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The mechanical stresses reduction in a thin-walled quasi-force-free magnetic system inserted into external crossed magnetic fields

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Received January 20, 2023
Revised February 20, 2023
Accepted February 27, 2023

The mechanical stresses in the thin-walled quasi-force-free winding of the solenoid, which arise when a strong pulsed magnetic field is produced, can be significantly reduced if the winding is inserted into the space between two magnets creating crossed fields. It is shown that, with a rational choice of the amplitude and shape of the external field pulses, the stresses can be reduced to values less than one tenth of the magnetic pressure of the generated field.

Keywords: strong magnetic fields, quasi-force-free field, force-balanced coil.

DOI: 10.21883/TPL.2023.05.56017.19510

Generation of fields of 50–100 T is connected with emergence of high mechanical stresses in the winding and insulation. Those stresses are among the main factors preventing obtaining strong magnetic fields. To ensure the strength of up-to-date magnets with record-high fields in them, coils with equally loaded multilayer windings are used [1–3]. In such magnetic systems, the outer radius is related with the internal by the ratio $\exp(B_0^2/2\mu_0\sigma_0)$, where σ_0 is the ultimate strength. Calculations show that a drastic increase in the coil dimensions and energy of the system takes place in case ultra-strong fields are generated [4]. An alternative variant is using quasi-force-free magnetic systems where electromagnetic forces are essentially reduced without enlarging the magnetic system dimensions and energy [4–7].

One of the possible realizations of such a magnetic system is a magnet with additional magnetic fields [8]. In this paper, we consider the applicability of this method for reducing mechanical stresses in a pulsed-mode system. Since this system provides the possibility to avoid impermissible heating of the winding, it attracts a great practical interest, for instance, in designing low-volume magnets [7].

In the stationary mode, it is possible to assume, ignoring the wall curvature, that the current density is constant over the winding conductor thickness, its absolute value being $\delta_0 = B_0/\mu_0\Delta$; here B_0 is the induction at the magnet axis, Δ is the solenoid wall thickness. Thereat, the distribution of the axial and azimuthal induction components over the wall thickness obeys the linear law: $B_z = B_0(1 - x/\Delta)$, $B_\varphi = B_0(x/\Delta)$, where $x = r\text{-}R$ (r is the point radial coordinate, R is the solenoid inner radius). Calculations via the elasticity-theory formulae accomplished in [6] and confirmed by numerical calculations showed that, in a thin-walled coil whose ends are fixed and winding turns are laid at 45° to the axis, the ratio of equivalent mechanical stress σ_M calculated via the von Mises formula to the magnetic field pressure at the axis $\eta = 2\mu_0\sigma_M/B_0^2$ takes a value of about 0.2. Additional reduction of this stress may be achieved in the magnetic system described in [8] where inside the solenoid with the quasi-force-free primary winding *1* there is an axial-current conductor *2* creating a field with induction $B_{\varphi 1}$, while outside the solenoid there is a magnet *3* creating an axial field with induction B_{z1} (Fig. 1).

When the boundary values of those inductions are equal ($B_{\varphi 1}(0) = B_{z1}(\Delta) = B_1$), the stationary distributions of induction, current density components, and electromagnetic force in the conductor introduced into the crossed external magnetic field have the following forms:

$$B_z = B_0(1 - x/\Delta) + B_1 x/\Delta,$$

$$B_\varphi = B_1(1 - x/\Delta) + B_0 x/\Delta,$$

$$\delta_z = \delta_\varphi = (B_0 - B_1)/\mu_0\Delta,$$

$$f(x) = (B_0 - B_1)^2(1 - 2x/\Delta)/\mu_0\Delta.$$

The characteristic quantity is the resultant of those forces, which is defined as

$$F(x) = \int_0^x f(x)dx = P_M(0) - P_M(x)$$
$$= (B_0 - B_1)^2(x - x^2/\Delta)/\mu_0\Delta. \quad (1)$$

where $P_M(x) = B(x)^2/2\mu_0$ is the magnetic pressure. This quantity takes the maximal value in the middle of the layer: $F_{\max} = (B_0 - B_1)^2/4\mu_0$. Given $\Delta \ll R$, components of the mechanical stress tensor are defined as follows [6,9]:

$$\sigma_r \approx -F(x) \approx (B_0 - B_1)^2(x - x^2/\Delta)/\mu_0\Delta, \quad (2)$$

