intensity in the field of FM wave intensity $I_{\text{lin}} = 10^7$ TW/cm ^{2} on frequency modulation coefficient.

Let us plot the average particle emission intensity as a function of the modulation factor δ_{FM} (Fig. 1) at wave intensity 10^7 TW/cm^2.

Maximum particle emission intensity $I_{\text{rad}} = 4.611 \cdot 10^6$ TW/cm ^{2} is reached at $\delta_{\text{FM}} = 1.6{-}2.8$. After the $\delta_{\text{FM}} = 2.8$ value, the radiation intensity begins to decrease, followed by periodic fluctuations. It follows that a wave with a coefficient of $\delta_{\text{FM}} \in [1.6, 2.8]$ will be most effective for particle acceleration.

Differentiating expression (8) with respect to the initial phase, we obtain the phase distribution of the particle radiation intensity in the field of the FM wave:

$$\frac{dI_{\text{rad}}}{d\Phi_{0n}} = I_{\text{lin}} \frac{\mu^2}{8} \frac{\sum\limits_{n=0}^{N} \frac{J_n^6(\delta_{\text{FM}})}{(1+n\eta)^4} \sin \Phi_{0n} \cos \Phi_{0n}}{\left[1 + \frac{\mu}{4} \sum\limits_{n=0}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2} (2\sin^2 \Phi_{0n} + 1)\right]^2}. \tag{10}$$

Let us depict the phase distribution of the radiation intensity ($dI_{\text{rad}}/d\Phi_{0n}$ along the ordinate axis and $\sin \Phi_{0n}$ along the x axis) at various frequency modulation coefficients for the intensity 10^7 TW/cm ^{2} of the electromagnetic wave on the phase plane.

Fig. 2 clearly shows that as the frequency modulation coefficient increases, the area of the phase intensity distribution decreases.

The instantaneous angular distribution of the radiation intensity has the form:

$$\frac{dI_{\text{rad}}}{d\Omega} = \frac{I_{\text{lin}}}{2\pi} \frac{\mu^2}{8} \frac{\sum\limits_{n=0}^{N} \frac{J_n^6(\delta_{\text{FM}})}{(1+n\eta)^4} \sin \Phi_{0n}}{\left[1 + \frac{\mu}{4} \sum\limits_{n=0}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2} (2\sin^2 \Phi_{0n} + 1)\right]^2} \tag{11}$$

Differentiating the distribution (11) with respect to Φ_{0n}, we obtain the phase-angle distribution of the particle's

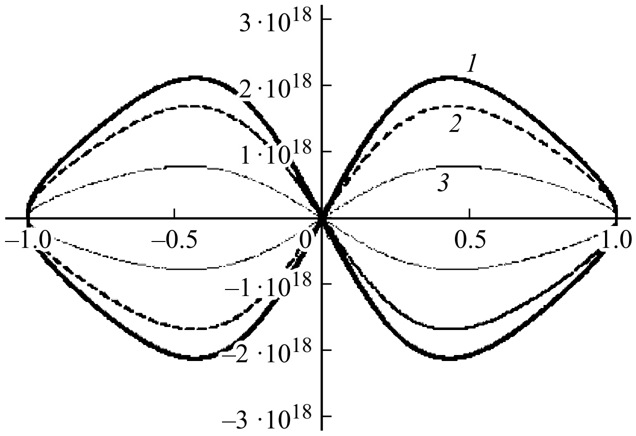

Figure 2. Phase distribution of the particle radiation intensity in the FM wave field at its intensity $I_{\text{lin}} = 10^7$ TW/cm ^{2} at different frequency modulation ratios (*1* — $\delta_{\text{FM}} = 0$, *2* — $\delta_{\text{FM}} = 0.5$, *3* — $\delta_{\text{FM}} = 1$).

