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Adiabatic guided modes of a three-layer integral optical waveguide

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The numerical solution of the problem of guided propagation of polarized light in a smooth junction of a planar waveguide is considered. Within the framework of the model of adiabatic guided modes, the system of Maxwell equations is reduced to a system of four ordinary differential equations and two algebraic equations for six components of the electromagnetic field in the zeroth approximation and the same number of equations in the first approximation. The multilayer structure of waveguides makes it possible to reduce the problem to a homogeneous system of linear algebraic equations, whose nontrivial solvability condition yields the dispersion equation. Auxiliary eigenvalue problems for describing the adiabatic modes of the waveguide are solved.

Keywords: smoothly irregular integrated-optical multilayer waveguides, eigenvalue and eigenvector problems, single-mode propagation of adiabatic waveguide modes.

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Introduction

The object of our consideration is the guided-wave propagation of monochromatic electromagnetic radiation of the optical range in thin-film integrated optical structures. Such structures are complex waveguide structures formed by applying additional guiding layers with various (smoothly irregular) geometric configurations onto a flat substrate. By a thin-film waveguide, we mean a waveguide whose guiding layer (core) thickness is comparable to the wavelength λ of the propagating radiation

Integrated optical structures are called smoothly irregular if they satisfy the inequalities specified by the geometry of the additional waveguide layer:

$$\left|\frac{\partial h}{\partial y}\right|, \quad \left|\frac{\partial h}{\partial z}\right| \ll 1.$$

The guided-wave propagation of monochromatic polarized electromagnetic radiation in integrated optical waveguides is described by the Maxwell equations.

In the absence of charges and currents, the scalar Maxwell equations follow from the vector ones, and the boundary conditions for normal components follow from the boundary conditions to the tangential components of the electromagnetic field [1,2]. In Cartesian coordinates corresponding to the geometry of the substrate (or the three-layered planar dielectric waveguide) the Maxwell equations have the form

$$\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z}=\frac{\varepsilon}{c}\frac{\partial E_x}{\partial t}, \quad \frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}=-\frac{\mu}{c}\frac{\partial H_x}{\partial t},$$
$$\frac{\partial H_x}{\partial z}-\frac{\partial H_z}{\partial x}=\frac{\varepsilon}{c}\frac{\partial E_y}{\partial t}, \quad \frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}=-\frac{\mu}{c}\frac{\partial H_y}{\partial t},$$
$$\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y}=\frac{\varepsilon}{c}\frac{\partial E_z}{\partial t}, \quad \frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}=-\frac{\mu}{c}\frac{\partial H_z}{\partial t}. \quad (1)$$

To construct the model of adiabatic guided modes (AGMs), we present the solutions of Eqs. (1) in terms of the locally normal guides modes of a locally planar reference waveguide (see [3,4]), which in the method of asymptotic expansion take the form:

$$\mathbf{E}(x;y,z,t)=\sum_{s=0}^{\infty}\frac{\mathbf{E}_s(x;y,z)}{(-i\omega)^{\gamma+s}}\exp\{i\omega t-ik_0\varphi(y,z)\},$$

$$\mathbf{H}(x;y,z,t)=\sum_{s=0}^{\infty}\frac{\mathbf{H}_s(x;y,z)}{(-i\omega)^{\gamma+s}}\exp\{i\omega t-ik_0\varphi(y,z)\}.$$

In the notation $\mathbf{E}_s(x;y,z)$, $\mathbf{H}_s(x;y,z)$ the separation of x from the rest arguments by a semicolon means the following assumption:

$$\frac{\partial \mathbf{E}_s(x;y,z)}{\partial y}, \quad \frac{\partial \mathbf{E}_s(x;y,z)}{\partial z}\sim\frac{1}{\omega}\frac{\partial \mathbf{E}_s(x;y,z)}{\partial x},$$
$$j=x,y,z,$$
$$\frac{\partial \mathbf{H}_s(x;y,z)}{\partial y}, \quad \frac{\partial \mathbf{H}_s(x;y,z)}{\partial z}\sim\frac{1}{\omega}\frac{\partial \mathbf{H}_s(x;y,z)}{\partial x},$$
$$j=x,y,z,$$

where ω is the circular frequency of the propagating monochromatic electromagnetic radiation.

