01

Diffraction of electromagnetic waves on one-dimensional diffraction gratings formed by slots in an absolutely absorbing screen

© A.M. Lerer, V.V. Makhno, V.I. Kravchenko

Southern Federal University, Rostov-on-Don, Russia
e-mail: lerer@sfedu.ru

Received December 30, 2022
Revised December 30, 2022
Accepted December 30, 2022

Two-sided approximate boundary conditions are obtained for an absolutely absorbing („black") layer lying on a multilayer dielectric. Paired summation equations (PSEs) are obtained for the tangent components of the electric and magnetic field strengths at the slots. These equations are solved by the Galerkin method with basis functions in the form of Chebyshev and Legendre polynomials. The resulting system of linear algebraic equations has fast internal convergence. To control the accuracy of the obtained solution, a dual problem is solved — a lattice of „black stripes". In this case, the unknowns in the PSU are the current density on the strips. The properties of lattices are analyzed.

Keywords: approximate boundary conditions, „black"screen, diffraction grating, pair summation equations, Galerkin's method.

DOI: 10.21883/TP.2023.04.55929.290-22

Introduction

This paper continues theoretical and experimental studies of X-ray beam diffraction by microchannel plates (MCPs) [1–3]. An MCP is a plate of leaden-silicate glass with a hexagonal lattice of round holes. Hole diameters are typically less than $12\,\mu$m. Within the energy range $100-500$ eV (the wavelength being nearly $2-10$ nm) the glass has a complex refractive index with the modulus lightly less than one. A solid glass plate is a nearly total absorber, it s reflection and transmission coefficients are less than -30 dB. The propagation of X-rays in channels is analogous to the propagation of optical and microwave electromagnetic radiation in waveguides. In Ref. [1] and other papers, the excitation, propagation, and emission of microchannel eigenwaves is studied using the Kirchhoff approximation. Due to the large number of waves, such a calculation is rather labor consuming. Due to the large number of waves, such a calculation is quite laborious. Therefore, the authors of Refs. [2,3] proposed a simplified approach of considering diffraction by holes in an opaque screen. The obtained results are in good agreement with experiment. Further development of the theory is the diffraction by holes in a totally absorbing „black"screen.

When studying diffraction by tin screens or bodies with thin-film coating, it is convenient to use the method of approximate boundary conditions (ABC). For example, in Ref. [4], impedance ABCs are obtained, including for a „black" coating, which were used to simulate the characteristics of wave scattering by bodies with absorbing coating.

In the case of thin screens, double-sided ABCs (DABCs) are introduced for dielectric layers [5], for thin films with curvature [6], for thin metallic gratings [7], etc.

The field of DABC application are screens whose thickness is much less than the wavelength. A modification of double-sided DABCs is possible for the case of screen thickness simply less than the wavelength. In Ref. [8], this is done for plasmonic optical gratings.

Although ABCs have been introduced for films unlimited in length and width, they are widely used for solving problems of scattering by diffraction gratings [8–12]. In the case of applying the ABC method, it is required to substantiate the reliability of the results obtained. One of possible approaches to this problem is to compare the results of calculations for the same structure by two methods. In Refs. [8,12,13], the problem of diffraction of electromagnetic waves in the optical and terahertz ranges by a two-dimensional lattice of metal and graphene strips was solved by two methods. The first method is based on the numerical-analytical method for solving a volume integro-differential equation (VIDE). The unknowns in VIDE are the components of the electric field strength inside the plasmonic strips. The second method uses approximate boundary conditions for a thin dielectric layer. Comparison of the results calculated by these methods substantiated the validity of applying the ABC method to reveal the main physical regularities.

The aim of this work is to modify the double-sided approximate boundary conditions (DABCs) of the impedance type for calculating the diffraction of electromagnetic waves by holes in totally absorbing screens; application of these boundary conditions to solve the boundary value problem of diffraction by a 1D periodic grating of holes in a „black" screen. In contrast to Refs. [1–3], we develop a rigorous theory in the wavelength range commensurate with the grating dimensions.

Sommerfeld defined a „black" body as the body with $\varepsilon = \mu$ having a large imaginary part [14]. Therefore, for calculating "black" screens it is reasonable to use the DABC [5] involving the permittivity and permeability of the screen.

The ADCs for black screens can be used also to calculate thin film superwideband absorbers, which find application in increasing the sensibility of optical instruments, in solar batteries, to counteract the means of visual-optical, optoelectronic and electronic intelligence, to improve electromagnetic compatibility in the microwave and terahertz ranges, in modulators and polarizers [15]. To absorb electromagnetic radiation in a certain frequency range, various materials are being developed and used. To date, a large number of composite polymeric radio-absorbing materials are known based on various forms of carbon particles, e.g., carbon nanotubes, as well as ferrite powders or nanoparticles. Modern absorbing structures must be thin and provide almost 100% absorption over a wide range of frequencies and incidence angles. In the optical range, coatings made of vertically oriented carbon nanotubes [16] based on titanium carbide [17] demonstrate the best results. Of particular interest are studies devoted to perfect absorbers based on metamaterials, such as periodic structures with layers of graphene [12,13,15,18–21], VO_2 [22], cylinders with complex permittivity and inhomogeneous shell [23].

