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One-dimensional dynamics of the domain boundary in a multilayer ferromagnetic structure

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Received January 11, 2023
Revised January 26, 2023
Accepted January 27, 2023

Using the example of a seven-layer ferromagnetic structure with three thin (nanoscale) and four wide magnetic layers, possible scenarios of domain boundary dynamics in a multilayer ferromagnet are considered. Significant differences in the dynamics of the domain boundary were found in the presence of thin magnetic layers with increased and decreased values of the magnetic anisotropy constant. A diagram of possible scenarios of the dynamics of the domain boundary is constructed depending on the initial velocity of its movement and the distance between thin magnetic layers. The presence of a critical distance between thin magnetic layers separating the dynamics of the domain boundary into two regions with qualitatively different behavior of the system is revealed.

Keywords: domain wall, dynamics, multilayer ferromagnet, nanostructure, spatial modulation of magnetic anisotropy.

DOI: 10.21883/PSS.2023.04.55993.4

1. Introduction

Achievements in the field of nanotechnology and methods of measurement of magnetization statics and dynamics allow coming back to earlier optical and magnetic studies of the domain wall (DW) dynamics from a new point of view and at a new level of experiments. Now there is the possibility to investigate experimentally nanometer motions of individual DWs and even the magnetic inhomogeneities localized in a nanovolume [1–5]. An interesting subject of such studies are multilayer magnetic structures. This is related to increasingly wider possibilities of their production and practical application [6–15]. Often, these are periodically alternating layers of two or more materials, including nanoscale layers, with different physical parameters. Even the creation of atomically flat magnetic interfaces has now become a common place in magnetic nanotechnology [11]. The investigation of one-dimensional models of dynamics of spin waves and magnetic inhomogeneities propagating in these systems normally to layer interfaces provides insights into the effect of the inhomogeneity of system parameters on the processes under consideration [7,8,12–16].

Often, the case of ultrathin metal nonmagnetic interface between two ferromagnetic or antiferromagnetic layers is considered [17–18]. In this case, to describe the magnetization dynamics in the layer, the Landau–Lifshitz equation with constant parameters of the material is used, and some boundary conditions are required to be met at the layer interfaces. According to another approach, the presence of layers, which are different from each other by values of one or more magnetic parameters, is taken into consideration by means of spatial modulation of magnetic parameters of the material (see, for example, [7,8,12,13,15,19]). It is shown in multilayer magnetic films, that the coercive force and the magnetic anisotropy of layers can be controlled by varying the modes of layer growing (see, for example, [20]). Often, in these systems thin (down to nanometer sizes) magnetic layers of one type are separated by wide layers of another type. If a thin magnetic layer has a decreased magnetic anisotropy, this layer is an effective „quantum well" (or „flat magnetic defect") for magnetic inhomogeneities (see, for example, [7,21,22]). The most interesting is the case, when the size of DW and the size that characterizes inhomogeneity of the anisotropy parameter, are of the same order of magnitude. In this case the DW should change its shape significantly when passing through the thin magnetic layer. In these systems pinning of domain walls, generation of localized magnetization waves, such as magnetic solitons and breathers, generation of spin waves are possible [7,12,14,23–24]. It is worth to note that two-dimensional magnetic inhomogeneities of magnetic vortex type near the inhomogeneities of the magnetic anisotropy also demonstrate a diversity of their behavior: pinning of the vortex by defect with explicit change in the rotation speed, reflection before defect with different motion paths, etc. [25]. The problem of description of one-dimensional dynamics of DW in three-layer and five-layer ferromagnetic structures, which are compositions of alternating wide and thin magnetic layers with different magnetic parameters of the anisotropy, can be reduced, under certain conditions, to the problem of interaction of kinks of the sine-Gordon equation (SGE) with impurities [7,16,26]. The case of two thin magnetic layers gives a large diversity of new multisoliton solutions for localized magnetic inhomogeneities and dynamic effects as compared to the case of one thin magnetic layer [27,28]. Even more diversity of solutions can be expected for possible types of magnetic inhomogeneities and dynamic effects in the presence of three or more thin magnetic layers in the system. This study investigates the dynamics of domain walls in a multilayer ferromagnetic structure with three thin magnetic layers.