Using the method of asymptotic expansion in dimensioned small parameter ω^{-1} [5–7], we obtain a system of homogeneous equations in the zeroth approximation:

$$-ik_0\frac{\partial\varphi}{\partial y}H_0^z+ik_0\frac{\partial\varphi}{\partial z}H_0^y=ik_0\varepsilon E_0^x, \quad (2)$$

$$-ik_0\frac{\partial\varphi}{\partial z}H_0^x-\frac{\partial H_0^z}{\partial x}=ik_0\varepsilon E_0^y, \quad (3)$$

$$\frac{\partial H_0^y}{\partial x} + ik_0 \frac{\partial \varphi}{\partial y} H_0^x = ik_0 \varepsilon E_0^z, \quad (4)$$

$$-ik_0 \frac{\partial \varphi}{\partial y} E_0^z + ik_0 \frac{\partial \varphi}{\partial z} E_0^y = -ik_0 \mu H_0^x, \quad (5)$$

$$-ik_0 \frac{\partial \varphi}{\partial z} E_0^x - \frac{\partial E_0^z}{\partial x} = -ik_0 \mu H_0^y, \quad (6)$$

$$\frac{\partial E_0^y}{\partial x} + ik_0 \frac{\partial \varphi}{\partial y} E_0^x = -ik_0 \mu H_0^z \quad (7)$$

and the system of equations in the first approximation of the method:

$$-ik_0 \frac{\partial \varphi}{\partial y} \frac{H_1^z}{(-i\omega)} + \frac{\partial H_0^z}{\partial y} + ik_0 \frac{\partial \varphi}{\partial z} \frac{H_1^y}{(-i\omega)} - \frac{\partial H_0^y}{\partial z} = ik_0 \varepsilon \frac{E_1^x}{(-i\omega)}, \quad (8)$$

$$-ik_0 \frac{\partial \varphi}{\partial z} \frac{H_1^x}{(-i\omega)} + \frac{\partial H_0^x}{\partial z} - \frac{\partial H_1^z}{\partial x} \frac{1}{-i\omega} = ik_0 \varepsilon \frac{E_1^y}{(-i\omega)}, \quad (9)$$

$$\frac{\partial H_1^y}{\partial x} \frac{1}{(-i\omega)} + ik_0 \frac{\partial \varphi}{\partial y} \frac{H_1^x}{(-i\omega)} - \frac{\partial H_0^x}{\partial y} = ik_0 \varepsilon \frac{E_1^z}{(-i\omega)}, \quad (10)$$

$$-ik_0 \frac{\partial \varphi}{\partial y} \frac{E_1^z}{(-i\omega)} + \frac{\partial E_0^z}{\partial y} + ik_0 \frac{\partial \varphi}{\partial z} \frac{E_1^y}{(-i\omega)} - \frac{\partial E_0^y}{\partial z} = -ik_0 \mu \frac{H_1^x}{(-i\omega)}, \quad (11)$$

$$-ik_0 \frac{\partial \varphi}{\partial z} \frac{E_1^x}{(-i\omega)} + \frac{\partial E_0^x}{\partial z} - \frac{\partial E_1^z}{\partial x} \frac{1}{-i\omega} = -ik_0 \mu \frac{H_1^y}{(-i\omega)}, \quad (12)$$

$$\frac{\partial E_1^y}{\partial x} \frac{1}{(-i\omega)} + ik_0 \frac{\partial \varphi}{\partial y} \frac{E_1^x}{(-i\omega)} - \frac{\partial E_0^x}{\partial y} = -ik_0 \mu \frac{H_1^z}{(-i\omega)}. \quad (13)$$

For a thin-film multilayer waveguide consisting of optically homogeneous layers, the matching conditions for the electromagnetic field hold at the interfaces between media

$$\mathbf{n} \times \mathbf{E}^- + \mathbf{n} \times \mathbf{E}^+ = 0, \quad (14)$$

$$\mathbf{n} \times \mathbf{H}^- + \mathbf{n} \times \mathbf{H}^+ = 0. \quad (15)$$

In addition, the asymptotic conditions should be valid

$$E_y^0, E_z^0, H_y^0, H_z^0 \underset{x \to \pm\infty}{\longrightarrow} 0, \quad (16)$$

which ensure the uniqueness of the solution of the problem (2)–(13).