1. Double-sided ABCs for a thin absorbing screen

The general form of double-sided ABCs for a thin dielectric screen is presented in the monograph [5]:

$$E^+_{x,z} - E^-_{x,z} = \pm\frac{\rho}{2}(H^+_{z,x} + H^-_{z,x}),$$

$$H^+_{x,z} - H^-_{x,z} = \mp\frac{\sigma}{2}(E^+_{z,x} + E^-_{z,x}).$$

Symbols $\pm$ denote the fields above and below the screen, the time dependence is $\exp(j\omega t)$:

$$\rho = j\omega(\mu - 1)\mu_0\tau, \ \ \sigma = j\omega(\varepsilon - 1)\varepsilon_0\tau, \tag{1}$$

where μ, ε, τ are the equivalent permittivity and permeability and the screen thickness.

For the two-dimensional case, the ABCs will take the following form
for E-polarization: **EHe**:

$$\mathbf{E}(0, 0, E), \mathbf{H}(H_x, H_y, 0), \mathbf{H} = \frac{j}{kZ_0\mu}\left(\mathbf{e}_x \frac{dE}{dy} - \mathbf{e}_y i k_x E\right),$$

$$E^+ = E^- = a\left(\frac{1}{\mu^+}\frac{dE^+}{dy} + \frac{1}{\mu^-}\frac{dE^-}{dy}\right);$$

$$\left(\frac{1}{\mu^+}\frac{dE^+}{dy} - \frac{1}{\mu^-}\frac{dE^-}{dy}\right) = -k^2 b(E^+ + E^-), \tag{2}$$

where

$$a = -\frac{\rho}{2}\frac{j}{kZ_0}, \ \ b = -jZ_0\frac{\sigma}{2k}, \tag{3}$$

where k, Z_0 are the wave number and the characteristic impedance in a vacuum;
for H-polarization:

$$\mathbf{H}(0, 0, E), \ \mathbf{E}(E_x, E_y, 0), \ \mathbf{H} = \frac{jZ_0}{k\varepsilon}\left(\mathbf{e}_x \frac{dE}{dy} - \mathbf{e}_y i k_x E\right);$$

it is necessary to make replacements $\rho \leftrightarrow \sigma$, $Z_0 \to 1/Z_0$, $\mu \to -\varepsilon$ in Eq. (1).

2. Double-sided ABCs for a totally absorbing screen

We solve the problem of reflection from a screen placed between two semi-infinite layers with the parameters $\varepsilon_1\mu_1$ (upper layer from which the wave is incident) and $\varepsilon_2 mu_2$. Double-sided ABCs (2) are fulfilled at the screen. It can be easily shown that the coefficients of reflection R and transmission T satisfy the system of linear equations

$$1 + R - T = ja(\xi_1 k_y^{(1)}(1 - R) + T\xi_2 k_y^{(2)}),$$

$$j\xi_1 k_y^{(1)}(1 - R) - Tj\xi_2 k_y^{(2)} = -k^2 b(1 + R + T),$$

where $k_y^{(1,2)}$ are the wave vector components normal to the screen in layers 1 and 2, $\xi_{1,2} = \frac{1}{\mu_{1,2}}$ for E-polarization, $\xi_{1,2} = \frac{1}{\varepsilon_{1,2}}$ for H-polarization.

The solution of the system of linear equations at $R = T = 0$ is possible with the coefficients

$$a = -\frac{j}{\xi_1 k_y^{(1)}}, \ \ b = -\frac{j\xi_1 k_y^{(1)}}{k^2}. \tag{4}$$

Let us substitute Eqs. (1) and (3) into Eq. (4) and assume that $\xi_1 = 1$ (vacuum). As a result, we get

$$(\mu - 1)d = -\frac{2j}{k_y^{(1)}}, \ \ (\varepsilon - 1)d = -2j\frac{k_y^{(1)}}{k^2}.$$

For normal incidence

$$(\varepsilon - 1)d = (\mu - 1)d = -\frac{2j}{k}, \tag{5}$$

which agrees with the Sommerfeld definition of a black body $\varepsilon = \mu$, with a large imaginary part [14].