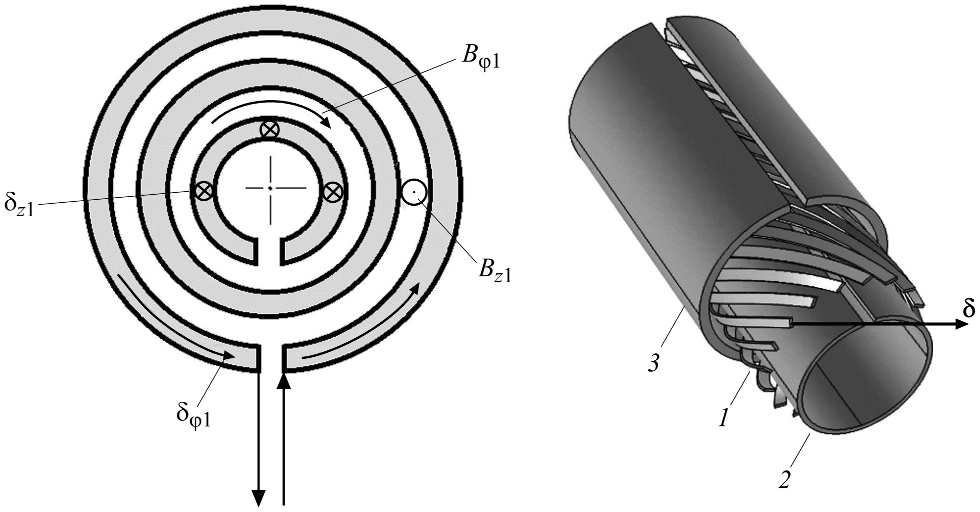

Figure 1. Magnetic system. *1* — solenoid with the quasi-force-free winding, *2* — conductor with axial current, *3* — magnet creating an axial field.

$$\sigma_\phi(x) \approx -\theta F(x) - \frac{1-\theta}{\Delta}\int_0^\Delta F(x)dx \approx -(B_0 - B_1)^2$$

$$\times \left[\theta\left(x - \frac{x^2}{\Delta}\right) + (1-\theta)\frac{\Delta}{6}\right] \Big/ \mu_0\Delta. \quad (3)$$

The ratios become valid when the solenoid inter-turn space is filled with a high-modulus dielectric [4,10,11]. For the winding with fixed ends, $\theta = \mu/(1-\mu)$, where μ is the Poisson coefficient assumed to be 0.3. The maximal values of stress moduli take place in the middle of the layer ($x = \Delta/2$):

$$|\sigma_{r,\max}| = F_{\max},$$

$$|\sigma_{\phi,\max}| = F_{\max}(\theta + 2)/3 = (2-\mu)/3(1-\mu)F_{\max}.$$

In the absence of crossed external fields, $B_1 = 0$ and $F_{\max} = B_0^2/4\mu_0$. Provided $B_1 = (\sqrt{2}-1)B_0 \approx 0.414B_0$, magnetic pressure at the middle point is $P_M(\Delta/2) = (1/2\mu_0)B_0^2$, while that at the boundary is $P_M(0) = (1/2\mu_0)\big(1 + (\sqrt{2}-1)^2\big)B_0^2$ (Fig. 2, *a*). Along with this, $F_{\max} = 0.172B_0^2/2\mu_0$, while the normalized stress becomes $\eta_{\max} \approx 0.07$. This effect gets achieved by using external fields with inductions significantly lower than field induction B_0 at the magnet axis.

According to the defined task, in this work we consider mechanical stresses arising in the pulsed mode when the time during which the current flows in the winding is comparable with the characteristic time of the magnetic field diffusion into the conductor ($\tau \approx \mu_0\Delta^2/\rho$, where ρ is the specific resistance).

