Semiconductors, 2023, Vol. 57, No. 2

Shubnikov-de Haas oscillations in 2D electron gas with anisotropic mobility

© D.V. Nomokonov, A.K. Bakarov, A.A. Bykov

Rzhanov Institute of Semiconductor Physics, Siberian Branch, Russian Academy of Sciences,
630090 Novosibirsk, Russia

E-mail: nomokonov@isp.nsc.ru

Received December 16, 2022
Revised February 22, 2023
Accepted March 2, 2023

Shubnikov-de Haas oscillations in selectively doped GaAs single quantum wells with AlAs/GaAs superlattice barriers has been studied at temperature $T = 4.2\,\mathrm{K}$ in magnetic fields $B < 1\,\mathrm{T}$. High-mobility heterostructures with thin spacer had been grown by molecular-beam epitaxy on (001) GaAs substrates. The mobilities of two-dimensional electron gas measured in two crystallographic directions [110] and $[\bar{1}10]$ differ from each other more than 50%. Properly adapted expression for Shubnikov-de Haas oscillations amplitudes in anisotropic samples has been used for correct analysis of this oscillations. It was stated that quantum life-time in our heterostructures as measured by Shubnikov-de Haas oscillations on Hall bars oriented in the directions [110] and $[\bar{1}10]$ varies less than 5%. Obtained results show that quantum life-time in two-dimensional electron system with anisotropic mobility is isotropic with aforementioned accuracy.

Keywords: Shubnikov-de Haas oscillations, anisotropic mobility, quantum life-time, superlattice barriers.

DOI: 10.21883/SC.2023.02.55952.4459

The low-temperature mobility of a two-dimensional (2D) electron gas in high-mobility selectively doped GaAs/AlGaAs heterostructures with a thick spacer ($d_S > 50\,\mathrm{nm}$) is limited by scattering not only by remote ionized donors and background impurities, but also by the heterointerface roughness [1]. Mobility μ of a 2D electron gas in such heterostructures grown by molecular beam epitaxy on (001) GaAs substrates is anisotropic. The electron mobility ($\mu = e\tau_t/m^*$, where τ_t is the transport scattering time and m^* is the effective electron mass) is maximal in crystallographic direction $[\bar{1}10]$ and minimal in direction [110]. The anisotropy of mobility μ in this case is due to scattering by the roughness of heterointerfaces extended in the $[\bar{1}10]$ direction and having a height of the order of an atomic monolayer [1].

Anisotropic scattering of electrons by the heterointerface roughness is manifested not only in high-mobility GaAs/AlGaAs heterostructures with a thick spacer and a correspondingly low 2D electron gas density ($n_e \sim 3 \cdot 10^{15}\,\mathrm{m}^{-2}$), but also in high-mobility GaAs/AlAs heterostructures with a thin spacer ($d_S < 50\,\mathrm{nm}$) and a higher electron density $n_e \sim 10^{16}\,\mathrm{m}^{-2}$ [2,3]. The studied GaAs/AlAs heterostructure is a single GaAs quantum well with short-period AlAs/GaAs superlattice barriers [4]. The scattering of 2D electrons by a random potential of ionized donors in this structure is suppressed due to both the spatial separation of doping and transport regions and the additional screening by X-electrons localized in AlAs layers [4–7]. Mobility μ_y in direction $[\bar{1}10]$ may be several times higher than mobility μ_x in direction [110] in such selectively doped heterostructures with a thin spacer grown on (001) GaAs substrates [3].

The processes of scattering of electrons of a 2D gas are characterized both by transport scattering time τ_t and quantum lifetime $\tau_q = 2\Gamma/\hbar$, where Γ is the half-width of quantum energy levels. In the general case, $\tau_t \neq \tau_q$, since τ_t is governed by large-angle scattering, while τ_q is affected by scattering by any angle [8,9]. In 2D systems with an anisotropic scattering potential, quantum lifetime τ_q measured using Shubnikov-de Haas (SdH) oscillations should be an effectively isotropic quantity [10]. This is the result of the averaging of outcomes of individual scattering events in the process of electron motion along cyclotron orbits [10]. This conclusions agrees with the results of examination of quantum transport in GaAs quantum wells with anisotropic mobility μ [11,12], but contradicts the data obtained for certain other 2D systems [13–15]. The causes of this discrepancy still remain debatable; further studies of SdH oscillations in various 2D electron systems with an anisotropic mobility are required. The study of anisotropic transport in single GaAs quantum wells with short-period AlAs/GaAs superlattice barriers is also made relevant by the fact that at present such structures are widely used in fundamental research [16] and in applications [17].