3. Diffraction by a grating of slits is an absorbing screen

The system of coordinates is chosen such that the grating is periodic with period d along the axis x (Fig. 1). The slits are directed along the axis z. The axis y is perpendicular to the grating plane. The screen with slits lies in the

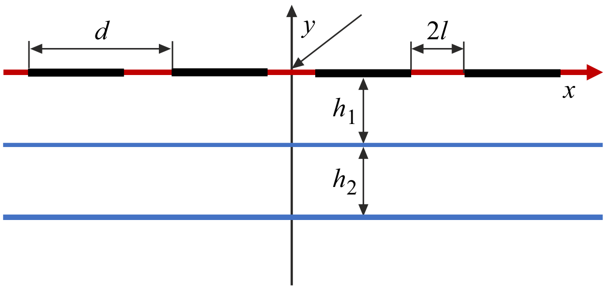

Figure 1. Slit grating

plane $y = 0$, the slit width is $2l$. The wave is incident from above from the medium with $\varepsilon = \mu = 1$, below the screen ($y \leq 0$) there is a multilayer substrate. The plane of incidence is perpendicular to the slits. Therefore, the polarization does not change upon diffraction. The angle of incidence is θ. Let us denote $U(x, y) = E_z(x, y)$ for E-polarization, $U(x, y) = H_z(x, y)$ for H-polarization and expand the unknown $U(x, y)$ in a Floquet series

$$U(x, y) = U^{ext}(x, y) + \frac{1}{d}\sum_{n=-\infty}^{\infty} \tilde{U}_n(y)\exp(j\alpha_n x),$$

$$\alpha_n = \frac{2n\pi}{d} + k_x, \tag{6}$$

where $k_x = k\sin\theta$ is the incident wave vector component tangent to the screen, $U^{ext}(x, y)$ is the external field, i.e., the incident field plus the field reflected from the screen and transmitted through the screen without slits. $U^{ext}(x, y)$ satisfies the ABCs (2). Functions $\tilde{U}_n(y)\exp(j\alpha_n x)$ satisfy the Helmholtz equation in each layer (see Appendix).

$$\tilde{U}_n(y) = \begin{cases} A_n U_n^+(y), & y \geq 0, \\ B_n U_n^-(y), & y \leq 0, \end{cases} \tag{7}$$

where $A_n B_n$ are unknown coefficient, $U_n^+(y) = \exp(-\gamma_n y)\gamma_n = \sqrt{\alpha_n^2 - k_x^2}$, functions $U_n^-(y)$ are defined in the Appendix. They satisfy the condition $U_n^-(0) = 1$.

Let us introduce the functions

$$f(x) = \begin{cases} U^+ - U^- - a\left(\xi_+ \frac{dU^+}{dy} + \xi_- \frac{dU^-}{dy}\right), & |x| \leq l, \\ 0, \ d \geq |x| \geq l, \end{cases}$$

$$g(x) = \begin{cases} \left(\xi_+ \frac{dU^+}{dy} - \xi_+ \frac{dU^-}{dy}\right) + k^2 b(U^+ + U^-), & |x| \leq l, \\ 0, \ d \geq |x| \geq l. \end{cases}$$

We substitute Eqs. (6), (7) into Eq. (13) As a result, we get

$$f_n = A_n - B_n - a(-\gamma_n A_n + \nu_{n,0} B_n);$$

$$g_n = (-\gamma_n A_n - \nu_{n,0} B_n) + k^2 b(A_n + B_n), \tag{8}$$

where f_n, g_n are coefficients in the Floquet series for functions $f(x)$, $g(x)$, $\nu_{n,0} = U'^-_n(0)$.

From Eq. (8), we find

$$\begin{pmatrix} A_n \\ B_n \end{pmatrix} = \hat{\mathbf{a}} \begin{pmatrix} f_n \\ g_n \end{pmatrix}. \tag{9}$$

Since calculating the elements of matrix $\hat{\mathbf{a}}$ is an elementary task, we do not present them here.

The next step of solution is to satisfy the continuity conditions for functions $U(x, y)$, $\frac{\partial}{\partial y}U(x, y)$ at a slit ($y = 0$, $|x| \leq l$):

$$E^{ext,+}(x, 0) - E^{ext,-}(x, 0) + \frac{1}{d}\sum_{n=-\infty}^{\infty}(A_n - B_n)\exp(j\alpha_n x) = 0,$$

$$E'^{ext,+}(x, 0) - E'^{ext,-}(x, 0) + \frac{1}{d}\sum_{n=-\infty}^{\infty}(\gamma_n A_n + \nu_{n,0} B_n)\exp(j\alpha_n x) = 0, \tag{10}$$

where

$$A_n - B_n = a_{11} f_n + a_{12} g_n - (\gamma_n A_n + \nu_{n,0} B_n)$$
$$= -\gamma_n a_{21} f_n - \nu_{n,0} a_{22} g_n, \quad |x| \leq l.$$

Thus, we obtained the first pair of the pair summatory equations with respect to f_n, g_n. The second pair follows from the definition of functions $f(x)$, $g(x)$

$$\frac{1}{d}\sum_{n=-\infty}^{\infty} f_n \exp(j\alpha_n x) = 0,$$

$$\frac{1}{d}\sum_{n=-\infty}^{\infty} g_n \exp(j\alpha_n x) = 0, \quad d \geq |x| \geq l. \tag{11}$$

4. Diffraction by a grating of absorbing strips

Naturally, a slit grating can be presented as a grating of strips as well. Hence, the solution considered below is of interest for checking the reliability and precision of results.