radiation intensity in the field of the FM wave:

$$\frac{d^2 I_{\text{rad}}}{d\Phi_{0n} d\Omega} = \frac{I_{\text{lin}}}{2\pi}$$
$$\times \left(\frac{\mu^2}{8} \frac{\sum\limits_{n=0}^{N} \frac{J_n^6(\delta_{\text{FM}})}{(1+n\eta)^4} \cos\Phi_{0n}}{\left[1 + \frac{\mu}{4}\sum\limits_{n=0}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2}(2\sin^2\Phi_{0n}+1)\right]^2} \right.$$
$$\left. - \frac{\mu^3}{4} \frac{\sum\limits_{n=0}^{N} \frac{J_n^8(\delta_{\text{FM}})}{(1+n\eta)^6} \sin^2\Phi_{0n}\cos\Phi_{0n}}{\left[1 + \frac{\mu}{4}\sum\limits_{n=0}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2}(2\sin^2\Phi_{0n}+1)\right]^3} \right). \tag{12}$$

Let us depict the phase-angle distribution of the radiation intensity ($d^2 I_{\text{rad}}/d\Omega d\Phi_{0n}$along the ordinate axis and $\sin\Phi_{0n}$ along the x axis) with different frequency modulation coefficients at the wave intensity 10^7 TW/cm^2 on the phase plane.

Fig. 3 shows that the area of the phase-angle distribution decreases as the frequency modulation coefficient increases.

3. Particle emission power in the field of an FM electromagnetic wave

The power dP transmitted through a surface element dl is equal to the modulus of the energy flux density vector and is defined by the formula:

$$\frac{dP}{dl} = |\mathbf{S}(t)|. \tag{13}$$

Entering the solid angle Ω into formula (13), we get the formula:

$$\frac{dP}{d\Omega} = r^2 |\mathbf{S}(t)|, \tag{14}$$

where $r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$ — the distance from the particle in space to its original position.

We take the coordinate values from [12] and write them in the following form:

$$x = x_0 + \sum_{n=-N}^{N} \frac{\chi_x}{\gamma k} \frac{\Phi_n}{1+n\eta}$$
$$- \frac{q b_x}{\gamma\omega k} \sum_{n=-N}^{N} \frac{J_n(\delta_{\text{FM}})}{(1+n\eta)^2} \cos\Phi_n + C_x,$$

$$y = y_0 + \sum_{n=-N}^{N} \frac{\chi_y}{\gamma k} \frac{\Phi_n}{1+n\eta}$$
$$- \frac{q b_y}{\gamma\omega k} \sum_{n=-N}^{N} \frac{j_n(\delta_{\text{FM}})}{(1+n\eta)^2} \cos\Phi_n + C_y,$$

$$z = z_0 + \sum_{n=-N}^{N} \frac{\Phi_n}{1+n\eta} - \frac{q^2(b_x^2 - b_y^2)}{8\gamma^2\omega^2 k}$$
$$\times \sum_{n=-N}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^3} \sin 2\Phi_n - \frac{q}{\gamma^2\omega k} \sum_{n=-N}^{N} \frac{J_n(\delta_{\text{FM}})}{(1+n\eta)^2}$$
$$\times [\chi_x b_x \cos\Phi_n + f\chi_y b_y \sin\Phi_n] + C_z,$$

where

$$C_x = -\sum_{n=-N}^{N} \frac{\chi_x}{\gamma k} \frac{\Phi_{0n}}{1+n\eta} + \frac{q b_x}{\gamma\omega k} \sum_{n=-N}^{N} \frac{J_n(\delta_{\text{FM}})}{(1+n\eta)^2} \cos\Phi_{0n},$$

$$C_y = -\sum_{n=-N}^{N} \frac{\chi_y}{\gamma k} \frac{\Phi_{0n}}{1+n\eta} + \frac{q b_y}{\gamma\omega k} \sum_{n=-N}^{N} \frac{J_n(\delta_{\text{FM}})}{(1+n\eta)^2} \sin\Phi_{0n},$$