Mobilities μ_x and μ_y in the main mutually perpendicular directions x, y are equal in isotropic 2D systems; consequently, dissipative resistance $\rho_d = \rho_{xx} = \rho_{yy}$. Weak-field SdH oscillations at $\omega_c\tau_q \sim 1$ are then described by the following relation [18–20]:

$$\rho_d^{\mathrm{SdH}} = 4\rho_{0d} X(T) \exp(-\pi/\omega_c\tau_q) \cos(2\pi\varepsilon_{\mathrm{F}}/\hbar\omega_c - \pi), \quad (1)$$

where ρ_{0d} is the resistance in zero magnetic field, $X(T) = (2\pi^2 k_{\mathrm{B}}T/\hbar\omega_c)/\sinh(2\pi^2 k_{\mathrm{B}}T/\hbar\omega_c)$, $\omega_c = eB/m^*$, and ε_{F} is the Fermi energy. The results reported in [19]

Parameters of heterostructures

Structure number	$d_{\rm SQW}$ (nm)	d_S (nm)	$n_{\rm Si}$ ($10^{16}\,{\rm m}^{-2}$)	n_e ($10^{15}\,{\rm m}^{-2}$)	μ_y ${\rm m}^2/({\rm V}\cdot{\rm s})$	μ_x ${\rm m}^2/({\rm V}\cdot{\rm s})$	μ_y/μ_x
1	14	32.5	2.3	6.59	287	170	1.69
2	13	29.4	3.2	7.48	124	80.5	1.54

Note. $d_{\rm SQW}$ — width of a single GaAs quantum well; d_S — spacer thickness; $n_{\rm Si}$ — net density of remote Si donors in thin δ-doped GaAs layers; n_e — density of a 2D electron gas in a single GaAs quantum well; μ_y and μ_x — mobilities in directions $[\bar{1}10]$ and $[110]$, respectively.

make it easy to generalize formula (1) to a 2D system with an anisotropic mobility ($\mu_x \neq \mu_y$). This generalization is performed by substituting the isotropic Drude conductivity in the initial expressions from [19] with an anisotropic conductivity:

$$\sigma_{xx}(B) = \frac{e^2}{m^*}\frac{\tau_x n}{1+\omega_c^2\tau_x\tau_y}, \quad \sigma_{yy}(B) = \frac{e^2}{m^*}\frac{\tau_y n}{1+\omega_c^2\tau_x\tau_y}, \tag{2}$$

$$\sigma_{xy}(B) = -\sigma_{yx}(B) = e\frac{\partial N}{\partial B} - \frac{e^2}{m^*}\frac{\omega_c\tau_x\tau_y n}{1+\omega_c^2\tau_x\tau_y}, \tag{3}$$

where $n = n_0(1+\Delta g/g_0)$, n_0 is the carrier density in zero magnetic field, $\Delta g/g_0$ is the relative perturbation of the density of states in a magnetic field, $\tau_x = \tau_{0x}(1-\Delta g/g_0)$ and $\tau_y = \tau_{0y}(1-\Delta g/g_0)$ are the transport lifetimes of an electron moving in directions x and y, τ_{0x} and τ_{0y} are the transport lifetimes in zero field, and N is the number of states below the Fermi energy per unit area.

Inverting a $\hat{\sigma}(B)$ conductivity tensor represented by formulae (2) and (3), one finds a $\hat{\rho}(B)$ resistivity tensor. Its diagonal components in the linear small-perturbation approximation ($\Delta g/g_0 \ll 1$) are

$$\rho_{xx}(B) = \frac{\sigma_{yy}(B)}{\det\hat{\sigma}} \approx \frac{m^*}{e^2\tau_x n}\left(1 + 2\frac{B}{n}\frac{\partial N}{\partial B}\right), \tag{4}$$

$$\rho_{yy}(B) = \frac{\sigma_{xx}(B)}{\det\hat{\sigma}} \approx \frac{m^*}{e^2\tau_y n}\left(1 + 2\frac{B}{n}\frac{\partial N}{\partial B}\right), \tag{5}$$

where $\det\hat{\sigma} \equiv \sigma_{xx}\sigma_{yy} + \sigma_{xy}^2$. Following [19], we use the fact that $\tau_x n = \tau_{0x}n_0$ and $\tau_y n = \tau_{0y}n_0$. The final result is

$$\rho_{xx}(B) \approx \rho_{0xx}(1 + 2\Delta g/g_0),$$

$$\rho_{yy}(B) \approx \rho_{0yy}(1 + 2\Delta g/g_0), \tag{6}$$

where

$$\rho_{0xx} = \frac{m^*}{e^2\tau_{0x}n_0} \quad \text{and} \quad \rho_{oyy} = \frac{m^*}{e^2\tau_{0y}n_0}$$

are the resistivities in zero magnetic field in directions x and y, respectively. It follows from expressions (6) that

$$\frac{\rho_{xx}(B)-\rho_{0xx}}{\rho_{0xx}} = \frac{\rho_{yy}(B)-\rho_{0yy}}{\rho_{0yy}} \approx 2\frac{\Delta g}{g_0}. \tag{7}$$