Specific features of the winding stress state formation in such a mode may be seen, for example, when external fields with limiting induction values at the boundaries B_1 get cut-off immediately after their establishment. As a result, absolute values of induction at both boundaries become equal to B_0. In the above-considered example, induction at the winding middle point retains its value for the time much shorter than the field diffusion time: $B_z(\Delta/2) = B_\varphi(\Delta/2) = B_0/\sqrt{2}$. Thereat, the magnetic pressure at the boundary takes the same value as at the middle point: $P_M(0) = P_M(\Delta/2) = B_0^2/2\mu_0$. At this point, the resultant and stresses become zero. Fig. 2, *b* shows that, after the external field is cut off, the normalized stress does not exceed 0.02 over the total winding turn thickness, i.e. it decreases by approximately 3 times as compared with its static-mode value. However, as time passes, the stresses increase along with the additional field attenuation in the entire volume of the conductor. This is demonstrated by the time dependences of the resultant and normalized stress (Fig. 2, *c*). By the moment $t \approx 0.07\tau$, the normalized stress reaches 0.07; then the system returns to the state typical of the static mode free of external fields ($\eta = 0.2$). Therefore, in implementing the pulsed-mode quasi-force-free magnet, it is reasonable to cut off the primary field prior to the moment of the stationary state establishment.

The possibility of reducing the pulsed-field mechanical stresses in practically valuable cases was considered via two examples.

1. The time dependence of the primary winding pulsed field had a form of one sine half-wave $2t_0$ in duration and B_0 in amplitude, while external fields were triggered simultaneously with the primary one, increased with time sinusoidally to amplitude B_1 during time t_1, and then attenuated exponentially with the time constant t_2 (the sin−crowbar mode).

2. The primary and external fields were triggered simultaneously, increased with time sinusoidally to amplitudes B_0 and B_1 for times t_0 and t_1, respectively, and then attenuated exponentially with the time constants t_3 and t_2, respectively (the crowbar−crowbar mode).

Calculations devoted to minimizing the maximum of normalized mechanical stresses $\eta_{\max}$ were performed based

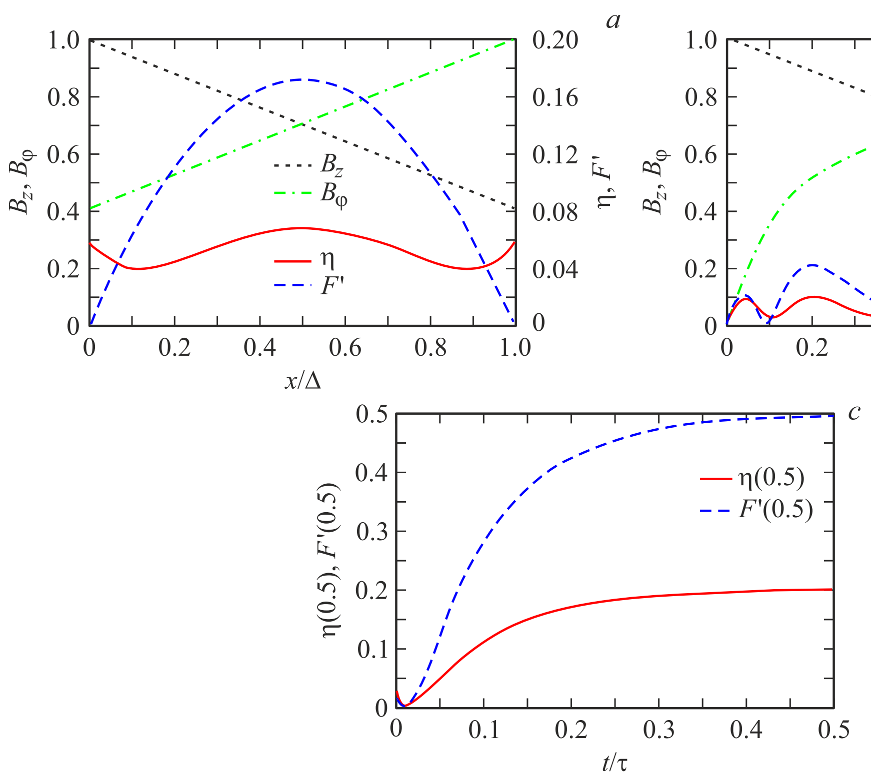

Figure 2. *a* — the case of a stationary field: B_z, B_φ are the resulting induction components, η and F' are the normalized equivalent mechanical stress and resultant ($F' = 2\mu_0 F/B_0^2$); *b* the same dependences at the moment $t = 0.005\tau$ after instantaneous cutoff of additional fields; *c* — time dependences of η and F' in the middle of the layer.