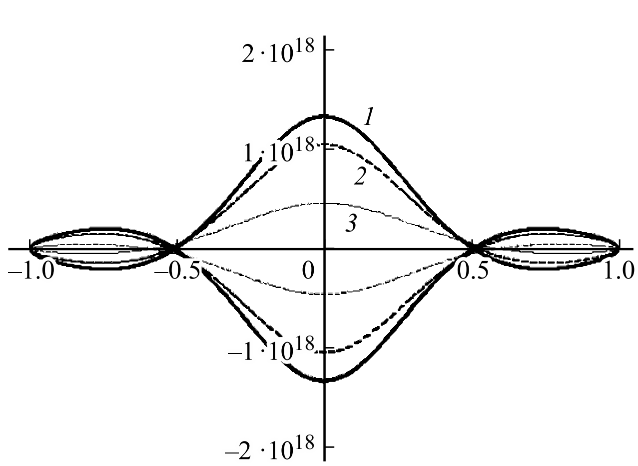

Figure 3. Phase-angle distribution of particle radiation intensity at wave intensity $I_{\text{lin}} = 10^7$ TW/cm^2 at different frequency modulation ratios (*1* — $\delta_{\text{FM}} = 0$, *2* — $\delta_{\text{FM}} = 0.5$, *3* — $\delta_{\text{FM}} = 1$).

$$C_z = -\sum_{n=-N}^{N} \frac{\Phi_{0n}}{1+n\eta} + \frac{q^2(b_x^2 - b_y^2)}{8\gamma^2\omega^2 k}$$
$$\times \sum_{n=-N}^{N} \frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\eta)^3} \sin 2\Phi_{0n}$$
$$+ \frac{q}{\gamma^2\omega k} \sum_{n=-N}^{N} \frac{J_n(\delta_{\mathrm{FM}})}{(1+n\eta)^2} [\chi_x b_x \cos\Phi_{0n} + f\chi_y b_y \sin\Phi_{0n}],$$

Circular polarization

By substituting the coordinates of the particle and the modulus of the Umov–Poynting vector (5) into formula (14), we obtain the following expression:

$$\frac{dP}{d\Omega} = I_{\mathrm{cir}} \sum_{n=0}^{N} J_n^2(\delta_{\mathrm{FM}}) \left\{ \left(\frac{\chi_x^2 + \chi_y^2}{\gamma^2} + h^2 \right) \sum_{n=0}^{N} \frac{\Phi_n^2}{(1+n\eta)^2 k^2} \right.$$
$$- 2\frac{qb(1+h)}{\sqrt{2}\gamma^2\omega k^2} \sum_{n=0}^{N} \frac{J_n(\delta_{\mathrm{FM}})}{(1+n\eta)^3} (\chi_x \cos\Phi_n + f\chi_y \sin\Phi_n)$$
$$+ 2\left(\frac{C_x\chi_x + C_y\chi_y}{\gamma} + hC_z \right) \sum_{n=0}^{N} \frac{\Phi_n}{(1+n\eta)k}$$
$$+ \frac{q^2b^2}{2\gamma^2\omega^2k^2} \sum_{n=0}^{N} \frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\eta)^4} \left[1 - \frac{(\chi_x \cos\Phi_n + f\chi_y \sin\Phi_n}{\gamma^2} \right]$$
$$+ C_x^2 + C_y^2 + C_z^2 - 2\frac{qb}{\sqrt{2}\gamma\omega k} \sum_{n=0}^{N} \frac{J_n(\delta_{\mathrm{FM}})}{(1+n\eta)^2}$$
$$\times \Big[(C_x \cos\Phi_n + fC_y \sin\Phi_n)$$
$$\left. + \frac{C_z}{\gamma}(\chi_x \cos\Phi_n) + f\chi_y \sin\Phi_n) \Big] \right\}. \qquad (15)$$

Integrating expression (15) over the solid angle $d\Omega = \cos\Phi_n d\Phi_n d\theta$, we obtain the total radiation power of the charge:

$$P = \frac{4\mu\pi^2}{k^2} I_{\mathrm{cir}} \sum_{n=0}^{N} \frac{J_n^4(\delta_{\mathrm{FM}})}{(1+n\eta)^4} \left(1 + \frac{\mu}{2} \sum_{n=0}^{N} \frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\eta)^2} \right)$$
$$\times \left[1 + \left(\cos\Phi_{0n} + \frac{\pi}{2} \sin\Phi_{0n} \right) \right].$$

The phase portrait of the total charge radiation power in the field of the FM wave will look like shown in Fig. 4.

