09

Measurement of distance by the maximum frequency of the interference signal with harmonic deviation of the wavelength of the self-mixing laser

© A.V. Skripal, S.Yu. Dobdin, M.G. Inkin, A.V. Dzhafarov

Saratov National Research State University, Saratov, Russia
e-mail: skripalav@info.sgu.ru

Received December 27, 2022
Revised December 27, 2022
Accepted December 27, 2022

A method is proposed for measuring absolute distances from the low-frequency spectrum of the interference signal of a frequency-modulated laser diode. The method of modulation of the self-mixing laser signal according to the harmonic law is used, in the spectrum of which a wide range of frequency components is observed. The connection between the maximum frequency of change of the interference signal and the absolute distance to the reflector is shown. A linear dependence of the frequency corresponding to the inflection region of the decay envelope of the interference signal spectrum on the distance is shown, which can be used to implement a non-contact distance measurement method with harmonic modulation of the self-mixing laser supply current.

Keywords: laser interferometry, self-mixing laser, semiconductor laser, laser radiation modulation, distance measurement, signal spectral analysis.

DOI: 10.21883/TP.2023.04.55940.282-22

Introduction

Semiconductor lasers with delayed feedback, referred to as laser autodynes [1,2], have wide applications for measuring micro-displacements [3], absolute distances [4–6], for vibrational analysis [7], measuring thickness and refractive index of thin films [8], for angular measurements [9], for determining velocity and acceleration at microdisplacements, including blood measurements in vivo [10–13], etc.

A distinctive feature of laser autodynes is the possibility of probe measurements at a considerable distance from the reflecting surface. The compactness of such autodynes can give a significant advantage in the implementation of measurements of microdisplacements and absolute distances in comparison with triangulation systems and optical interferometers, despite their much higher accuracy [14,15].

Measurements of absolute distances using frequency modulation of the laser diode supply current began with very low accuracy values (a few millimeters) [4] and have now reached values of several hundreds and even tens of microns [5,6]. The main progress in improving the accuracy of measuring absolute distances is associated with the expansion of the range of laser wavelength deviation from hundredths of a nanometer to several nanometers. Such deviation values are obtained by varying the laser diode supply current over a wide range. This leads to a nonlinear dependence of the laser diode radiation wavelength on its supply current and is to be taken into account when measuring the distance.

Earlier, we have proposed a method of harmonic modulation of laser radiation based on measuring the amplitudes of the autodyne signal spectral components, which allows measuring absolute values of the distance to reflector at small deviations of the laser diode wavelength [16]. The advantages of measuring absolute distances up to 10 cm with harmonic modulation of the laser autodyne wavelength in comparison with the triangular modulation method were previously discussed in Ref. [17]. The disadvantage of the method based on measuring the amplitudes of the spectral components of the autodyne signal is the need to search for a uniqueness region when solving the inverse problem.

In this paper, we propose a method for measuring absolute distances in the case of harmonic frequency modulation of laser radiation with respect to the frequency determined by approximating the decline of the frequency spectrum of an autodyne signal.

1. Interference signal model for harmonic modulation of the laser autodyne wavelength

When the laser wavelength is modulated, the semiconductor laser power P can be expressed in terms of the amplitude and phase components, depending on the pump current density $j(t)$ [18]:

$$P(j(t)) = P_1(j(t)) + P_2\cos(\omega(j(t))\tau), \qquad (1)$$

where P_1 is the direct power component, P_2 is the amplitude power component depending on the wave phase incursion $\omega(j(t))\tau$ in a system with external reflector, $\tau = 2 \cdot L/c$ is the time it takes the laser radiation to travel the distance to the external reflector and back, $\omega(j(t))$ is the semiconductor laser radiation frequency depending on the pump current density $j(t)$ and the feedback level.

The autodyne signal parameters are affected by the level of external optical feedback [19–21]. As shown earlier, it is possible to choose the feedback level at which the semiconductor laser radiation frequency will not change significantly and thereby introduce distortions into the shape of the interference signal [16,22].