For the strip grating, the external field is the field in the structure without strips. As in the previous case,we write the fields in the form (6), (7). We introduce functions $f(x)$, $g(x)$, defined at $y = 0$

$$f(x) = \begin{cases} U^+ - U^-, & |x| \leq \bar{l}, \\ 0, & d \geq |x| \geq \bar{l}, \end{cases}$$

$$g(x) = \begin{cases} \left(\xi_+ \frac{dU^+}{dy} - \xi_- \frac{dU^-}{dy}\right), & |x| \leq \bar{l}, \\ 0, & d \geq |x| \geq \bar{l}, \end{cases}$$

where $2\bar{l} = d - 2l$ is the strip width.

We express A_n, B_n in terms of f_n, g_n:

$$f_n = A_n - B_n, \quad g_n = (-\gamma_n A_n - \nu_{n,0} B_n).$$

Hence we get

$$\begin{pmatrix} A_n \\ B_n \end{pmatrix} = \mathbf{b} \begin{pmatrix} f_n \\ g_n \end{pmatrix}, \quad (12)$$

$$b_{11} = \frac{\nu_{n,0}}{\gamma_n + \nu_{n,0}}, \quad b_{12} = -\frac{1}{\gamma_n + \nu_{n,0}},$$

$$b_{21} = -\frac{\gamma_n}{\gamma_n + \nu_{n,0}}, \quad b_{22} = -\frac{1}{\gamma_n + \nu_{n,0}}.$$

The next step is to satisfy the ABCs (2) at the strips. We substitute Eqs. (12) into them. As a result, we arrive at the first pair of summatory equations at $|x| \leq \bar{l}$. The second pair is (10), but at $d \geq |x| \geq \bar{l}$.

5. Solving the PSE by Galerkin method

We seek the solution in the form

$$f(x) = \begin{cases} \sum_{m=0}^{\infty} X_m^{(f)} B_m(x), & |x| \leq l, \\ 0, & d \geq |x| \geq l; \end{cases}$$

$$g(x) = \begin{cases} \sum_{m=0}^{\infty} X_m^{(g)} B_m(x), & |x| \leq l, \\ 0, & d \geq |x| \geq l, \end{cases} \quad (13)$$

where $X_m^{(f)}, X_m^{(g)}$ are unknown coefficients, $B_m(x)$ are the basis functions (BFs). As BFs, we use the Chebyshev polynomials T_m, U_m and Legendre polynomials P_m [8]:

$$B_m(x) = P_m(x/l), \text{ or } B_m(x) = T_m(x/l), \quad (14)$$

$$B_m(x) = U_m(x/l)\sqrt{l^2 - x^2}. \quad (15)$$

For the strip grating, the basis is the same, but with the replacement $l \to \bar{l}$.

We use basis (14) a) for slit grating and H-polarization, b) for strip grating and E-polarization. Basis (15) is used a) for slit grating and E-polarization, b) for strip grating and H-polarization. The Floquet transforms of BFs are equal to a constant, which then will enter the unknown coefficients,

$$\tilde{B}_{m,n}(x) = \frac{J_{m+1/2}(\alpha_n l)}{\alpha_n^{1/2}},$$

or $\tilde{B}_{m,n}(x) = J_m(\alpha_n l)$ for the basis (14),

$$\tilde{B}_{m,n}(x) = \frac{J_{m+1}(\alpha_n l)}{\alpha_n},$$

for the basis (15).

We substitute Eqs. (14), (15) into Eq. (13)

$$f_n = \sum_{m=0}^{\infty} \bar{X}_m^{(f)} \tilde{B}_{m,n}, \quad g_n = \sum_{m=0}^{\infty} \bar{X}_m^{(g)} \tilde{B}_{m,n},$$

where $\bar{X}_m^{(f)}, \bar{X}_m^{(g)}$ are unknown coefficients differing from the old ones $X_m^{(f)}, X_m^{(g)}$ by constants. The obtained f_n, g_n satisfy Eq. (10).

Then we substitute Eq. (12) into Eq. (9) and project the resulting equations on BFs (14) or (15). As a result, we get a system of linear algebraic equations (SLAE) for unknown $\bar{X}_m^{(f)}, \bar{X}_m^{(g)}$. The SLAE possesses fast internal convergence, so that in series (13) it is sufficient to consider 5–15 terms to calculate the complex amplitudes of spatial harmonics with an error less than 0.5%.