From the figure you can see a tendency to a sharp decrease in the area of phase distribution with increasing modulation coefficient δ_{FM}.

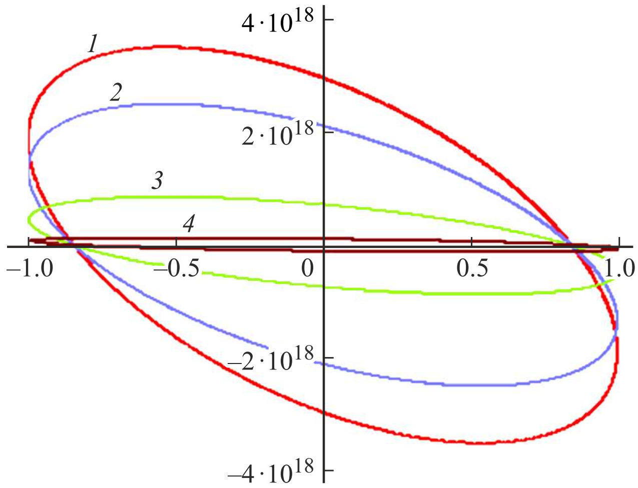

Figure 4. Phase distribution of particle radiation power in the field of FM wave intensity $I_{\mathrm{cir}} = 10^7\,\mathrm{TW/cm^2}$ at different frequency modulation coefficients (*1* — $\delta_{\mathrm{FM}} = 0$, *2* — $\delta_{\mathrm{FM}} = 0.5$, *3* — $\delta_{\mathrm{FM}} = 1$, *4* — $\delta_{\mathrm{FM}} = 1.5$).

Linear polarization

Substituting the coordinates of the particle and the modulus of the Umov–Poynting vector (7) into formula (14), we obtain the following expression:

$$\frac{dP}{d\Omega} = I_{\mathrm{lin}} \sum_{n=0}^{N} J_n^2(\delta_{\mathrm{FM}}) \cos^2\Phi_n \left\{ \left(\frac{\chi_x^2}{\gamma^2} + h^2 \right) \right.$$
$$\times \sum_{n=0}^{N} \frac{\Phi_n^2}{(1+n\eta)^2k^2} - 2\frac{qb\chi_x(1+h)}{\gamma^2\omega K^2} \sum_{n=0}^{N} \frac{J_n(\delta_{\mathrm{FM}})}{(1+n\eta)^3}$$
$$\Phi_n \cos\Phi_n + 2\left(\frac{C_x\chi_x}{\gamma} + hC_z \right) \sum_{n=0}^{N} \frac{\Phi_n}{(1+n\eta)k}$$
$$- \frac{q^2b^2}{4\gamma^2\omega k^2} \frac{h}{k} \sum_{n=0}^{N} \frac{J_n(\delta_{\mathrm{FM}})}{(1+n\eta)^4} \Phi_n \sin 2\Phi_n$$
$$+ \frac{q^3b^3\chi_x}{4\gamma^4\omega^3k^3} \sum_{n=0}^{N} \frac{J_n^3(\delta_{\mathrm{FM}})}{(1+n\eta)^5} \cos\Phi_n \sin 2\Phi_n$$
$$+ \frac{q^2b^2}{\gamma^2\omega^2k^2} \left(1 + \frac{\chi_x^2}{\gamma^2} \right) \sum_{n=0}^{N} \frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\eta)^4} \cos^2\Phi_n$$
$$+ \frac{q^4b^4}{64\gamma^4\omega^4k^4} \sum_{n=0}^{N} \frac{J_n^4(\delta_{\mathrm{FM}})}{(1+n\eta)^6} \sin^2 2\Phi_n$$
$$+ C_x^2 + C_z^2 - 2\frac{qb}{\gamma\omega k} \left(C_x + \frac{C_z\chi_x}{\gamma} \right) \sum_{n=0}^{N} \frac{J_n(\delta_{\mathrm{FM}})}{(1+n\eta)^2} \cos\Phi_n$$
$$\left. - C_z \frac{q^2b^2}{4\gamma^2\omega^2k^2} \sum_{n=0}^{N} \frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\eta)^3} \sin 3\Phi_n \right\} \qquad (16)$$