2. The case of thin magnetic layer in the form of „potential barrier“

Let us consider a seven-layer ferromagnetic structure composed of four wide layers separated by three thin layers (with a size of an order of magnitude of the domain wall width) located at a distance of d from each other. Wide and thin magnetic layers are different from each other by their magnetic anisotropy constants. Let us consider a simple case of inertial motion without attenuation. Parameters of the anisotropy are considered to be functions of y coordinate directed normally to the layer interface. Let us use spherical coordinates $\mathbf{M}(\cos\varphi\sin\theta, \sin\varphi, \cos\varphi\cos\theta)$ to describe dynamics of the magnetization, where $0 \le \theta \le 2\pi$ is angle in the xz plane between the direction of magnetic moment vector and the easy magnetization axis (the Oz axis), $-\pi/2 < \varphi < \pi/2$ is angle that describes outgo of the $\mathbf{M}$ from the plane of DW. By taking into account the exchange interaction and the anisotropy in the energy density of the magnetic material and assuming $\varphi \ll 1$, the dimensionless equation of motion for the magnetization in angular variables in the one-dimensional case can be represented as follows [7,16,26,29]:

$$\frac{\partial^2\theta}{\partial y^2} - \ddot{\theta} - \frac{1}{2} f(y)\sin 2\theta = 0, \tag{1}$$

where $f(y) = K(y) = K_1(y)/K_1^0$ is a function that determines spatial modulation of the anisotropy constant, K_1^0 is the anisotropy constant in thick layers. Time t is normalized to $4\pi M_S\gamma\sqrt{Q}$, where $Q = K_1/(2\pi M_S^2)$ is quality factor of the material. x coordinate is normalized to δ_0, where δ_0 is width of the static Bloch DW. Equation (1) is derived in the assumption that $K_1 \ll 2\pi M_S^2$. Equation (1) with zero right member and $K(y) = 1$ is transformed to the known sine-Gordon equation [28].

The $K(y)$ function is modelled by a rectangular shaped function

$$f(x) = \begin{cases} 1, & |y| > W/2,\ |y+d| > W/2,\ |y-d| > W/2, \\ K, & |y| < W/2,\ |y+d| < W/2,\ |y-d| < W/2, \end{cases} \tag{2}$$

where W is parameter that characterizes width of the thin magnetic layer, K is constant of the magnetic anisotropy in the region of the thin magnetic layer. It is worth to note that thin layers with increased magnetic anisotropy as compared to the homogeneous state are potential barriers for the moving DW. In the extreme case, when thin layers can be considered infinitely thin, equation (1) can be solved analytically [30] using the collective variables technique [28]. With arbitrary W and K equation (1) can only be solved numerically. Equation (1) was solved numerically using an explicit scheme [12,16,26]. The equation was discretized by the standard five-point scheme of „cross“ type that possesses the stability condition of $(\Delta t/\Delta y)^2 \le 0.5$, where Δt is time step, Δy is coordinate step. At the initial moment of time we have a Bloch-type DW moving at a constant speed of υ_0, and boundary conditions are as follows: $\theta(-\infty, t) = 0$, $\theta(+\infty, t) = \pi$, $\theta'(\pm\infty, t) = 0$. In addition, the scheme used is convenient because it is a „single-step“ scheme that uses relatively small number of calls to memory and has a potential for optimization of the computational algorithm. In the process of numerical experiment, the DW crosses the regions of thin layers and at every moment of time the DW structure and its main dynamic characteristics are calculated: position of its center, speed and trajectory of motion.

The coordinate origin is placed in the center of the second barrier for definiteness, and centers of other barriers are located on both sides of the origin with dimensionless coordinates of $y_1 = -3$ and $y_3 = 3$. W and K parameters are taken as $W = 1$, $K = 2$. Let the DW moves from the infinity toward potential barriers. To exclude the interaction between the DW and barriers at the initial moment of time, the initial position of the DW center must be set at a sufficient distance from the barriers. The numerical analysis has shown that for this purpose it is sufficient to consider the case with the initial DW center coordinate of $y = -10$. With initial DW speed less or equal to 0.59, it reflects from the first barrier and moves back with the same modulus of speed (curve *1* in Fig. 1). Therefore, this