6. Calculation results

First, let us consider the dependence of grating parameters on the wavelength (Fig. 2, 3). The grating with the dimensions $2/d = 0.75$ is placed on a double-layered dielectric. Th upper layer has the thickness $h_1 = 0.95d$ and the refractive index 1.5, the lower layer is a semi-infinite substrate with the refractive index 1.77. The incidence is normal. We choose the wavelength λ_0, for which the conditions of total absorption are satisfied. Figures 2, 3

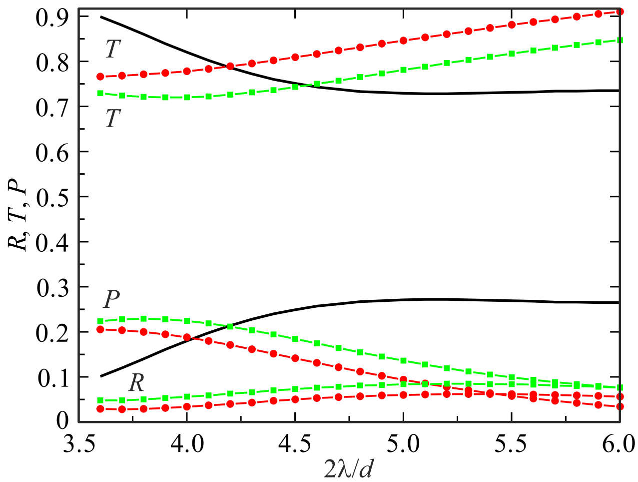

Figure 2. Dependence of grating parameters on the wavelength E-polarization

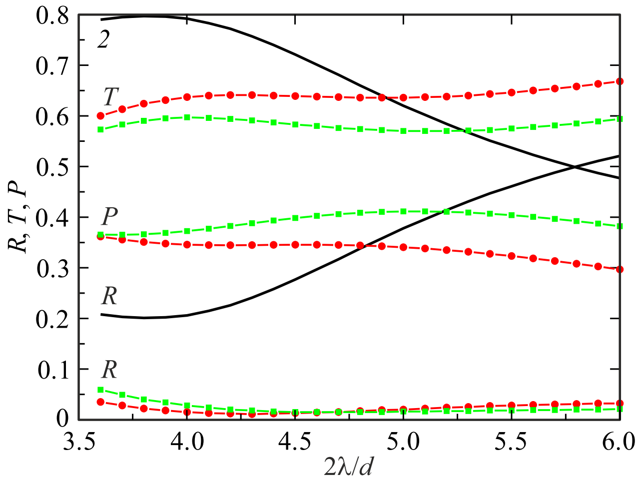

Figure 3. Dependence of grating parameters on the wavelength H-polarization

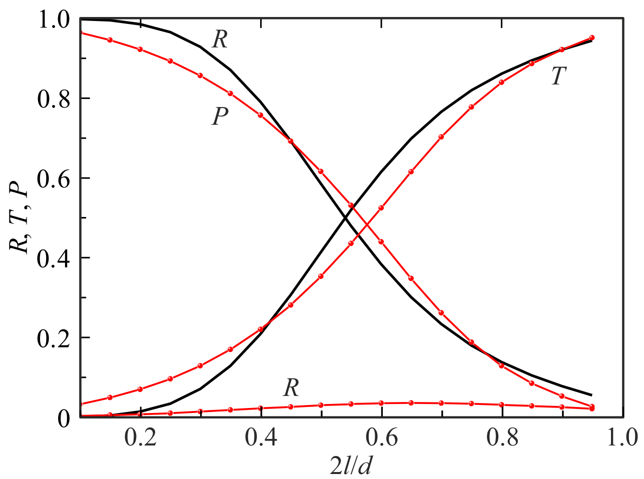

Figure 4. Dependence of grating parameters on the wavelength E-Polarization. $2\lambda/d = 2\lambda_0/d = 4$.

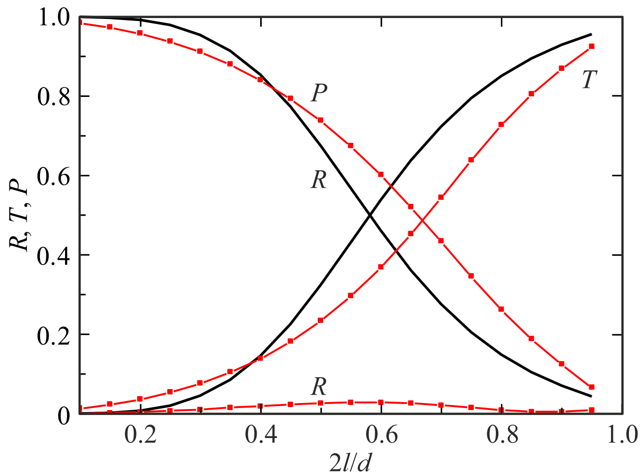

Figure 5. Dependence of grating parameters on the wavelength H-Polarization. $2\lambda/d = 2\lambda_0/d = 4$.

show the wavelength dependences of the power reflection coefficient R, power transmission coefficient T and losses $P = 1 - R - T$. Solid lines correspond to $a = 0$, $\mathrm{Im} b \to \infty$, i.e. perfectly transmitting screen, $P \to 0$, the lines with symbols represent an absorbing screen. Circles correspond to $2\lambda_0/d = 4$, squares to $2\lambda_0/d = 6$.

The results for the gratings with absorbing screen demonstrate the following distinctive features as compared to those for non-absorbing screen:

- the reflection coefficient is substantially decreased;
- the wavelength dependence of reflection and transmission coefficients is weaker;
- the transmission coefficient either grows with wavelength, or has a small extremum.