Formulae (7) suggest that relevant values of ρ_{0d} need to be used to construct Dingle plots for each direction. Depending on the considered direction, ρ_d in an anisotropic structure is substituted with ρ_{xx} or ρ_{yy}, and ρ_{0d} in formula (1) is substituted with ρ_{0xx} or ρ_{0yy}. This is the modification of known formula (1) [18–20] that takes into account the anisotropy of a 2D electron gas. Normalized amplitudes of SdH oscillations in an anisotropic system are then written as

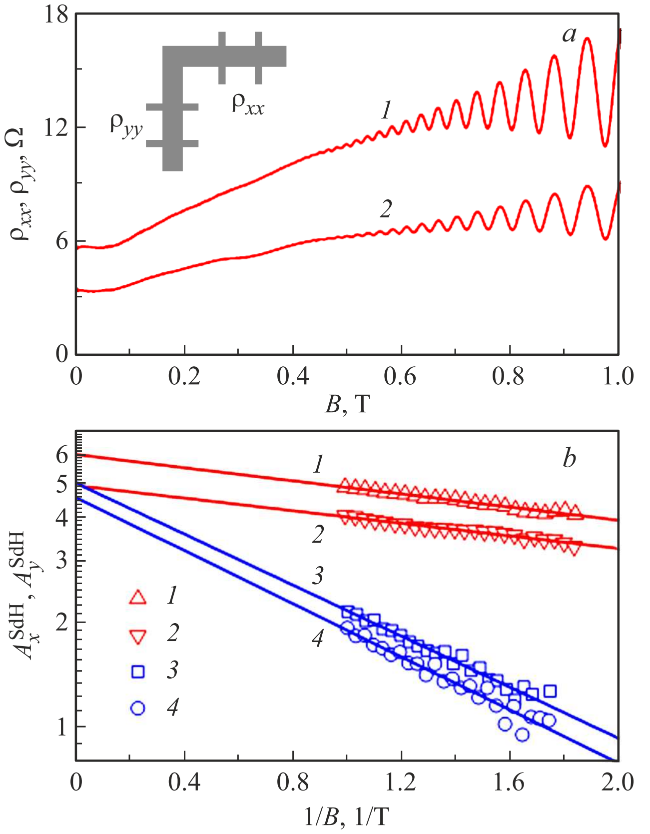
$$A_x^{\rm SdH} = \Delta\rho_x^{\rm SdH}/\rho_{0xx}X(T) = A_{0x}^{\rm SdH}\exp(-\pi/\omega_c\tau_{qx}), \tag{8}$$

$$A_y^{\rm SdH} = \Delta\rho_y^{\rm SdH}/\rho_{0yy}X(T) = A_{0y}^{\rm SdH}\exp(-\pi/\omega_c\tau_{qy}), \tag{9}$$

where $\Delta\rho_x^{\rm SdH}$ and $\Delta\rho_y^{\rm SdH}$ are the amplitudes of SdH oscillations measured in the corresponding directions; $A_{0x}^{\rm SdH} = A_{0y}^{\rm SdH} = 4$. According to (8) and (9), dependences $A_x^{\rm SdH}(1/B)$ and $A_y^{\rm SdH}(1/B)$ in semi-log scale are linear with initial points $A_x^{\rm SdH}(1/B=0) = A_y^{\rm SdH}(1/B=0) = 4$ and with slopes defined by quantities τ_{qx} and τ_{qy}.

The studied selectively doped GaAs/AlAs heterostructures were grown by molecular beam epitaxy on semi-insulating GaAs (100) substrates. These heterostructures were single GaAs quantum wells with width $d_{\rm SQW}$ and short-period AlAs/GaAs superlattice barriers consisting of thin AlAs and GaAs layers [4]. Two Si δ-layers located within thin GaAs layers at distance d_S from the upper and lower heterointerfaces of single GaAs quantum wells served as sources of free electrons. L-shaped Hall bars (see the inset in the figure) oriented along the [110] and $[\bar{1}10]$ crystallographic directions were fabricated based on the grown heterostructures via optical lithography and liquid etching. The AC resistance of bars ($I_{\rm ac} < 1\,\mu$A, $f_{\rm ac} \sim 0.5\,$kHz) was measured at $T = 4.2\,$K in magnetic fields $B < 1\,$T. The parameters of the studied heterostructures are listed in the table.