1. Two-dimensional dielectric waveguides

In the absence of „y“dependence, the Maxwell equations are simplified and separated into two independent subsystems for TE and TM polarizations. In particular, for TE polarization the following equations are valid in the zeroth order:

$$\frac{dE_0^y}{dx} + ik_0 \mu H_0^z = 0, \quad (17)$$

$$\frac{dH_0^z}{dx} - \frac{ik_0}{\mu} \beta^2(z) E_0^y + ik_0 E_0^y = 0, \quad (18)$$

$$H_0^x = -\frac{1}{\mu} \beta(z) E_0^y, \quad (19)$$

and in the first order:

$$\frac{\partial E_1^y}{\partial x} + ik_0 \mu H_1^z = 0 \quad (20)$$

$$-\frac{\partial H_1^z}{\partial x} + \frac{ik_0}{\mu} \beta^2(z) E_1^y - ik_0 \varepsilon E_1^y = i\omega \frac{\partial H_0^x}{\partial z} - \frac{i\omega}{\mu} \beta(z) \frac{\partial E_0^y}{\partial z}, \quad (21)$$

$$H_1^x = -\frac{1}{\mu} \left(\beta(z) E_1^y + \frac{\omega}{k_0} \left(\frac{\partial E_0^y}{\partial z} \right) \right). \quad (22)$$

At horizontal boundaries, relations (14),(15) reduce to the equality of the horizontal electric field components. At the slanted part of the interface between the waveguide layers $x = h(z)$, the tangent plane is specified by the equation $dx - (dh/dz)dz = 0$, and the matching conditions for the electromagnetic fields at point $(h(z), z)'$ of the slanted interface have the form

$$[\mathbf{n} \times \mathbf{E}] = \left(E_y \frac{\partial h}{\partial z} - E_z \frac{\partial h}{\partial y}; -E_z - E_x \frac{\partial h}{\partial z}; E_y - E_x \frac{\partial h}{\partial y} \right)^T, \quad (23)$$

$$[\mathbf{n} \times \mathbf{H}] = \left(H_y \frac{\partial h}{\partial z} - H_z \frac{\partial h}{\partial y}; -H_z - E_x \frac{\partial h}{\partial z}; H_y - H_x \frac{\partial h}{\partial y} \right)^T. \quad (24)$$

They are fully specified by pairs of independent components E_y^τ, E_z^τ and H_y^τ, H_z^τ.

Let us formulate the problem of finding solutions to Eqs. (17)–(24) that decrease at infinity:

$$\lim_{n \to \pm\infty} |E_0^y(x; z)| = 0, \quad \lim_{x \to \pm\infty} |E_1^y(x; z)| = 0.$$

We approach its solution by means of an auxiliary spectral problem

$$\left(\frac{d^2}{dx^2} + k_0 \varepsilon \mu \right) E_{k_0}^y = k_0^2 \beta_k^2 E_{k_0}^y,$$

$$\mu H_{k_0}^x = -\beta_k E_{k_0}^x,$$

$$H_{k_0}^x = \frac{i}{k_0 \mu} \frac{dE_{k_0}^y}{dx},$$

$$\left(\frac{d^2}{dx^2} + k_0^2 \varepsilon \mu \right) E_{k_1}^y - k_0^2 \beta_k^2 E_{k_1}^y = k_0 \mu \omega \left(\frac{\beta_k}{\mu} \frac{dE_{k_0}^y}{dz} - - \frac{dH_{k_0}^x}{dz} \right),$$

$$H_{k_1}^x + \frac{\beta_k}{\mu} E_{k_1}^y = -\frac{\omega}{k_0 \mu} \frac{dE_{k_0}^y}{dz},$$

$$H^z_{k_0} = \frac{i}{k_0\mu}\frac{dE_{k_0}}{d}x$$

with asymptotic conditions

$$\lim_{n\to\pm\infty}|E^y_k(x;z)| = 0, \qquad \lim_{x\to\pm\infty}|E^y_1(x;z)| = 0$$

and normalization conditions

$$E^y_k, E^y_k \equiv \int_{-\infty}^{\infty} E^y_k(x,z)\overline{E^y_k}(x,z)dx = 1,$$

$$E^y_k, E^y_k \equiv \int_{-\infty}^{\infty} E^y_k(x,z)\overline{E^y_k}(x,z)dx = 1.$$

To find the electromagnetic field of the adiabatic guided modes, let us consider the solution of an auxiliary problem.