In this case, upon harmonic modulation of the pump current density $j(t)$, the semiconductor laser radiation frequency takes the form

$$\omega(j(t)) = \omega_0 + \omega_A \sin(2\pi\nu_1 t), \quad (2)$$

where ω_0 is the natural frequency of the semiconductor laser diode; ω_A is the frequency deviation of the semiconductor laser diode; ν_1 is the frequency of supply current modulation. Expression (3) for the power of radiation of the frequency modulated semiconductor laser will take the form ,

$$P(j(t)) = P_1(j(t)) + P_2 \cos(\omega_0\tau + \omega_A\tau \cdot \sin(\Omega t)), \quad (3)$$

where $\theta = \omega_0\tau$ is the steady-state phase of the autodyne signal, $\sigma = \omega_A\tau$ is the amplitude of the current modulation phase, $\Omega = 2\pi\nu_1$ is the circular frequency of the laser diode supply current modulation.

Since only the phase component of the multifrequency autodyne signal is used to determine the distance, Eq. (3) can be written in the form

$$P(j(t)) = \cos\left(\theta + 4\pi L\frac{\Delta\lambda}{\lambda^2}\sin(\Omega t)\right). \quad (4)$$

To analyze the autodyne signal under conditions of harmonic modulation of the laser diode radiation wavelength, we will use the representation of the signal in the form of an expansion in a series in Bessel functions of the first kind J_n and in a Fourier series with amplitudes of the spectral components S_n. In this case, without considering the constant component, $P(t)$ takes the form

$$(P(t) = \cos(\theta)J_0(\sigma) + 2\cos(\theta)\sum_{n=1}^{\infty} J_{2n}(\sigma)\cos(2n\Omega t)$$

$$- 2\sin(\theta)\sum_{n=1}^{\infty} J_{2n-1}(\sigma)\cos((2n-1)(\Omega t)). \quad (5)$$

To analyze the spectrum of an autodyne signal, we use the relationship between the frequency of the spectral harmonic and the instantaneous velocity of the reflector. Let us assume that the object is moving uniformly in a straight line with a constant speed ϑ in the observation interval t. In this case, the time dependence of the external resonator roundtrip time for laser radiation will take the following form:

$$\tau(t) = \frac{2}{c}(L_0 + \vartheta \cdot t). \quad (6)$$

Then the normalized component of the autodyne signal can be presented in the form

$$P = \cos\left(\frac{2\omega_0}{c}(L_0 + \vartheta \cdot t)\right), \quad (7)$$

and keeping in mind that $\omega_0 = 2\pi \cdot c/\lambda_0$, we get

$$P = \cos\left(\frac{4\pi \cdot \vartheta \cdot t}{\lambda_0} + \frac{4\pi \cdot L_0}{\lambda_0}\right). \quad (8)$$

Comparing expression (6) for the normalized variable component of the autodyne signal with the harmonic cosine function

$$P = \cos(\Omega \cdot t + \varepsilon), \quad (9)$$

where $\varepsilon = 4\pi \cdot L_0/\lambda_0$ is the initial phase, and taking into account that $\Omega = 2\pi \cdot \nu_n$, where ν_n is the frequency of the autodyne signal variation at constant variation rate of the laser radiation wavelength, we get

$$\nu_n = \frac{2 \cdot \vartheta}{\lambda_0},$$

from where the rate is

$$\vartheta = \nu_n \cdot \lambda_0/2, \quad (10)$$

that is, for the harmonic modulation of the laser radiation wavelength, it is possible to obtain the instantaneous rate of wavelength variation by determining the frequency of normalized variable component of the autodyne signal spectrum by means of Eq. (10).

Equating the instantaneous rate (10) obtained from the autodyne signal spectrum to the derivative of the frequency deviation amplitude of the semiconductor laser diode

$$\frac{d\left(4\pi L\frac{\Delta\lambda}{\lambda^2}\sin(2\pi\nu_1 t)\right)}{dt} = \frac{\nu_n \cdot \lambda_0}{2},$$

it is possible to obtain the relationship between the distance to the reflector with the spectral harmonic frequency ν_n:

$$L = \frac{\lambda^2}{4\pi\Delta\lambda}\frac{\nu_n}{\nu_1}. \quad (11)$$

With the properties of Fourier transform for threshold-restricted functions taken into account, the maximum instantaneous rate of deviation $\Delta\lambda$ will correspond to an inflection point of the envelope of the autodyne signal decline. Hence, it is the frequency corresponding to the inflection region of the autodyne signal inflection region that should be taken as the value of maximum frequency of the variation of the interference signal ν_n.