Experimental dependences $\rho_{xx}(B)$ and $\rho_{yy}(B)$ for structure 1 (panel *a* of the figure) demonstrate that SdH oscillations in the examined 2D electron system at $T = 4.2\,$K are manifested at $B > 0.5\,$T. A quantum positive magnetoresistance [21,22] and oscillations induced by scattering of a 2D electron gas by acoustical phonons [23,24] are observed in weaker magnetic fields. In agreement with formulae (8) and (9), the amplitudes of SdH oscillations for ρ_{xx} and ρ_{yy} differ. Experimental (symbols) and calculated (solid lines) dependences of $A_x^{\rm SdH}$ and $A_y^{\rm SdH}$ on $1/B$ plotted in semi-log scale for structures 1 and 2 are shown in panel *b* of the figure. Dependences $A_x^{\rm SdH}(1/B)$ and $A_y^{\rm SdH}(1/B)$ are linear in semi-log scale and have equal (to within 5%) slopes for each structure. The values of $A_{0x}^{\rm SdH}$ and $A_{0y}^{\rm SdH}$ differ insignificantly from the theoretical value $A_{0x}^{\rm SdH} = A_{0y}^{\rm SdH} = 4$.



a — Experimental dependences of ρ_{xx} and ρ_{yy} on B measured with an L-shaped bar at $T = 4.2\,\mathrm{K}$ for structure 1: *1* — ρ_{xx}, *2* — ρ_{yy}. The geometry of an L-shaped bar is shown in the inset. *b* — Dependences of $A_x^{\rm SdH}$ and $A_y^{\rm SdH}$ on $1/B$: *1* and *2* — structure 1; *3* and *4* — structure 2. Symbols denote experimental data. Solid lines represent the results of calculation performed using formulae (8) and (9): *1* — $A_{0x}^{\rm SdH} = 6.05$, $\tau_{qx} = 5.61\,\mathrm{ps}$; *2* — $A_{0y}^{\rm SdH} = 4.94$, $\tau_{qy} = 5.85\,\mathrm{ps}$; *3* — $A_{0x}^{\rm SdH} = 5.02$, $\tau_{qx} = 1.44\,\mathrm{ps}$; and *4* — $A_{0y}^{\rm SdH} = 4.57$, $\tau_{qy} = 1.38\,\mathrm{ps}$.

The equality of slopes of dependences $A_y^{\rm SdH}(1/B)$ and $A_x^{\rm SdH}(1/B)$ indicates that quantum lifetime τ_q measured using SdH oscillations is isotropic (to within 5%) in the examined system. The quantum lifetime determined by analyzing SdH oscillations is necessarily effectively isotropic, since it is averaged over cyclotron orbits of motion of an electron in a magnetic field [10]. Therefore, the anisotropy of τ_q reported in [13,15] is, in our view, the result of an incorrect application of the van der Pauw method [25]. In systems with an anisotropic conductivity, this method provides an opportunity to measure ρ_{xx} and ρ_{yy} only at $B = 0$ [26]. It is not possible to derive dependences $\rho_{xx}(B)$ and $\rho_{yy}(B)$ in an anisotropic system directly from resistance dependences $R_{xx}(B)$ and $R_{yy}(B)$ measured using the van der Pauw method. In order to determine such dependences, one needs first to solve the corresponding non-standard boundary value problem for each magnitude of magnetic field B and find the potential distribution [26]. Therefore, the van der Pauw method is not suitable for the determination of τ_q from SdH oscillation data in the presence of anisotropy and a magnetic field. A correct characterization of conductivity anisotropy is needed in the study of magnetotransport characteristics of structures not only in the van der Pauw geometry, but also when anisotropic Corbino disks are used [27]. The anisotropy of τ_q, which was revealed in [14] in the analysis of SdH oscillations measured in a Hall bar geometry, also appears dubious. In our opinion, inaccuracies in data processing and a relatively narrow probed range of fields are the probable reasons why an erroneous conclusion regarding the τ_q anisotropy was made.

Thus, Shubnikov-de Haas oscillations in a 2D electron gas with an anisotropic conductivity in selectively doped single GaAs quantum wells with short-period GaAs/AlAs superlattice barriers were studied. Mobility anisotropy μ_y/μ_x in the studied heterostructures in crystallographic directions $[\bar{1}10]$ and $[110]$ was greater than 1.5. A properly adapted formula for the amplitude of SdH oscillations was used to perform a correct analysis of Shubnikov-de Haas oscillations in a 2D electron gas. It was found experimentally that the quantum lifetime values measured by means of the Shubnikov-de Haas effect with Hall bars oriented along directions $[\bar{1}10]$ and $[110]$ are equal to within 5%.