Integrating expression (15) over the solid angle $d\Omega = \cos\Phi_n d\Phi_n d\theta$, we obtain the total radiation power of

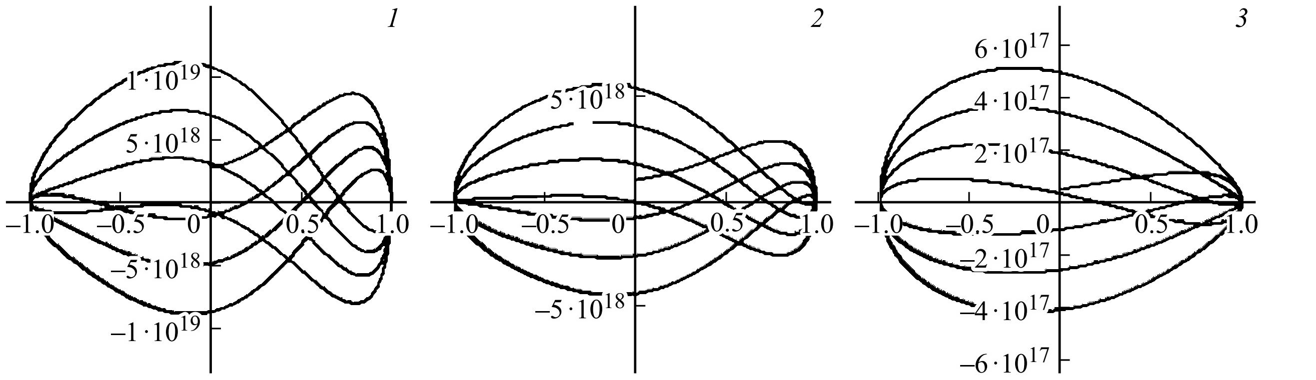

Figure 5. Phase distribution of particle radiation power in the field of FM wave intensity $I_{\text{lin}} = 10^7$ TW/cm ^{2} at different frequency modulation ratios (*1* — $\delta_{\text{FM}} = 0$, *2* — $\delta_{\text{FM}} = 1$, *3* — $\delta_{\text{FM}} = 3$).

the charge:

$$P = \frac{q^2}{\pi m^2 c^5} I_{\text{lin}}^2 \sum_{n=0}^{N} J_n^2(\delta_{\text{FM}}) \left\{ \frac{14}{9} \sum_{n=0}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2} \right.$$
$$\times \left(\frac{\mu}{16} \sum_{n=-N}^{N} \frac{J_n^2(\delta_{\text{VM}})}{(1+n\eta)^2} (2\sin^2\Phi_{0n}+1)^2 + \sin\Phi_{0n} \right)$$
$$+ \frac{\pi 3}{4} \sum_{n=0}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^4}$$
$$\times \left(\frac{\mu}{4} \sum_{n=-N}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2} (2\sin^2\Phi_{0n}+1) + 1 \right) \sin\Phi_{0n}$$
$$- \frac{3}{4} \sum_{n=0}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^4} (\Phi_{0n}\sin\phi_{0n} + \cos\Phi_{0n})$$
$$\left. + \frac{\mu}{40} \sum_{n=-N}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2} (2\sin^2\Phi_{0n}+1) \right\},$$

The phase portrait of the total charge radiation power in the field of the FM wave will look like shown in Fig. 5.