2. Adiabatic guided TE modes in the zeroth and first approximation

Let us consider the amplitudes of the TE mode electromagnetic field components in subdomains of thin homogeneous films of three-layered parts of the waveguides. We express the general solutions for E_y in terms of parameters

$$\gamma_z = k_0\sqrt{\beta^2(z) - n_s^2},$$

$$\chi_f(z) = k_0\sqrt{n_f^2 - \beta^2(z)},$$

$$\gamma_c = k_0\sqrt{\beta^2(z) - n_c^2}.$$

At the interfaces between the layers, the electromagnetic field matching conditions in the zeroth approximation for the TE mode take the form of a homogeneous system of linear algebraic equations (SLAE) for the amplitude coefficients $A^c_0(z)$, $A^s_0(z)$, $A^{f+}_0(z)$, $A^{f-}_0(z)$. The solvability condition determined by the dependence between $\beta(z)$ and $h(z) = a_2(z) - a_1$ leads to finding the solution parameters $\beta_0(z)$, $\gamma^c_0(z)$, $\gamma^s_0(z)$, $\chi^f_0(z)$. The solution parameters $\beta_0(z)$ and $\gamma^c_0(z)$, $\gamma^s_0(z)$, $\chi^f_0(z)$, as well as the nontrivial SLAE solutions themselves, are searched for in such a way, that the coefficients $A^c_0(z)$, $A^s_0(z)$, $A^{f+}_0(z)$, $A^{f-}_0(z)$ would be continuously differentiable functions of the argument. In this case, the particular first-order equations depending on their derivatives will be valid.

With the relations $\beta_0(z)$ $A^c_0(z)$, $A^s_0(z)$, $A^{f+}_0(z)$, $A^{f-}_0(z)$ taken into account, the studied Eqs. (20)−(22) of the first order of smallness in the three layers take the form:

$$\left(\frac{d^2}{dx^2} + k_0^2\varepsilon\mu - k_0^2\beta_k^2\right)E^y_{k1} = 2k_0\beta_k\omega\frac{d}{dz}$$

$$\times\left\{\begin{array}{c} A_s(z)\exp\{\gamma_s(z)(x-a_1)\} \\ \left(A^+_f(z)\exp\{i\chi_f(z)(x-a_1)\}+\right. \\ \left.+A^-_f(z)\exp\{-i\chi_f(z)(x-a_1)\}\right) \\ A_c(z)\exp\{-\gamma_c(z)(x-a_1)\} \end{array}\right\} \tag{25}$$

$$H^x_1 + \frac{\beta z}{\mu}E^y_1 = -\frac{\omega}{\mu k_0}\frac{\partial}{\partial z}$$

$$\times\left\{\begin{array}{c} A_s(z)\exp\{\gamma_s(z)(x-a_1)\} \\ A^+_f(z)\exp\{i\chi_f(z)(x-a_1)\}+ \\ +A^-_f(z)\exp\{-i\chi_f(z)(x-a_1)\} \\ A_c(z)\exp\{-\gamma_c(z)(x-a_1)\} \end{array}\right\} \tag{26}$$

$$\frac{\partial E^y_1}{\partial x} + ik_0\mu H^z_1 = 0. \tag{27}$$

The solutions of the system of equations (25)-(27) have the form of a sum of solutions of the homogeneous parts of these equations and partial solutions of the complete inhomogeneous equations.

The solution of a homogeneous system in the first approximation has the form:

$$E^y_s(z) = A_{s1}(z)\exp\big(\gamma_{s1}(z)(x-a_1)\big),$$

$$E^y_f(z) = A^+_{f1}(z)\exp\big(i\chi_{f1}(z)(x-a_1)\big) + A^-_{f1}(z)\exp\big(-i\chi_{f1}(z)(x-a_1)\big),$$

$$E^y_c(z) = A_{c1}(z)\exp(\gamma_{c1}(z)(x-a_1)),$$

$$H^x_s(z) = -\frac{\beta_1(z)}{\mu}A_{s1}(z)\exp(\gamma_{s1}(z)(x-a_1)),$$

$$H^x_f(z) = -\frac{\beta_1(z)}{\mu}\big(A^+_{f1}(z)\exp(i\chi_{f1}(z)(x-a_1)) + A^-_{f1}(z)\exp(-i\chi_{f1}(z)(x-a_1))\big),$$

$$H^x_c(z) = -\frac{\beta_1(z)}{\mu}A_{c1}(z)\exp(\gamma_{c1}(z)(x-a_1)),$$

$$H^z_s(z) = \frac{i\gamma_{s1}(z)}{k_0\mu}A_{s1}(z)\exp(\gamma_{s1}(z)(x-a_1)),$$

$$H^z_f = -\frac{\chi_{f1}}{k_0\mu}\Big(A^+_{f1}(z)\exp(i\chi_{f1}(z)(x-a_1)\Big) + A^-_{f1}(z)\exp\left(-i\chi_{f1}(z)(x-a_1)\right)\Big),$$

$$H^z_c(z) = \frac{i\gamma_{c1}(z)}{k_0\mu}A_{c1}(z)\exp(\gamma_{c1}(z)(x-a_1)).$$

We search for a particular solution of the inhomogeneous system of ODEs (25)-(27) using the Wronskian by the method developed for second-order ODEs. In this case, complete general solutions that do not necessarily satisfy the asymptotic conditions (16) are used, and only after obtaining the final formal expressions, we nullify the terms growing at infinity.