Funding

This study was supported by the Russian Foundation for Basic Research, project No. 20-02-00309.

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] Y. Tokura, T. Saku, S. Tarucha, Y. Horikoshi. Phys. Rev. B, **46**, 15558 (1992).
[2] A.A. Bykov, A.K. Bakarov, A.V. Goran, A.V. Latyshev, A.I. Toropov. JETP Lett., **74**, 164 (2001).
[3] K.-J. Friedland, R. Hey, O. Bierwagen, H. Kostial, Y. Hirayama, K.H. Ploog. Physica E, **13**, 642 (2002).
[4] K.-J. Friedland, R. Hey, H. Kostial, R. Klann, K. Ploog. Phys. Rev. Lett., **77**, 4616 (1996).
[5] D.V. Dmitriev, I.S. Strygin, A.A. Bykov, S. Dietrich, S.A. Vitkalov. JETP Lett., **95**, 420 (2012).
[6] M. Sammon, M.A. Zudov, B.I. Shklovskii. Phys. Rev. Mater., **2**, 064604 (2018).
[7] A.A. Bykov, I.S. Strygin, A.V. Goran, D.V. Nomokonov, A.K. Bakarov. JETP Lett., **112**, 437 (2020).
[8] J.P. Harrang, R.J. Higgins, R.K. Goodall, P.R. Jay, M. Laviron, P. Delescluse. Phys. Rev. B, **32**, 8126 (1985).
[9] S. Das Sarma, Frank Stern. Rev. B, **32**, 8442 (1985).
[10] A.C. Churchill, G.H. Kim, A. Kurobe, M.Y. Simmons, D.A. Ritchie, M. Pepper, G.A.C. Jones. J. Phys.: Condens. Matter, **6**, 6131 (1994).
[11] A.A. Bykov, D.R. Islamov, A.V. Goran, A.K. Bakarov. JETP Lett., **86**, 779 (2008).

[12] A.A. Bykov, A.V. Goran, W. Mayer, S.A. Vitkalov. JETP Lett., **98**, 717 (2014).
[13] S. Elhamri, W.C. Mitchel, W.D. Mitchell, R. Berney, G.R. Landis. J. Electron. Mater., **35**, 599 (2006).
[14] C. Morrison, M. Myronov. Appl. Phys. Lett., **111**, 192103 (2017).
[15] L. Yang, J. Wang, T. Wang, M. Wu, P. Wang, D. Wang, X. Yang, F. Xu, W. Ge, X. Wu, X. Wang, B. Shen. Appl. Phys. Lett., **115**, 152107 (2019).
[16] M.L. Savchenko, A. Shuvaev, I.A. Dmitriev, S.D. Ganichev, Z.D. Kvon, A. Pimenov. Phys. Rev. B, **106**, L161408 (2022).
[17] A.B. Pashkovskii, S.A. Bogdanov, A.K. Bakarov, K.S. Zhuravlev, V.G. Lapin, V.M. Lukashin, S.N. Karpov, D.Ju. Protasov, I.A. Rogachev, E.V. Tereshkin. Tech. Phys. Lett., **48** (6), 44 (2022).
[18] I.M. Lifshitz, A.M. Kosevich. Sov. Phys. JETP, **2**, 636 (1956).
[19] A. Isihara, L. Smrcka. J. Phys. C: Solid State Phys., **19**, 6777 (1986).
[20] P.T. Coleridge. Phys. Rev. B, **44**, 3793 (1991).
[21] M.G. Vavilov, I.L. Aleiner. Phys. Rev. B, **69**, 035303 (2004).
[22] S. Dietrich, S. Vitkalov, D.V. Dmitriev, A.A. Bykov. Phys. Rev. B, **85**, 115312 (2012).
[23] A.A. Bykov, A.K. Kalagin, A.K. Bakarov. JETP Lett., **81**, 523 (2005).
[24] O.E. Raichev. Phys. Rev. B, **80**, 075318 (2009).
[25] S.H. Simon. Phys. Rev. Lett., **83**, 4223 (1999).
[26] O. Bierwagen, R. Pomraenke, S. Eilers, W.T. Masselink. Phys. Rev. B, **70**, 165307 (2004).
[27] D.V. Nomokonov, A.V. Goran, A.A. Bykov. J. Appl. Phys., **125**, 164301 (2019).

Translated by D.Safin