2. Computer simulation of the interference signal spectrum with harmonic modulation of the laser autodyne wavelength

The simulation of the autodyne signal was carried out with the following parameters: $\lambda = 650$ nm, deviation of the semiconductor laser diode radiation wavelength $\Delta\lambda = 0.067$ nm, distance to the object $L = 50$ mm, modulation frequency of the laser pump current $\nu_1 = 100$ Hz.

Figure 1, 2 show the model of the interference signal and its Fourier spectrum for different values of the radiation wavelength deviation $\Delta\lambda$ and the stationary phase of the autodyne signal θ.

In Fig. 1, one can observe low-frequency modulation due to the modulation of the laser diode radiation power at the modulation frequency of the laser supply current, as well as high-frequency modulation due to the interference of the reflected wave with the incident wave, forming a standing wave in the external resonator. With the deviation of the laser radiation wavelength, a displacement of the nodes and antinodes of the standing wave is observed, because of which the intensity of the laser radiation will change. The change in the radiation intensity is recorded by a photodetector built in on the reverse side of the emitting face of the laser autodyne.

As can be seen from Fig. 2, the interference signal spectrum consists of harmonics whose frequencies are multiples of the deviation frequency of the laser diode power supply current. The maximum deviation frequency of the laser supply current will correspond to the frequency ν_n, corresponding to the inflection region of the decline envelope of the spectrum of the interference signal, determined in the region of the minimum of the first derivative of the envelope function.

To improve the accuracy of determining the frequency ν_n, the decline of the interference signal spectrum was approximated by a polynomial of the form:

$$A_i(x) = a_0^{(i)} + a_1^{(i)}(x - x_i) + a_2^{(i)}(x - x_i)^2 + a_3^{(i)}(x - x_i)^3 + \dots a_n^{(i)}(x - x_i)^n.$$

Tables 1, 2 present the results of reconstructing the distance value from the results of approximating the decline of the interference signal spectrum by polynomials of various degrees.

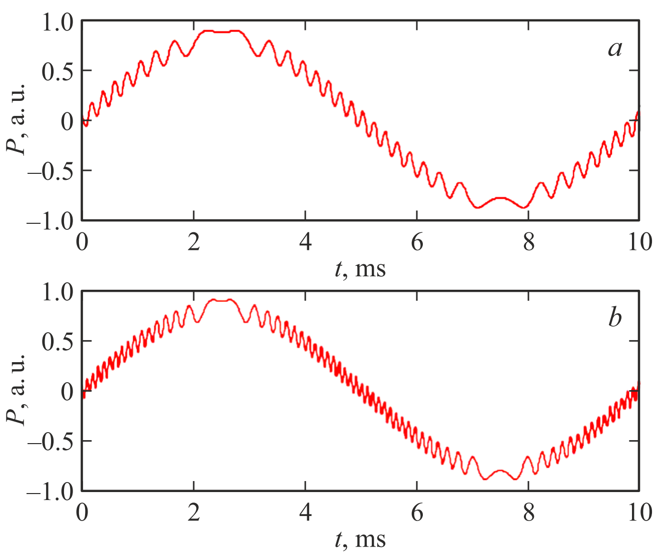

Figure 1. Autodyne signal at various deviations of wavelength $\Delta\lambda$: *a* — 0.067, *b* — 0.034 nm.