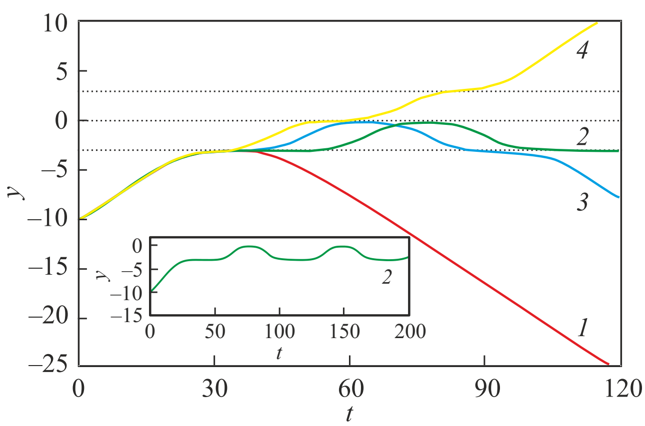

Figure 1. y coordinate of the DW center as a function of time at $W = 1$, $K = 2.0$. Lines: (*1*) — $\upsilon = 0.59$: (*2*) — $\upsilon = 0.595$; (*3*) — $\upsilon = 0.59855$; (*4*) — $\upsilon = 0.5986$; (*5*) — $\upsilon = 0.602$.

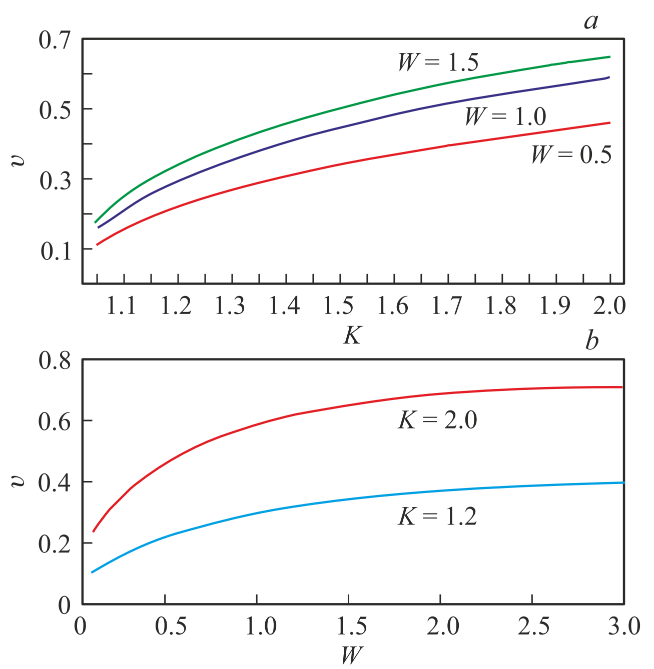

Figure 2. Dependence of the maximum DW speed when reflecting from the first barrier: *a*) in on value of the K at different W. *b*) in on value of W at different K.

speed $v = 0.59$ will be considered as maximum DW speed required to realize the scenario of its reflection from all barriers at given parameters. With increase in K and W this threshold maximum speed of DW reflection from barriers increases non-linearly (Fig. 2). With approximation of this and with a small size of the potential barrier the following dependence is derived: $v \sim \sqrt{K}$ and $v \sim \sqrt{W}$. However, with sufficiently large W, K (for example, at $K > 2$ in Fig. 2, *a*) nearly linear dependence on K is observed for this threshold speed. Dependence of this speed on d is shown in Fig. 3. It can be seen that with an increase in d (when the collective effect of potential barriers on the DW dynamics is already lost) it tends to a value equal to the threshold speed for the case of one barrier. As d tends to zero, three potential barriers have an effect on the DW dynamics equal to that of one wider barrier, and the threshold speed in this case is nearly equal to the value obtained for the case of one barrier with $3W$ [31].