In Fig. 5, just as in the previous one, we can trace the tendency of the phase distribution area to decrease with increasing modulation coefficient δ_{FM}, and the right part of the function decreases more rapidly.

4. Fourier transform of the electric field strength of a particle in the field of an FM wave

The radiation spectrum of a charged particle in the field of a plane FM electromagnetic wave can be represented as a sum of an infinite number of waves:

$$E(\mathbf{r}, t) = \sum_{\omega=-\infty}^{\infty} \mathbf{E}_\omega(\mathbf{r})$$
$$\times \exp\left\{ -i[\omega\xi + \delta_{\text{FM}} \sin(\omega'\xi + \alpha_0) + \varphi_0] \right\}.$$

The Fourier component can be represented as a periodic function with period $\tilde{T} = (1+h)2\pi/\omega$:

$$E_\omega(\mathbf{r}) = \frac{1}{\tilde{T}} \int_{t}^{\bar{t}} \mathbf{E}(\mathbf{r}, t)$$
$$\times \exp\left\{ -i[\omega\xi + \delta_{\text{FM}} \sin(\omega'\xi + \alpha_0) + \varphi_0] \right\} dt. \tag{17}$$

From equation (1) we express $\mathbf{E}(\mathbf{r}, t)$, substitute it into function (17) and go from integrating over time to integrating over the space-time component ξ:

$$\mathbf{E}_\omega(\mathbf{r}) = \frac{1}{\tilde{T}} \int_{\xi(t)}^{\xi(\bar{t})} \left(\frac{1}{q}\frac{d\mathbf{p}}{dt} - \frac{1}{c}[\mathbf{v}] \times \mathbf{H} \right)$$
$$\times \sum_{n=0}^{N} J_n(\delta_{\text{FM}}) \cos\Phi_n (1+g) d\xi.$$

Then for the real part of $\mathbf{E}_\omega(\mathbf{r})$ we obtain the following components:

$$\text{Re}(\mathbf{E}_{\omega,x}) = \frac{b_x}{2(1+h)} \sum_{n=0}^{N} J_n^2(\delta_{\text{FM}})$$
$$\times \left[(1+h) - \frac{q^2(b_x^2 - b_y^2)}{8\gamma^2\omega^2} \sum_{n=0}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\eta)^2} \right], \tag{18}$$

$$\text{Re}(\mathbf{E}_{\omega,y}) = \text{Re}(\mathbf{E}_{\omega,z}) = 0.$$

Linear polarization

In this case, from (18) for the Fourier transform, we get

$$\mathrm{Re}(\mathbf{E}_\omega) = b\sum_{n=0}^{N} J_n^2(\delta_{\mathrm{FM}})$$

$$\times\left\{\frac{8+\mu\sum\limits_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\eta)^2}[4(2\sin^2\Phi_n+1)-1]}{16+8\mu\sum\limits_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\eta)^2}(2\sin^2\Phi_n+1)}\right\}. \quad (19)$$

By substituting (19) into formula (17), we get the radiation spectrum of the particle at the initial moment of time:

$$\mathrm{Re}(\mathbf{E}(\mathbf{r},t_0)) = b\sum_{\omega=-\infty}^{\omega=\infty}\sum_{n=0}^{N} J_n^2(\delta_{\mathrm{FM}})\frac{A}{B}. \quad (20)$$

where

$$A = 8+\mu\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\eta)^2}\left[4(2\sin^2\Phi_n+1)-1\right],$$

$$B = 16+8\mu\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\eta)^2}(2\sin^2\Phi_n+1).$$

The radiation spectrum has the following phase distribution:

$$\mathrm{Re}\left(\left|\frac{d\mathbf{E}(\mathbf{r},t_0)}{d\Phi_0}\right|\right) = b\sum_{\omega=-\infty}^{\omega=\infty}\sum_{n=0}^{N} J_n^2(\delta_{\mathrm{FM}})$$

$$\times\left|\left(1-2\frac{A}{B}\right)\frac{8\mu}{B}\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})}{(1_n\eta)^2}\sin 2\Phi_n\right|. \quad (21)$$