The general solution in the first approximation has the form:

$$E_y = E^{homog}_y + E^p_y,$$

$$H_x = H^{homog}_x + H^p_x,$$

$$H_z = H^{homog}_z.$$

The boundary conditions at the planar boundary take the form

$$E_s^y(x = a_1) = E_f^y(x = a_1),$$

$$H_s^z(x = a_1) = H_f^z(x = a_1).$$

At the curved boundary $x = a_2(z)$ (in this case, the boundary conditions $a_2(z) - a_1 = h(z)$ take the form:

$$E_f^y(x = a_2(z)) = E_c^y(x = a_2(z)),$$

$$H_f^z(x = a_2(z)) + \frac{\partial h}{\partial z} H_f^x(x = a_2(z))$$

$$= H_c^z(x = a_2(z)) + \frac{\partial h}{\partial z} H_c^x(x = a_2(z)).$$

Finally, we obtain an inhomogeneous system of four equations with the unknown coefficients $(A_{s1}(z))$, A_{f1}^+, A_{f1}^-, $A_{c1}(z))$ and unknown parameter $\beta_1(z)$.

3. Discussion and conclusion

To formulate the system of first-order ODEs, it is necessary to solve, first, a homogeneous system of zero-order ODEs. After that, for each zero-order solution, we write down an inhomogeneous system of first-order ODEs.

In a three-layer thin-film waveguide, the system of homogeneous ODEs (17)−(19) for zero-order contributions to adiabatic waveguide modes is reduced to a homogeneous SLAE

$$\hat{M}(\beta_0(z))\mathbf{A}_0(\beta_0(z)) = \mathbf{0}. \quad (28)$$

The condition for the solvability of SLAE (1) is

$$det\hat{M}(\beta_0(z)) = 0 \quad (29)$$

at any $z \in [z_0, z_1]$ in the interval of the initial problem solution.

The system of inhomogeneous ODEs (20)-(22) for the first-order contributions to AGMs comprises analytical expressions depending on derivatives of $\mathbf{A}_0(\beta_0(z))$ and $\beta_0(z)$, $\chi_0^f(z)$, $\gamma_0^c(z)$, $\gamma_0^s(z)$ in the right-hand side. Thus, for the specific notation of the right-hand side, depending on the solutions (28) and (29), these solutions should belong to the class of continuously differentiable functions. In order to find such zero-order solutions, the method described in [8] is proposed.

After the explicitly expressing $\mathbf{A}_0(\beta_0(z))$ depending on the numerical solution $\beta_0 \in C^1[z_0, z_1]$ by means of a symbolic calculation software tool, it becomes possible to reduce the system of inhomogeneous ODEs (25)−(27) for the first-order contributions to a system of inhomogeneous SLAE

$$\hat{M}(\beta_1(z))\mathbf{A}_1(\beta_1(z)) = \mathbf{F}\left(\frac{\partial \beta_0}{\partial z}, \frac{\partial \mathbf{A}_0}{\partial z}\right) \quad (30)$$

with a similar matrix as in the zero-order contributions, but depending the other parameter $\beta_1(z)$ and, therefore, on other $\chi_1^f(z)$, $\gamma_1^c(z)$, $\gamma_1^s(z)$. As above, in this case it is also necessary to require the solvability of the inhomogeneous SLAE (30).

After obtaining the solutions to the zero- and first-order equations for electromagnetic fields of the AGM model in a closed form, we can use them to express the electric and magnetic field in the first (plus zero) approximation

$$\mathbf{E}(x;y,z) = \mathbf{E}_0(x;y,z) + \frac{i}{\omega}\mathbf{E}_1(x;y,z),$$

$$\mathbf{H}(x;y,z) = \mathbf{H}_0(x;y,z) + \frac{i}{\omega}\mathbf{H}_1(x;y,z).$$

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Conflict of interest

The authors declare that they have no conflict of interest.

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