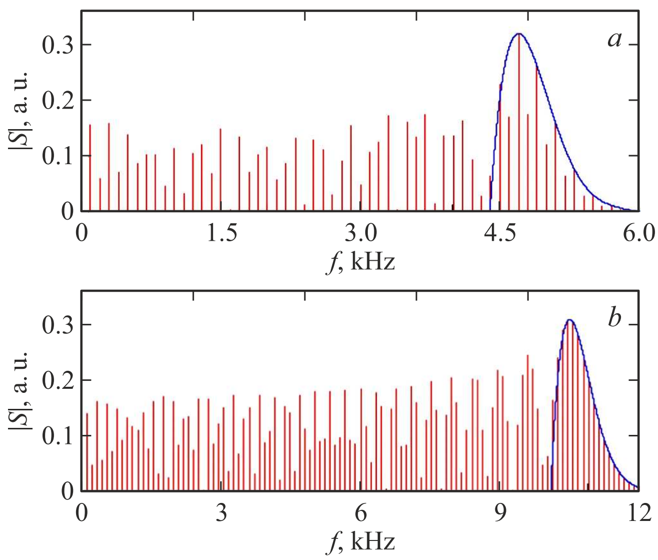

Figure 2. Spectrum of the interference signal upon modulation of the radiation power of the laser diode for various values of the wavelength deviation and the stationary phase: *a* — $\Delta\lambda = 0.067$ nm and $\theta = \pi/4$; *b* — $\Delta\lambda = 0.034$ nm and $\theta = \pi/3$.

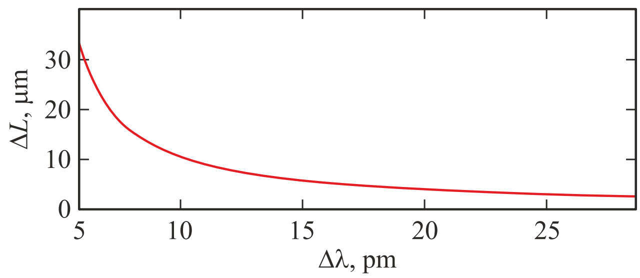

Figure 3. Dependence of the error of reconstructing the distance by the results of approximating the interference signal spectrum decline with a polynomial of the 5th order, calculated from the maximum frequency of the interference signal variation on the wavelength deviation $\Delta\lambda$.

It follows from the above tables that the accuracy of measurements can be improved by using the approximation of the interference signal spectrum decline by mixed harmonics at the stationary phase equal to $\theta = \pi/4$, as well as by increasing the deviation of the laser radiation wavelength. The highest accuracy of solving the inverse problem of determining the distance by means of Eq. (11) is achieved by choosing a polynomial of the 5th order ($n = 5$). The average distance determination accuracy in the range 50–100 mm was 28.5 μm, the minimum and maximum accuracy was 1.3 μm and 56.3 μm, respectively.

Figure 3 shows the dependence of the error in reconstructing the distance on the laser wavelength deviation, obtained by approximating the decline of the interference signal spectrum by a polynomial of the 5th degree, calculated from the maximum frequency of the interference signal variation.

Table 1. Error values when restoring the distance based on approximating the decline in the interference signal spectrum by polynomials of various degrees at $\Delta\lambda = 34$ pm and $\theta = \pi/4$

View of approximation	Error		
	Odd harmonics	Even harmonics	Mixed-parity harmonics
4th degree	300 μm	1090 μm	220 μm
5th degree	15 μm	60 μm	10 μm
6th degree	100 μm	70 μm	80 μm
7th degree	70 μm	80 μm	40 μm

Table 2. Error values when restoring the distance based on approximating the decline in the interference signal spectrum by polynomials of various degrees at $\Delta\lambda = 67$ pm and $\theta = \pi/4$

View of approximation	Error		
	Odd harmonics	Even harmonics	Mixed-parity harmonics
4th degree	75 μm	330 μm	50 μm
5th degree	9 μm	20 μm	5 μm
6th degree	35 μm	25 μm	30 μm
7th degree	25 μm	20 μm	10 μm

As can be seen from Fig. 3, there is a decrease in the average measurement error with an increase in the laser wavelength deviation, which can be explained by the enrichment of the spectrum of the interference signal and the use of higher-order harmonics to determine the absolute distance.