With a small increase in DW motion speed it can pass through the first potential barrier but reflects from the second barrier (curve *3* in Fig. 1). Two scenarios are possible in this case. If the DW moves with a speed of 0.59855, it passes the first barrier and at $t \approx 30$ it reflects from the second barrier and, moving back, passes the first barrier again and goes out to the infinity (curve *3* in Fig. 1). However, in a small speed interval of $0.59 < v < 0.59855$, the DW, after having passed the first barrier, looses part of its energy for the interaction with the barrier. Then, when it reflects from the second barrier, it has no sufficient energy to pass the first barrier in the opposite direction, so it reflects from the first barrier again. Thus, at a certain initial speed within this interval, for example, at a speed of 0.595, the DW will oscillate within the region limited by the first barrier and the second barrier (curve *2* in Fig. 1). By increasing the speed, for example, up to 0.5986, another possible scenario can be observed. DW (curve *4* in Fig. 1). In this case the DW, after having passed the second barrier, reflects from the third barrier and starts oscillating between them. It is worth to note that the DW oscillation between the first and the second barriers, between the second and the third barriers is obviously non-harmonic. With further increase in the initial speed (starting from the speed of 0.602) the DW can pass all three barriers and go out to the infinity (curve *5* in Fig. 1). It is worth to note that when passing the middle barrier, the speed drops nearly down to zero.

The dependence of this minimum speed of barrier passing v_{cr} on different values of K, W can be obtained. For example, the dependence of v_{cr} on K is shown in Fig. 4. It can be seen in the figure, that this dependence has the same behavior as that for the threshold maximum speed of reflection. It can be seen from the ratio between the maximum speed of reflection from all barriers and the minimum speed of passing the barriers, that these speeds are insignificantly different for the considered cases: from 1% of difference to 2% of difference for large values of ($K > 2$, $W > 1.5$). For the case of one barrier [32]

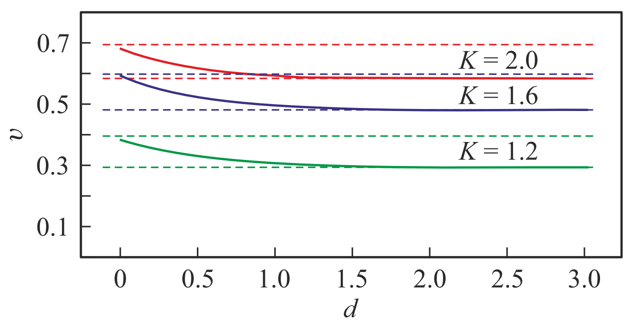

Figure 3. Maximum speed of DW reflection from the first barrier as a function of d at different values of K. Dashed line shows this speed for the case of one barrier.

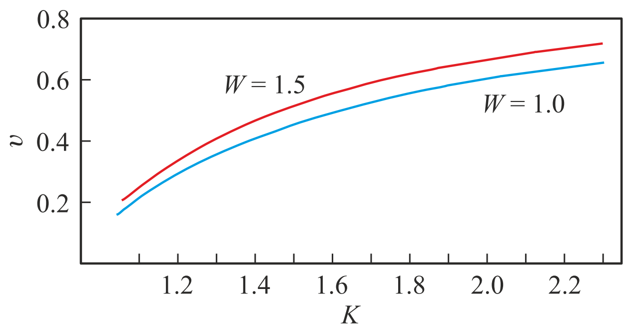

Figure 4. Minimum speed of DW passing through barriers v_{cr} as a function of K at different values of W.

an analytical formula for the threshold minimum speed of passing is suggested

$$v_{cr}^2 = \frac{1}{2}(K-1)\tanh\left(\frac{W}{2}\right). \quad (3)$$

For the case of small values of W equation (3) can be simplified as follows:

$$v_{cr} \approx \frac{1}{2}\sqrt{(K-1)W}. \quad (4)$$

From Fig. 4 at $W = 1$, $K = 1.1$ it follows that $v_{cr} = 0.215$, and by applying formula (4), we get $v_{cr} \approx 0.16$. The 25% difference between these values is indicative that formula (4) also can qualitatively describe the dependence of the critical speed in question on K, W.

The comparison of speeds before and after interaction with barriers shows that these speeds are nearly the same. The start to be noticeably different only at large values of K, W. For the threshold speed of reflection, starting from $K > 2$ and $W > 1$ a decrease in speed by less than 1% takes place as compared to the initial value. For the minimum speed of barrier passing, from $K > 2$ and $W > 1$ the DW speed will be 3.5–4% less than the speed of the DW initially approaching the barriers. Such a decrease in speed will be related to the fact that the DW reflection and passing from/through barriers is additionally accompanied by generation of low-amplitude volume waves. The total energy of the system in the case under consideration is always constant.