The reflection coefficient R is minimum at the layer thickness equal to a quarter of wavelength in the dielectric. When the thickness is greater than half-wave, R is practically independent of the thickness.

Figures 4, 5 present the dependences of the grating parameters on the slit width and Figs. 6, 7 — on the angle of incidence. Solid lines correspond to a totally transparent screen, $P \to 0$, lines with symbols are for an absorbing screen. It should be noted that the transmission coefficient values T are commensurable for the absorbing and totally transparent screens. The character of angular dependences $R(\theta)$, $T(\theta)$ in these two types of screens, is naturally the same, namely, the reflection coefficient sharply grows upon increasing he incidence angle. Although the condition of total absorption (5) is valid for normal incidence, the reflection coefficient is less than 0.15 up to $\theta = 50^\circ$ for E-polarization and $\theta = 70^\circ$ for H-polarization.

Figures 8, 9 illustrate the calculation results for another type of screen made of magneto-dielectric with $\varepsilon > 1$, $\mu > 1$. We substitute Eq. (1) into Eq. (3). As a result, we get

$$a = \frac{(\mu - 1)\tau}{2}, \quad b = \frac{(\varepsilon - 1)\tau}{2}.$$

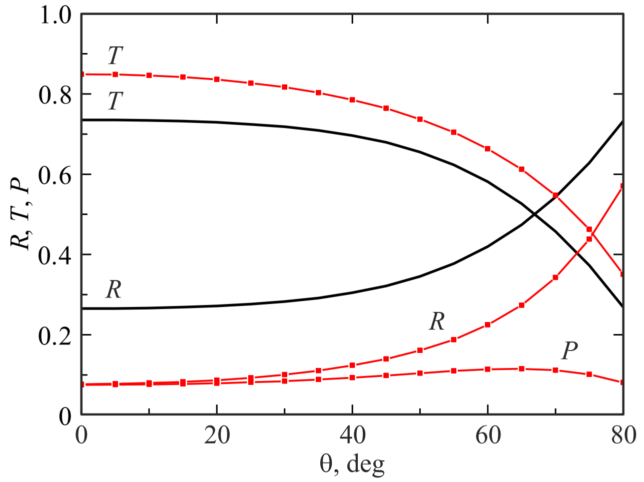

Figure 6. Dependence of grating parameters on the incidence angle E-Polarization. $2\lambda/d = 2\lambda_0/d = 6$.

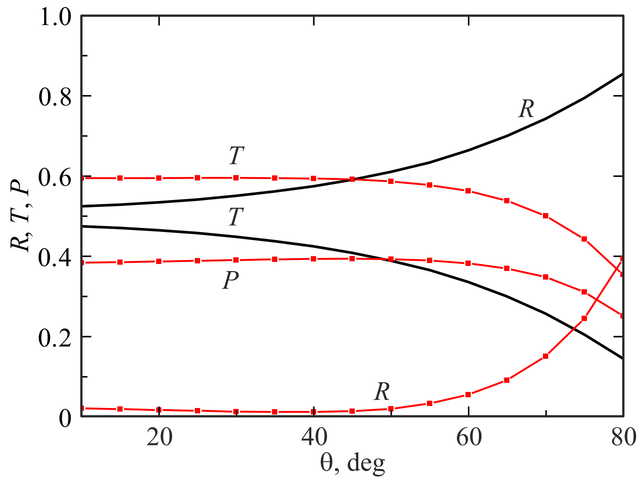

Figure 7. Dependence of grating parameters on the incidence angle H-Polarization. $2\lambda/d = 2\lambda_0/d = 6$.

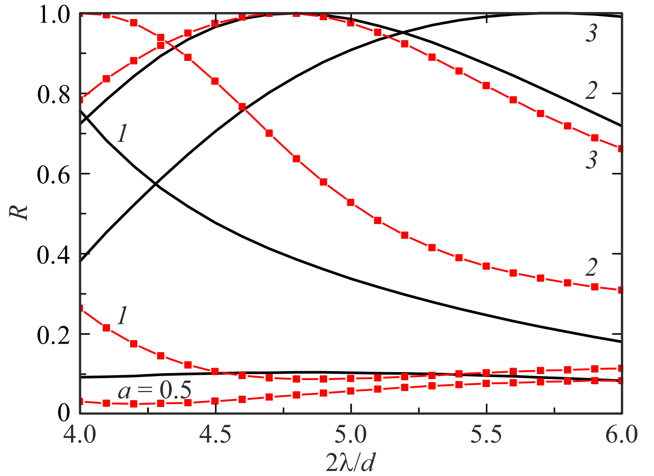

Figure 8. Dependence of the grating reflection coefficient on the wavelength at changing the parameter a. $2l/d = 0.75$, $b = a$. Curves without symbols — H-polarization, with symbols — E-polarization.