3. Experimental part

The measurements were carried out using a setup, whose block diagram is shown in Fig. 4. The setup included the semiconductor laser module HLDP 650a, based on quantum-dimensional structures with a diffraction-limited single spatial mode and a wavelength of 654 nm *1*. The laser autodyne radiation was focused on the surface of the object *2*, the diameter of the laser radiation spot on the object surface being 1 mm. The operating mode of the laser diode was specified by the supply current control unit *5*. The laser wavelength was modulated using a signal generator built into the laboratory station of virtual instruments NIELVIS 3 (National Instruments, USA).

The radiation reflected from an external reflector was directed into the laser cavity, the variation of its intensity was recorded by photodetector *4*. The detected signal passed through the signal amplifier *6* and the ac signal filter *7* and arrived at the input of the analog-to-digital convertor *8* with a sampling frequency of 150 kHz, connected to the computer *9*. The frequency modulation of the laser was implemented by changing the supply current. The deviation was measured by means of the echelle spectrometer SHR (Solar LS, Belarus).

The laser diode radiation was focused by a lens with the numerical aperture $NA = 0.25$. The reflector was located in the plane of laser beam focusing. The maximal coefficient of autodyne amplification is known to occur near the threshold values of the supply current of a semiconductor laser. Due to this fact, the supply current of laser autodyne was set at a level of 1.2 of the threshold current value ($I_{th} = 25$ mA), in this case the laser diode radiation power decreased to 2 mW, the nominal operating power being 5 mW.

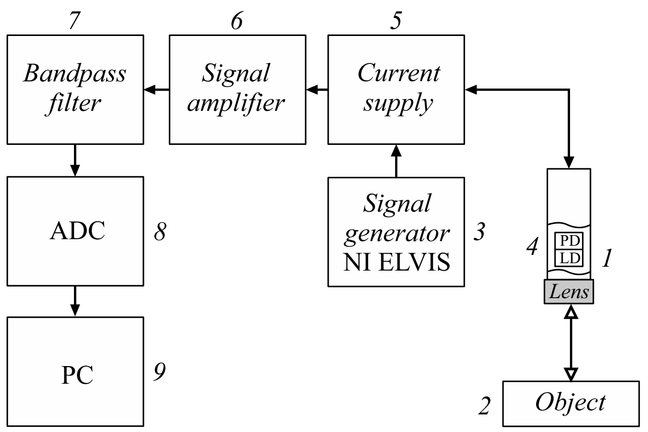

Figure 4. Block diagram of the experimental setup: *1* — laser autodyne, *2* — object, *3* — NIELVIS generator of signals, *4* — photo detector, *5* — supply current control unit, *6* — signal amplifier, *7* — bandpass filter, *8* — ADC, *9* — computer.

Table 3. Absolute distance values reconstructed from the results of approximating the interference signal spectrum decline by a polynomial of the 5-th degree for different values of deviation of the laser autodyne wavelength

№	Deviation of laser radiation wavelength					
	39 pm		58 pm		73 pm	
	Distance, mm	MSE, μm	Distance, mm	MSE, μm	Distance, mm	MSE, μm
1	51.64	33	51.6	27	51.58	25
2	51.69		51.54		51.57	
3	51.7		51.57		51.54	
4	51.73		51.61		51.62	
5	51.72		51.63		51.6	
6	51.68		51.6		51.61	
7	51.75		51.57		51.56	
8	51.71		51.61		51.58	

To reduce the feedback level, controlled defocusing of the laser beam was used. Using sets of different spectral components of the autodyne signal, the feedback level was determined, which did not exceed $C < 0.2$ during measurements. When the feedback level was chosen, the autodyne operation mode was controlled using a high-resolution SHR echelle spectrometer, in the spectrum of which, even in the presence of feedback, only one lasing frequency was observed, which excluded the occurrence of a chaotic power instability mode and laser diode radiation frequency deviation.

The autodyne signal was filtered using a third-order Butterworth filter. Signal processing was carried out in Python using the scipy library.

