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Analysis of the cooperative dynamics of nonlinear systems based on joint singularity spectrum

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A generalization of the wavelet-transform modulus maxima method to the case of multifractal analysis is proposed, in which the cooperative dynamics of subsystems and the change in the interaction between them are characterized using a joint singularity spectrum. On the example of the phenomenon of chaotic synchronization in the model of interacting Lorenz systems, the possibility of diagnosing a change in the functioning regime in terms of the wavelet-based multifractal formalism is illustrated.

Keywords: multifractal analysis, random process, scaling, singularity spectrum.

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Multifractal analysis is used widely to quantitatively characterize complex scaling, which is typical of the dynamics of various systems in nature [1–4]. A considerable number of experimentally recorded signals with a very complex structure belong to the class of multifractal objects. The use of multifractal formalism concepts is one of the methods for their statistical analysis (especially relevant to strongly inhomogeneous and nonstationary processes [5]). Several approaches to quantitative description of complex signals in terms of a singularity spectrum or a scaling spectrum are used at present [6–8]. One of such approaches is the wavelet transform modulus maxima method [5]. It is relevant to the study of cooperative dynamics of systems (e.g., complex networks) to extend the concept of multifractal analysis of signal structures to the examination of mutual dynamics of subsystems (or network fragments) over several simultaneously recorded processes. In the present study, we consider a generalization of the method from [5] to the case of multifractal analysis with a version of a singularity spectrum characterizing the cooperative dynamics of subsystems and changes in the interaction between them.

The proposed method involves the calculation of wavelet transforms [9,10] of signals $x(t)$ and $y(t)$ of interacting subsystems in accordance with the following formulae:

$$W_x(a,b) = \frac{1}{a}\int_{-\infty}^{\infty} x(t)\psi\left(\frac{t-b}{a}\right)dt,$$

$$W_y(a,b) = \frac{1}{a}\int_{-\infty}^{\infty} y(t)\psi\left(\frac{t-b}{a}\right)dt, \qquad (1)$$

where parameters a and b characterize the scale and translation of basis function ψ, and a real function (e.g., MHAT-wavelet) is often chosen as a wavelet for analysis. Following the calculation of coefficients $W_x(a,b)$ and $W_y(a,b)$, one extracts skeletons (lines of local maxima and minima of wavelet transforms that contain the key information on them).

A modification of the algorithm for calculation of generalized partition functions [6] is proposed to be used to calculate the joint singularity spectrum characterizing the cooperative dynamics of subsystems:

$$Z^2(q,a) = \sum_{i\in L1(a)} \left(\sup_{\acute{a}\leqslant a}\left|W_x\left(\acute{a}, b_i(\acute{a})\right)\right|\right)^q$$

$$\times \sum_{j\in L2(a)} \left(\sup_{\grave{a}\leqslant a}\left|W_y\left(\grave{a}, b_j(\grave{a})\right)\right|\right)^q, \qquad (2)$$

where $L1(a)$, $L2(a)$ are the sets of all skeleton lines of wavelet transforms $W_x(a,b)$, $W_y(a,b)$ on scale a and $b_i(\acute{a})$, $b_j(\grave{a})$ represent the positioning of maxima corresponding to lines i, j on scales $\acute{a} \leqslant a$, $\grave{a} \leqslant a$. As in the algorithm for a scalar signal [6], power-law dependence

$$Z(q,a) \sim a^{\tau(q)}, \qquad (3)$$

is analyzed, and scaling exponent $\tau(q)$ and the singularity spectrum are calculated in accordance with it based on the Legendre transform:

$$D(h) = qh - \tau(q),$$

$$h = \frac{d\tau(q)}{dq}. \qquad (4)$$

In contrast to [5,6], the $D(h)$ singularity spectrum calculated this way is representative of the joint dynamics of subsystems producing signals $x(t)$ and $y(t)$. Let us illustrate the application of the proposed method using the example of synchronization of chaotic oscillations in the dynamics of interacting Lorenz systems