It is worth to note that a dynamic DW behavior like this has been found through numerical calculations for other values of W and K as well. It can be expected that a change in the form of function (2) (which has already been investigated previously for the case of three-layer structure (see, for example, [16,32])), also will not result in a qualitative change in the dynamic behavior of the DW.

3. The case of thin magnetic layer in the form of „quantum well“

Now let us consider the case of three thin magnetic layers, which are „quantum wells“ for the DW. Let us consider the $K(y)$ function of form (2) with $K = 0.5$, $W = 1$, $d = 2$. The numerical calculation shows a qualitative difference between the observed scenarios of DW dynamics and the case of „potential barriers“ considered above. At a certain initial DW speed of v_0 below the critical speed of passing through three wells v_{cr}, pinning of the DW is observed at the first, the second and the third wells (curves *1, 2, 3* in Fig. 5, *a*). The basic frequency of oscillations can be determined by Fourier analysis. In our case these frequencies are $\omega_1 = 0.373$, $\omega_2 = 0.319$, $\omega_3 = 0.319$. Also, pinning scenarios were observed for the case of DW hopping from one „quantum well“ to another (see Fig. 6). Such DW hopping is due to the loss of energy for radiation, the excitation of inner degrees of freedom of the DW (for example, pulsation mode), the excitation of localized oscillations of breather type in „quantum wells“ and their interaction with each other. It is worth to note that oscillation frequencies are different from each other despite the fact that thin layers are identical ($\omega_a = 0.301$, $\omega_b = 0.318$, $\omega_c = 0.301$).