Figure 5 shows the measured autodyne signal *a* and its spectrum *b* for the case of voltage sweep according to the harmonic law over a time of 10 ms ($\nu_1 = 100$ Hz) in the range of laser radiation wavelength deviation from 0 to 38 pm. To increase the accuracy of measurements, the cycles of voltage sweep were multiply repeated.

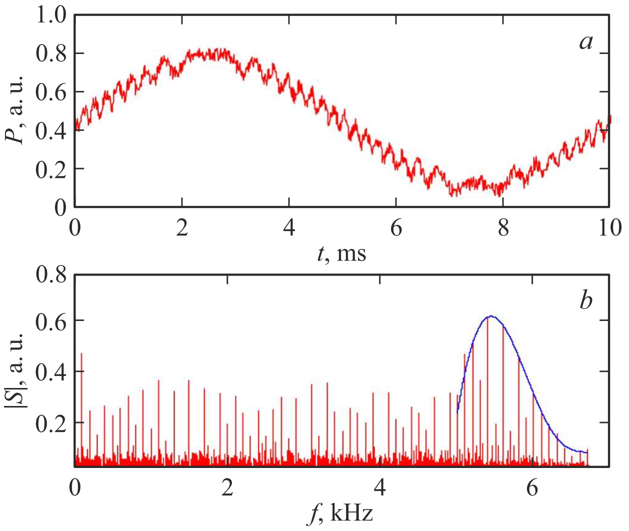

Figure 5. The measured autodyne signal (a) and its spectrum (b) for the case of harmonic voltage sweep over a time of 10 ms, the deviation of laser radiation wavelength deviation was 38 pm.

The decline of the autodyne signal was approximated by a 5th-order polynomial in the region of higher spectral components of the autodyne signal (solid curve). The distance to the reflector was determined from the value of the frequency ν_n corresponding to the inflection point of the approximating curve in the region of higher spectral components of the autodyne signal using relation (11). The values of the absolute distance, reconstructed from the results of the approximation of the interference signal spectrum decline by a linear polynomial of the 5th degree, are given in Table 3 for several measurements corresponding to random values of the stationary phase Θ, due to an uncontrolled change in the temperature of the laser crystal. The results of the calculation of the root-mean-square (RMS) deviation at a fixed distance and the deviation of the laser radiation wavelength are also presented here.

The average value of the absolute distance amounted to 51.63 mm. The minimum scatter value of 25 μm is observed at a deviation of 73 pm. The average scatter of measurements at different values of the deviation of the laser radiation wavelength was 76 μm.

Conclusions

Compared to the well-known method for determining the distance from the spectrum of a frequency-modulated laser autodyne with triangular modulation, the harmonic frequency modulation of laser radiation makes it possible to increase the distance sensitivity of the method due to the possibility of measuring the frequency of the spectral component, which is not a multiple of the external modulation frequency. Harmonic frequency modulation of laser radiation gives rise to a set of spectral components in the autodyne signal spectrum. Based on the results of approximating the interference signal spectrum decline by a polynomial, the frequency of the interference signal is calculated, corresponding to the inflection region of the decline envelope of the interference signal spectrum, which is determined in the region of the minimum of the first derivative of the envelope function. The maximum frequency of the interference signal variation determined in this way is used to calculate the absolute distance with an accuracy of tens of microns.

Experimental measurements confirmed the possibility of measuring the distance by the proposed method with high accuracy, which increases with growing laser radiation wavelength deviation and in measurements with a stationary phase incursion equal to $\pi/4$. The linear dependence of the frequency corresponding to the inflection region of the decay envelope of the interference signal spectrum on distance can be used to implement a non-contact method for measuring the absolute distance with harmonic current modulation of the laser radiation wavelength.

Funding

The work was financially supported by the Innovation Promotion Fund (project № 171GSSS15-L/78935).