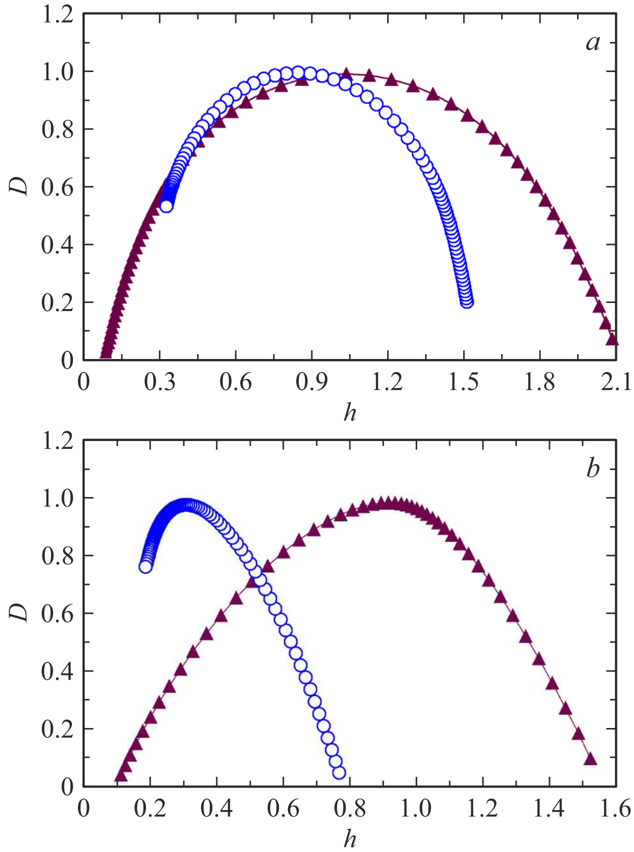
$$\frac{dx_{1,2}}{dt} = \sigma(y_{1,2} - x_{1,2}) + \gamma(x_{2,1} - x_{1,2}),$$

$$\frac{dy_{1,2}}{dt} = r_{1,2}x_{1,2} - x_{1,2}z_{1,2} - y_{1,2},$$

$$\frac{dz_{1,2}}{dt} = x_{1,2}y_{1,2} - z_{1,2}c \tag{5}$$

with parameters $\sigma = 10$, $r_1 = 28.8$, $r_2 = 28$, and $c = 8/3$ and coupling parameter γ being varied. Series of return times to secant planes $x_1^2 + y_1^2 = 30$, $x_2^2 + y_2^2 = 30$, $z_1 = 30$, and $z_2 = 30$ served as the input signals for the considered modified algorithm. The dynamics of model (5) is specific in that it behaves atypically with an increase in the coupling parameter: desynchronization becomes more pronounced in the $\gamma < 2$ range, but gives way to synchronous dynamics as parameter γ grows further. As was demonstrated in [11], if each Lorenz system is characterized by a certain frequency of rotation (around one of the equilibrium states) and switching frequency (with systems regarded as bistable ones), the difference between the corresponding frequencies (of both rotation and switching) for each system of the model increases as γ grows approximately to $\gamma = 2$, and their expected tuning with an increase in the coupling parameter occurs only at $\gamma > 2$. Let us compare the nonsynchronous ($\gamma = 2$) and synchronous ($\gamma = 7$) dynamics modes of model (5) using the series of return times to Poincaré sections $x_1^2 + y_1^2 = 30$ and $x_2^2 + y_2^2 = 30$ as analyzed signals of the proposed method. The results of calculation of joint singularity spectra are presented in the figure (panel *a*). One may note that, first, the average Hölder exponent value corresponding to the maximum of the singularity spectrum decreases and, second, this spectrum becomes narrower upon synchronization. This agrees with the results of earlier studies for one of the state variables of interacting systems with self-sustained oscillations [12]; however, in the present case, functions $D(h)$ characterize the mutual dynamics of subsystems of model (5). The average Hölder exponent value for the chosen secant planes decreases from 1.03 to 0.85 (i.e., by 17%), and the singularity spectrum width decreases from 1.99 to 1.17 (by 41%). If one examines the series of return times to Poincaré sections $z_1 = 30$ and $z_2 = 30$, the changes in singularity spectra become more apparent (see the figure, panel *b*): the average Hölder exponent value decreases from 0.93 to 0.31 (by 67%), and the spectrum width goes down from 1.41 to 0.59 (a 58% reduction).

Thus, it may be concluded that the proposed generalization of the multifractal formalism to cooperative dynamics of interacting systems provides an opportunity to detect changes both in correlation characteristics, which are characterized quantitatively by Hölder exponents [13]), and in the complexity of mutual dynamics, which is characterized by the range of variation of Hölder exponents (singularity spectrum width). The proposed method, which was illustrated in the present study using the example of



Joint singularity spectra for nonsynchronous (triangles) and synchronous (circles) oscillations in the dynamics of interacting Lorenz systems with the series of return times to Poincaré sections $x_1^2 + y_1^2 = 30$, $x_2^2 + y_2^2 = 30$ (*a*) and $z_1 = 30$, $z_2 = 30$ (*b*) serving as analyzed signals.

a pair of coupled systems with self-sustained oscillations, may be applied in the examination of behavior of more complex objects (e.g., complex networks of interacting elements). It is relevant to the analysis of dynamics of complex systems based on experimental data, including, in particular, multichannel electroencephalograms recorded in different body states. In this latter case, complexity estimates may be used to perform various diagnostic tasks, such as identifying the effects of healthy aging in motor functional tests [14,15].

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Conflict of interest

The authors declare that they have no conflict of interest.

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