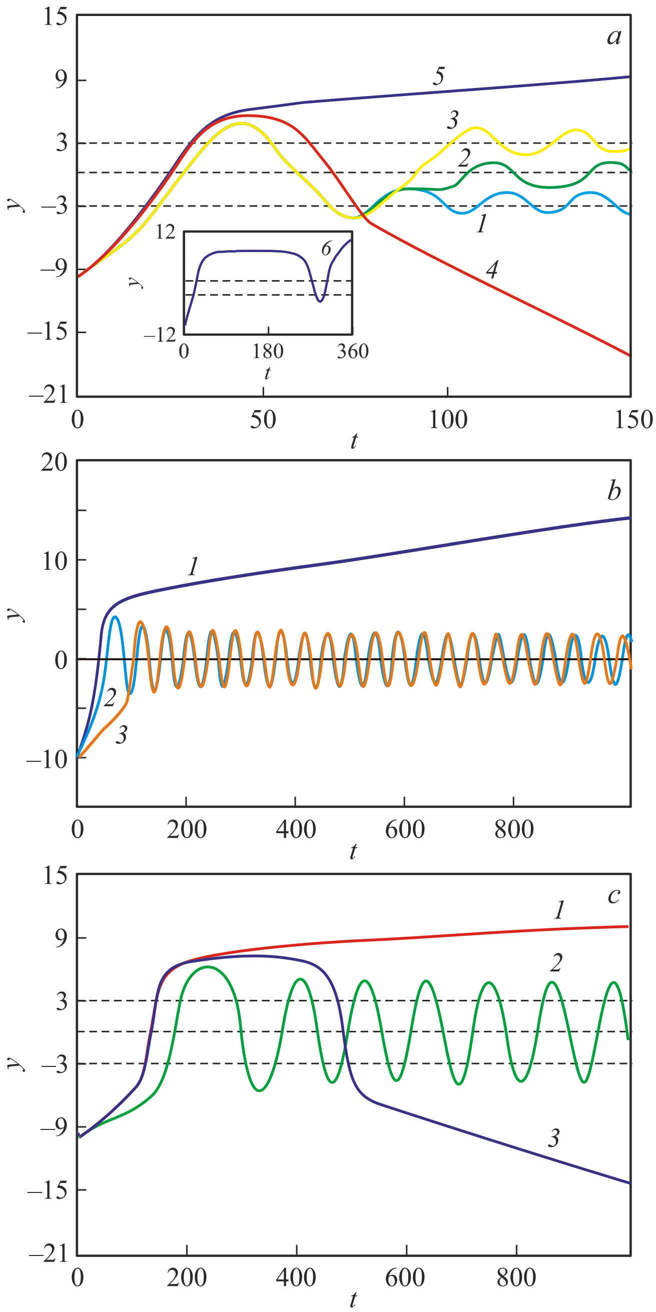

Figure 5. DW center coordinate y as a function of time: *a*) at $W = 1$, $K = 0.5$, $d = 2$, curves: (*1*) — $v_0 = 0.28$; (*2*) — $v_0 = 0.283043899$; (*3*) — $v_0 = 0.2849$; (*4*) — $v_0 = 0.33$; (*5*) — $v_0 = 0.343$; (*6*) — $v_0 = 0.3426$; *b*) for $W = 1$, $K = 0.5$, $d = 1$, curves: (*1*) — $v_0 - 0.201$; (*2*) — $v_0 = 0.14$; (*3*) — $v_0 = 0.06$; *c*) $W = 1$, $K = 0.8$, $d = 2$, curves: (*1*) — $v_0 = 0.0413$; (*2*) —$v_0 = 0.0275$; (*3*) — $v_0 = 0.0403$.

As for the case of one and two thin layers [27,28,31], at certain values of speeds below v_{cr}, an interesting dynamic effect is observed: the resonance reflection of the DW from thin magnetic layers which are quantum wells (curve *4*, Fig. 5, *a*). In this case the DW, after having passed the regions of thin magnetic layers, stops, then it starts moving back and goes away in the direction opposite to the initial direction with a speed of 0.19. This effect, as it has been shown previously for one and two thin magnetic layers, has a resonance behavior related to the interaction between the DW and the breather-type localized magnetization waves emerging on the thin magnetic layers. The detection of these resonance speed is associated with certain difficulties in the case of numerical calculations and requires at least an order of magnitude higher accuracy of calculations. As in the case of two thin magnetic layers [27], one more resonance effect was observed, i.e. „quasitunneling". In this case the DW, having its speed below the minimum required to pass the regions of three thin magnetic layers, passes through them (curve *6*, Fig. 5, *a*). With further increase in the DW speed up to a certain values of v_{cr} (curve *5*, Fig. 5, *a*) it goes away to the infinity.

Fig. 7 shows v_{cr} as a function of K and W. It can be seen from the figure, that with a decrease of K down to approximately 0.75, this critical speed is low and at all considered values of W remains nearly unchanged. Although the presence of a small local maximum is observed. For example, with $W = 1$ and 1.3 this maximum is achieved at $K \approx 0.72$. With further decrease in K, at all considered values of W a sharp increase in $W = 1.3$ is observed. Thus, for the case of $W = 1.3$ this sharp

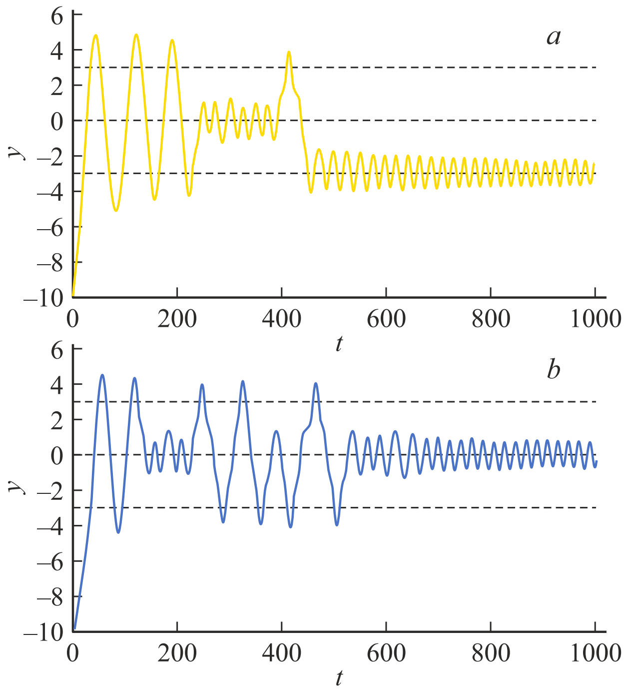

Figure 6. DW center coordinate y as a function of time at $W = 1$, $K = 0.5$. (*a*) — $v_0 = 0.3$; (*b*) — $v_0 = 0.2$; (*c*) — $v_0 = 0.27$.