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] A.P. Napartovich, A.G. Sukharev. Kvant. elektron., **34** (7), 630 (2004) (in Russian).
V.Ya. Noskov, S.M. Smolsky. Radiotekhnika, **8**, 91 (2013).
[3] H. Wang, Y. Ruan, Y. Yu, Q. Guo, J. Xi,, J. Tong. IEEE Access, **8**, 123253 (2020). DOI: 10.1109/ACCESS.2020.3007516
[4] M.C. Amann, T. Bosch, M. Lescure, R. Myllyla, M. Rioux. Opt. Engineer., **40** (1), 10 (2001). DOI: 10.1117/1.1330700
[5] M. Norgia, A. Magnani, A. Pesatori. Rev. Sci. Instrum., **83** (4), 045113 (2012). DOI: 10.1063/1.3703311
[6] K. Ke, K. Kou, X. Li, L. Li, H. Xiang. Appl. Opt., **53** (27), 6280 (2014). DOI: 10.1364/AO.53.006280
[7] Z. Zhang, C. Li, Z. Huang. Opti. Commun., **436**, 192 (2019). DOI: 10.1016/j.optcom.2018.12.032
[8] S. Donati. Laser Photon., **6** (3), 393 (2012). DOI: 10.1002/lpor.201100002
[9] S. Donati. Proceed. SPIE, 10150, 101501I (2016). DOI: 10.1117/12.2248912
[10] J. Chen, X. Wang, Y. Wu, Y. Yang, M. Qiu, M. Wang, Y. Li. Appl. Opt., 61, 4687 (2022). DOI: 10.1364/AO.455671
[11] F.F. Mul, M.H. Koelink, A.L. Weijers, J. Greve, J.G. Aarnoudse, R. Graaff, A.C.M. Dassel. Appl. Opt., **31** (27), 5844 (1992). DOI: 10.1364/AO.31.005844
[12] K. Meigas, H. Hinrikus, R. Kattai, J. Lass. J. Biomed. Opt., **8** (1), 152 (2003). DOI: 10.1117/1.1528949
[13] S.K. Ozdemir, S. Takamiya, S. Ito, S. Shinohara, H. Yoshida. IEEE Transactions on Instrument. Measurement, **49** (5), 1029 (2000). DOI: 10.1109/19.872925
[14] Z. Dong, X. Sun, W. Liu, H. Yang. Sensors, **18** (10), 3527 (2018). DOI: 10.3390/s18103527
[15] T. Wu, Y. Hui, Z. Yan, Z. Li, Q. Li. Opt. Laser Technol., 89, 196 (2017). DOI: 10.1016/j.optlastec.2016.09.034
[16] A.V. Skripal, S.Yu. Dobdin, A.V. Jafarov, K.A. Sadchikova, V.B. Feklistov. Izvestiya Saratovskogo universiteta Novaya seriya. Seriya: Fizika, **20** (2), 84 (2020) (in Russian). DOI: 10.18500/1817-3020-2020-20-2-84-91
[17] D.A. Usanov, A.V. Skripal, S.Yu. Dobdin, A.V. Jafarov, I.S. Sokolenko. Komp'yuternaya optika, **43** (5), 797 (2019) (in Russian). DOI: 10.18287/2412-6179-2019-43-5-796-802
D.A. Usanov, A.V. Skripal, S.Yu. Dobdin, E.I. Astakhov, I.Yu. Kostyuchenko, A.V. Jafarov. Izvestiya Saratovskogo universiteta Novaya seriya. Seriya: Fizika, **18** (3), 189 (2018) (in Russian).
[19] H. Olesen, J. H. Osmundsen, B. Tromborg. IEEE J. Quantum Electron., **22** (6), 762 (1986). DOI: 10.1109/JQE.1986.1073061
[20] N. Schunk, K. Petermann. IEEE J. Quantum Electron, **24** (7), 1242 (1988). DOI: 10.1109/3.960
[21] V.Ya. Noskov, S.M. Smolsky, K.A. Ignatkov, A.P. Chupakhin. Ural'skiy radiotekhnicheskiy zhurnal, **3** (1), 7 (2019). DOI: 10.15826/urej.2019.3.1.001
[22] G. Giuliani, M. Norgia, S. Donati, T. Bosch. J. Opt. A: Pure Appl. Opt., 4, 283 (2002). DOI: 10.1088/1464-4258/4/6/371

Translated by Ego Translating