The radiation spectrum of a particle at a unit solid angle is defined by the formula

$$\mathrm{Re}\left(\left|\frac{d\mathbf{E}(\mathbf{r},t_0)}{d\Omega}\right|\right) = \frac{b}{2\pi}\sum_{\omega=-\infty}^{\omega=\infty}\sum_{n=0}^{N} J_n^2(\delta_{\mathrm{FM}})$$

$$\times\left|\left(1-2\frac{A}{B}\right)\frac{16\mu}{B}\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\eta)^2}\sin^2\Phi_n\right|. \quad (22)$$

The phase-angular distribution of this radiation spectrum is as follows

$$\mathrm{Re}\left(\left|\frac{d\mathbf{E}(\mathbf{r},t_0)}{d\Phi_0 d\Omega}\right|\right) = \frac{b}{2\pi}\sum_{\omega=-\infty}^{\omega=\infty}\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\eta)^2}$$

$$\times\left|\frac{16\mu}{B}\left(1-2\frac{A}{B}\right)\right.$$

$$\left.\times\left(1-\frac{16\mu}{B}\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\eta)^2}\sin^2\Phi_n\right)\cos\Phi_n\right|. \quad (23)$$

Let us represent the function that characterize spectral density of the radiation:

$$\mathrm{Re}(S(\omega)) = \mathrm{Re}\left(|\mathbf{E}(\mathbf{r},t_0)|^2\right) = b2\left|\sum_{\omega=-\infty}^{\omega=\infty}\sum_{n=0}^{N} J_n^4(\delta_{\mathrm{FM}})\frac{A^2}{B^2}\right|. \quad (24)$$

The phase distribution of the spectral density of radiation is expressed by the formula

$$\frac{dS(\omega)}{d\Phi_0} = 32\mu^2 b^2\left|\sum_{\omega=-\infty}^{\omega=\infty}\sum_{n=-N}^{N}\frac{J_n^8(\delta_{\mathrm{FM}})}{(1+n\eta)^4}\frac{A}{B^3}\sin 2\Phi_n\right|. \quad (25)$$

The spectral density of radiation per unit solid angle is determined by the expression

$$\frac{dS(\omega)}{d\Omega} = \frac{32\mu^2 b^2}{\pi}\left|\sum_{\omega=-\infty}^{\omega=\infty}\sum_{n=-N}^{N}\frac{J_n^8(\delta_{\mathrm{FM}})}{(1+n\eta)^4}\frac{A}{B^3}\sin\Phi_n\right|. \quad (26)$$

By differentiating relationship (26) with respect to Φ_0, we get the phase-angular distribution:

$$\frac{d^2S(\omega)}{d\Phi_0 d\Omega} = \frac{32\mu^2 b^2}{\pi}\left|\sum_{\omega-\omega}^{\omega=\infty}\sum_{n=-N}^{N}\frac{J_n^8(\delta_{\mathrm{FM}})}{(1+n\eta)^4}\right.$$

$$\times\left[\frac{A}{B^3}\cos\Phi_n-\frac{16\mu}{B^4}\left(B-3\mu\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\eta)^2}\right)\right.$$

$$\left.\left.\times\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\eta)^2}\sin^2\Phi_n\sin\Phi_n\right]\right|. \quad (27)$$

Conclusion

The issue of spectral-angular radiation characteristics of a charged particle in the field of a frequency-modulated electromagnetic wave is investigated. Expressions for intensity and power of relativistic charge radiation in the case of circularly and linearly polarized electromagnetic wave are obtained. The phase distributions of intensity and power of radiation of a particle moving in the FM electromagnetic wave with intensity 10^{19} W/cm ^{2} were obtained. Fourier transform of the electric field strength of particle radiation in the field of FM electromagnetic wave of linear polarization is calculated. Results of this study can be used for the mathematical interpretation of experiments on the interaction of modulated laser radiation with magnetoplasma.

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