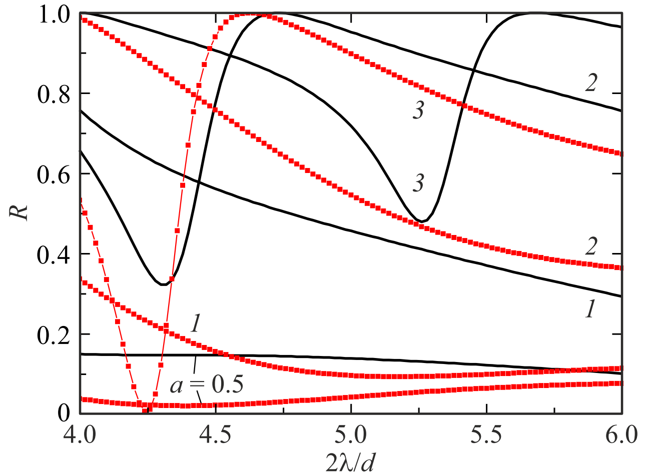

Figure 9. Dependence of the grating reflection coefficient on the wavelength. $2l/d = 0.5$, $b = a$. Curves without symbols — H-polarization, with symbols — E-polarization.

Assume ε, μ to be real-valued. Then there are no losses and the reflection coefficient $T = 1 - R$. In the figures, a total reflection effect is seen, which arises upon the field resonance in magneto-dielectric strips. The width of the strips is $2w = d - 2l$, therefore, in Fig. 7 $2w/d = 0.25$, and in Fig. 8 twice as much. The resonances in Fig. 7 are of the first order and in Fig. 8 of the second order. The resonance wavelength, naturally, increases with an increase in parameters $a/d, d/d$. The resonance of the transverse electric field (H-polarization of the incident wave) are longer-wavelength than the resonances of the longitudinal field (E-polarization).

Thus, in this work, from the solution to the problem of the electromagnetic wave reflection from a screen satisfying the double-sided boundary conditions of the impedance type, the conditions for the complete absorption of the wave by the screen are obtained. The obtained boundary conditions are applied to solve the problem of diffraction by a grating made of slits and stripes in a totally absorbing screen. The problem is reduced to solving paired summatory equations, for which the Galerkin method is used. A comparison with diffraction gratings in a totally transparent screen revealed a sharp decrease in the reflection coefficient in a wide range of wavelengths and angles of incidence with a comparable transmission coefficient and weaker dependence of the reflection and transmission coefficients on the wavelength.

Funding

This work was supported financially by the Ministry of Science and Higher Education of the Russian Federation (State Assignment in the field of scientific activity № FENW-2022-0001).

Conflict of interest

The authors declare that they have no conflict of interest.

Appendix

Let us define the function $U_n^-(y)$ in Eq. (7) for the substrate of the diffraction grating. The substrate is located at $y \le 0$ (Fig. 1). It consists $N+1$ of layers (in Fig. 1 $N = 2$), the layer with the number $N+1$ being semi-infinite. The layers are numbered from top to bottom. The thickness of the layer is $p(p = 1, \dots N)h_p$, its permittivity is ε_p. The permeability of all layers is equal to one. The coordinates of the layer boundaries are $y = y_p$, where $y_1 = 0$, $y_{p+1} = y_p - h_p$, $p = 1, \dots N+1$.

The function$U(x, y)$ (6) is a solution to the Helmholtz equation in each layer

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\varepsilon(y)\right)U(x, y) = 0.$$

Therefore, functions $U_n^-(y)$ (see Eqs. (6), (7)) satisfy the equations

$$\left(\frac{\partial^2}{\partial y^2} + k^2\varepsilon(y) - \alpha_n^2\right)U_n^-(y) = 0. \qquad \text{(A.1)}$$

The boundary conditions imply the continuity at $y = y_p$, $p(p = 2, \dots N+1)$ of functions a) $U_n^-(y)$ and b) $\xi \frac{dU_n^-(y)}{dy}$ ($\xi = 1$ for E-polarization, $\xi = \frac{1}{\varepsilon}$ for H-polarization). Besides that, the condition c) $U_N^-(0) = 1$ must be fulfilled.

To simplify the fulfilment of the boundary conditions for the solutions of Eqs. (A. 1), let us write them in the form: at $y_p \ge y \ge y_{p+1}$, $(p = 1, \dots N)$

$$U_n^-(y) = \frac{X}{\operatorname{sh}\kappa_{n,p}h_P}\Big\{-D_{p+1}\operatorname{sh}[\kappa_{n,p}(y - y_p)] + D_p\operatorname{sh}[\kappa_{n,p}(y - y_{p+1})]\Big\}, \qquad \text{(A.2)}$$

at $y \geq y_{N+1}$

$$U_n^-(y) = -XD_{N+1} \exp[\kappa_{n,N+1}(y - y_{N+1})], \qquad (A.3)$$

where X, D_p are unknown coefficients, $\kappa_{n,p} = \sqrt{\alpha_n^2 - k^2\varepsilon_p}$.

The functions (A. 2), (A. 3) satisfy the boundary condition a), i.e., they are continuous at layer interfaces.