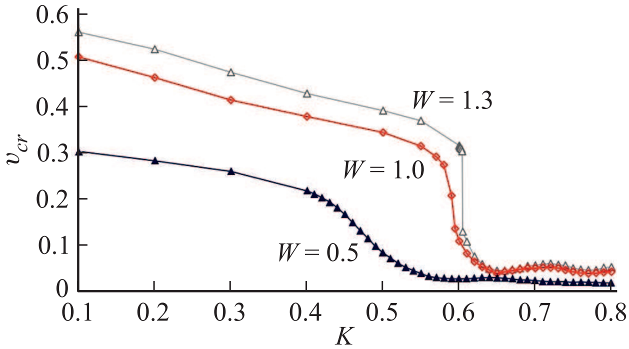

Figure 7. Critical speed of DW passing v_{cr} as a function of W and K.

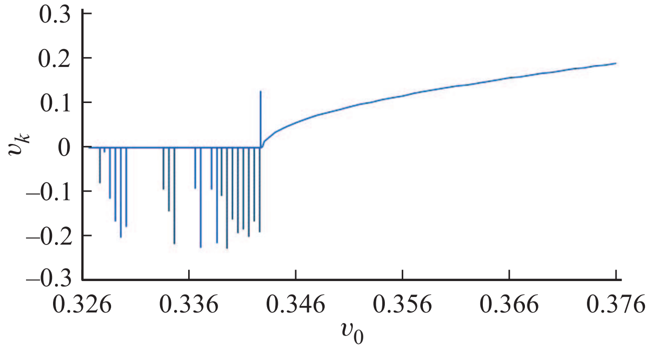

Figure 8. Final speed of the DW as a function of its initial speed at $W = 1$, $K = 0.5$, $d = 2$.

increase is observed in the region of $K \approx 0.6$, for $W = 1$ it is observed at $K \approx 0.59$, for $W = 0.5$ it is observed at $K \approx 0.4$. It is worth to note that with a decrease in W the region of sharp increase in v_{cr} is stretched. For example, for $W = 0.5$ this region starts at $K \approx 0.4$ and ends at $K \approx 0.55$. For W greater than unit this region becomes very small. Then, with increase in K the critical speed increases almost linearly. Results of the studies of critical speed of passing through multilayer regions (both quantum wells and barriers) can be used to determine the effective coercive force for multilayer ferromagnetic materials. For example, it can be done by determining the magnetic field required to accelerate the DW to the critical speed in the case of homogeneous material.

Fig. 8 shows the final speed of DW as a function of its initial speed. It can be seen from the figure, that there is a set of resonance speeds, which, as it has been shown for the case of one and two thin magnetic layers, appear with a certain periodicity, and with approach to the critical speed their number grows. The last vertical line in Fig. 8 near the critical speed of passing corresponds to the „quasitunneling" scenario. If the v_{cr} is exceeded, the final DW speed increases non-linearly, however, as soon as the $v_0 = 0.356$ is exceeded, the final speed increases almost linearly with growth of the initial speed. The same dependence is also typical for the case of one and two quantum wells [27,28,31,33]. For the case of one thin

Possible scenarios of pinning and frequencies of DW oscillations for different initial speeds and d. $W = 1$, $K = 0.5$, $\omega_{theor} = 0.311$

Initial speed v_0	Scenario of DW dynamics	Frequency ω	$\Delta\omega$, ($\lvert\omega-\omega_{theor}\rvert$)
$d = 10$			
0.07	Pinning at the 1-st well	0.338	0.027
0.1	« at the 1-st well	0.352	0.041
0.11	« at the 1-st well	0.330	0.019
0.12	« at the 2-nd well	0.339	0.028
0.13	« at the 3-rd well	0.339	0.028
$d = 5$			
0.05	Pinning at the 1-st well	0.344	0.033
0.15	« at the 2-nd well	0.344	0.033
0.17	« at the 3-rd well	0.324	0.013
$d = 2$			
0.28	Pinning at the 1-st well	0.373	0.062
0.283043899	« at the 2-nd well	0.319	0.008
0.2849	« at the 3-rd well	0.319	0.008

magnetic layer [34] a formula is suggested that relates the final DW speed with its initial speed exceeding v_{cr}:

$$v_k^2 = c(v_0^2 - v_{\min}^2). \tag{5}$$

It can be noticed for the considered case of $W = 1$, $K = 0.5$, $d = 2$, that at a coefficient $c = 1.47$ formula (5) gives a good description of the final speed for our case as well. By making an assumption that the c coefficient in our case is a function of W, K, d and n being number of thin layers, this dependence can also be approximately represented as follows

$$c_{theor} = \frac{WKdn}{2}.$$

For example, for the above-considered case: $c_{theor} = 1.5$, which is with a quite high accuracy coincides with the value obtained from the numerical experiment.