Normalized mechanical stresses

B_1/B_0	t_0/τ	sin—crowbar			crowbar—crowbar			
		t_1/τ	t_2/τ	η_{max}	t_1/τ	t_2/τ	t_3/τ	η_{max}
0.5	1.5	0.85	2.06	0.073	0.86	1.78	1.46	0.074
0.5	2	1.37	1.75	0.063	1.37	1.90	2.00	0.063
0.5	2.5	1.84	1.68	0.057	1.84	1.62	2.16	0.057
0.4	1.5	1.00	1.70	0.086	1.00	1.66	1.50	0.086
0.4	2	1.45	1.24	0.080	1.45	1.28	1.73	0.080
0.4	2.5	1.93	1.66	0.077	1.93	1.91	1.78	0.077
0.3	1.5	1.07	1.61	0.111	1.08	1.65	1.30	0.111
0.3	2	1.54	1.92	0.105	1.54	1.90	1.54	0.105
0.3	2.5	2.05	2.00	0.103	2.04	2.00	1.81	0.103

on the preset values of B_0, t_0 and B_1. Values of parameters t_1/τ, t_2/τ, t_3/τ ensuring the minimal η_{max} are given in the table. These data allow selecting the pulse parameters based on the acceptable level of η_{max}. Fig. 3 presents the examples of time dependences of quantities characterizing formation of mechanical stresses in the primary winding. Time dependences of those quantities presented for pulses with parameters $t_0 = 1.5\tau$, $B_1 = 0.4B_0$ show that, in those cases, deviation of the maximal value of normalized equivalent mechanical stresses does not exceed $\eta_{max} = 0.086$ over the entire pulse duration; this value scarcely differs from the result of static-mode calculations.

The two modes differ from each other only slightly. The main factor affecting the normalized equivalent stresses η_{max} is the amplitude ratio B_1/B_0. Even weak field $(B_1/B_0 = 0.3)$ reduces η_{max} approximately twice as compared with that in the case of the field absence $(B_1/B_0 = 0)$.

Thus, the goal of the study is achieved; its results confirm the possibility of essentially reducing mechanical stresses in the quasi-force-free winding of the solenoid placed in crossed external fields, not only in the static field but also in the pulsed one. In the case considered in Fig. 3, the equivalent mechanical stress of 1 GPa takes place at $\eta_{max} = 0.086$ in the field with induction of about

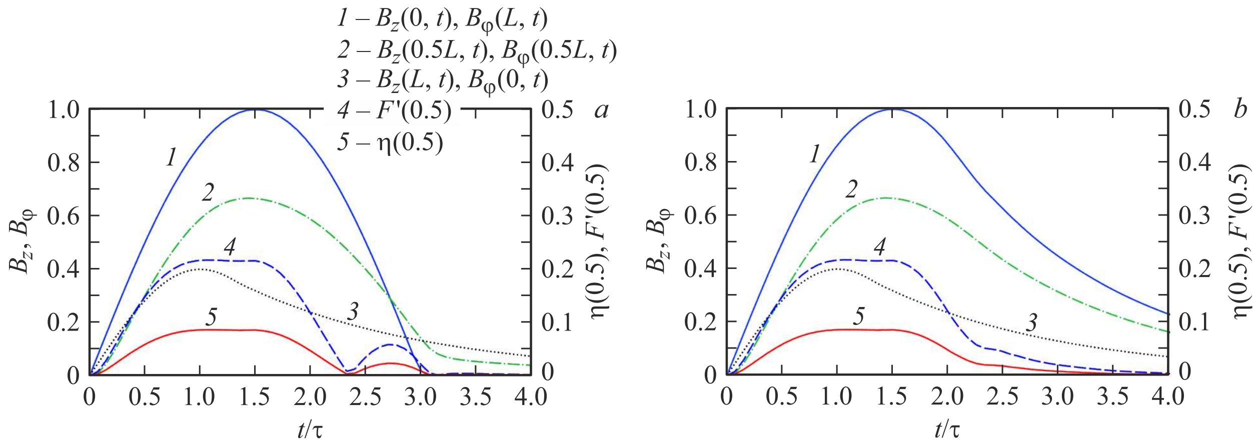

Figure 3. Time dependences of inductions B_z, B_φ at the conductive layer boundaries and in its middle, and also those of normalized mechanical stresses η and resultant F' in the middle of the layer. *a* — sin–crowbar, *b* — crowbar–crowbar.

170 T. This shows that the described method is promising for obtaining megagauss pulsed magnetic fields in non-destructible magnets.

Financial support

The study was supported by the Russian Science Foundation (project № 18-19-00230).

Conflict of interests

The authors declare that they have no conflict of interests.

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Translated by Solonitsyna Anna