Applying the second boundary condition b), we arrive at the recurrent scheme

$$D_p Q_{n,p} = D_{p+1}(T_{n,p} + T_{n,p+1}) - D_{p+2}Q_{n,p+1}, \quad p = N, N-1, \ldots 1, \qquad (A.4)$$

where

$$D_{N+2} = 0, \quad Q_{n,p} = \frac{\xi_p \kappa_{n,p}}{\mathrm{sh}(\kappa_{n,p} h_P)},$$

$$T_{n,p} = \begin{cases} \xi_p \kappa_{n,p} \,\mathrm{cth}(\kappa_{n,p} h_p), & p \neq N+1, \\ \xi_P \kappa_{nj,p}, & p = N+1. \end{cases}$$

We put in Eq. (A. 4) $D_{N+1} = 1$ and find all D_p. Finally, we find the unknown coefficient X from the condition c)

$$U_n^-(o) = XD_1 = 1.$$

Thus, all unknown coefficients in Eqs. (A. 2), (A. 3) are determined.

References

[1] M.I. Mazuritskiy, A.M. Lerer, P.V. Makhno. JETP, **123** (6), 942 (2016). DOI: 10.7868/S0044451016120038
[2] M.I. Mazuritskiy, A.M. Lerer. JETP Lett., **112** (3), 38 (2020). DOI: 10.1134/S0021364020150072
[3] M.I. Mazuritskiy, A.M. Lerer, A. Marcelli, S.B. Dabagov. J. Synchrotron Radiation, **29** (3), 355 (2022). DOI: 10.1107/S1600577521012893
[4] A.G. Kyurkchan, D.B. Demin. ZhTF,. **74** (2), 24 (2004) (in Russian).
[5] L.A. Vaynshteyn. *Otkrytyye rezonatory i otkrytyye volnovody* (Sov. radio, M., 1966) (in Russian).
[6] Ya.R. Grinberg. Radiotekhnika i elektronika, **26** (12), 2493 (1981) (in Russian).
[7] L.A. Vainshtein. *K elektrodinamicheskoy teorii reshetok. V sb. „Elektronika bol'shikh moshchnostey"* (M.: Nauka, 1963) (in Russian)
[8] A.M. Lerer, I.N. Ivanova. J. Commun. Technol. Electron., **61** (5), 486 (2016). DOI: 10.7868/S0033849416050089
[9] T.L. Zinenko, M. Marciniak, A.I. Nosich, IEEE J. Sel. Top. Quantum Electron., **19**, 9000108 (2013).
[10] O.V. Shapoval, R. Sauleau, A.I. Nosich. IEEE Trans. Antennas Propag., **59**, 3339 (2011).
[11] O.V. Shapoval, A.I. Nosich, AIP Adv., **3**, 042120 (2013).
[12] A.M. Lerer, G.S. Makeeva., Tech. Phys. Lett., **44** (9), 852 (2018). DOI: 10.1134/S1063785018090225
[13] A.M. Lerer, I.N. Ivanova, G.S. Makeeva, V.V. Cherepanov. Opt. Spectr., **129** (3), 339 (2021). DOI: 10.1134/S0030400X21030097
[14] L.N. Zakhar'yev, A.A. Limanskiy *Rasseyaniye voln „chernymi" telami* (M.: Sovetskoye radio, 1972) (in Russian).
[15] C. Zeng, H. Lu, D. Mao, Y. Du, H. Hua, W. Zhao, J. Zhao. Opto-Electron. Adv., **5** (4), 200098-1 (2022). DOI: 10.29026/oea.2022.200098
[16] V. Ghai, H. Singh, P.K. Agnihotri, ACS Appl. Nano Mater., **2**, 7951 (2019). DOI: 10.1021/acsanm.9b01950]
[17] H.M.T. Raza, A.S. Rana. *Third International Conference on Latest trends in Electrical Engineering and Computing Technologies* (INTELLECT), (2022), DOI: 10.1109/INTELLECT55495.2022.9969393
[18] K.T. Lin, H. Lin, T.S. Yang, B.H. Jia. Nat Commun., **11**, 1389 (2020).
[19] X. Chen, Z. Tian, Y. Lu, Y. Xu, X. Zhang, C. Ouyang, J. Gu, J. Han, W. Zhang. Adv. Opt. Mater., **8** (3), 1900660 (2020). DOI: 10.1002/adom.201900660
[20] J. Chen, R. You, X. Bian, L. Zhu, H. Wang. IEEE Sensors, 1−4 (2022). DOI: 10.1109/SENSORS52175.2022.9967319
[21] M. Rahmanzadeh, H. Rajabalipanah, A. Abdolali. IEEE Trans. Plasma Sci., **45**, 945 (2017).
[22] S. Barzegar-Parizi, Z. Vafapour, IEEE Transactions Plasma Sci., **50** (12), 5038 (2022). DOI: 10.1109/TPS.2022.3224479
[23] Ya.I. Chizhevskaya, O.N. Smol'nikova, S.P. Skobelev. ZhTF, **91** (2), 326 (2021). DOI: 10.21883/JTF.2021.02.50369.191-20

Translated by Ego Translating