Frequencies of DW oscillations in the case of pinning at different thin layers can be determined with the use of Fourier analysis. Let us compare the numerically obtained frequencies to the frequency determined by the following formula

$$\omega_{theor}^2 = (1 - K)\operatorname{sech}^2(W)\tanh(W/2), \tag{6}$$

suggested in [34] for the case of one thin magnetic layer (see the table). Frequencies of DW oscillations at different thin layers obtained through calculation are not always equal to each other and are different from the theoretical frequency of $\omega_{theor} = 0.311$ by $\Delta\omega$. The differences of frequencies from each other and from the value obtained by formula (6) may be related to the interaction between the DW and the oscillations localized in the region of thin magnetic layers, as well as to the fact that maximum amplitude of non-linear oscillations at pinning may be different and dependent on the DW speed. It can be seen from the table, that differences between numerically obtained frequencies are not more than 3%, and differences from the frequency determined by formula (6) can be from 3 to 20%.

With a decrease in d below a certain critical value, the number of possible scenarios with DW pinning decreases considerably. With initial speeds below the critical level, pinning of the DW oscillating in the region of all three thin magnetic layers is observed (lines *2* and *3*, Fig. 5, *b*). That is in this case three thin layers „work“ as a single effective layer. Frequencies of DW oscillations in this case are independent from the initial speed of the DW and equal to $\omega = 0.141$. It is worth to note that the frequency determined by formula (6) for the case of one impurity with $W = 3$ is 0.067. When the critical speed is achieved, the DW goes away to the infinity (line *1*, Fig. 5, *b*). The same scenario of DW pinning can be observed by decreasing K without changes in d (Fig. 5, *c*). The frequency in this case is also independent from the initial speed of the DW and equal to $\omega = 0.060$. It is worth to note that similar dynamic behavior of the DW has been obtained previously for the case of two thin magnetic layers as well [31].

4. Conclusion

The emergence of both known and new scenarios of DW dynamics in a multilayer ferromagnetic with three thin (nanoscale) and four wide magnetic layers as compared with previously considered cases of one and two thin magnetic layers is shown. A large difference is observed in the dynamics of the domain wall between the cases of thin magnetic layers with increased and decreased constant of magnetic anisotropy. In the second case the DW dynamics is accompanied by generation of localized waves. The interaction with them results, as in the case of one and two thin magnetic layers, in resonance effects of DW reflection from quantum wells and DW passing through them at an initial speed below the critical level. Non-linear dependencies of this critical speed of passing through regions of three thin layers on the layer sizes and anisotropy are determined. For the case with increased constant of magnetic anisotropy in the thin layer the final DW speed is nearly the same as the initial speed, i. e. the additional increase in the number of layers up to three does not result in any noticeable change in the speed. For the case with decreased constant of magnetic anisotropy these speeds may be significantly different from each other, and with increase in the number of layers this critical speed increases. A formula is suggested that contains a constant dependent on the number of thin layers. The formula relates the final DW speed to the initial speed, which is above the critical level. Results of the studies of critical speed of passing through multilayer regions can be used to determine the effective coercive force for multilayer ferromagnetic materials. It is clear, that the effect of strengthening of the „collective influence" of thin layers, which are „quantum wells" for the domain wall, with increase in the number of these layers will continue to result in emergence of new physical effects and scenarios of DW dynamics. Study of the case of periodic thin magnetic layers, that has been previously considered within the studying of spin wave dynamics, is of special interest.

Funding

This study was carried out with the support from the Russian Foundation for Basic Research, grant No. 20–31-90048 and within the scope of state assignment #AAAA-A19-119022290052-9.

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by Y.Alekseev