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Self-Trapping Phenomenon in an Exciton-Polariton System When the Lower Polariton Branch is Pumped by Two Laser Pulses with Close Frequencies

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Received: March 18, 2022
Revised May 20, 2022
Accepted September 15, 2022

The dynamics of exciton-polariton states in a semiconductor microcavity is studied under pumping of the state corresponding to the lower polariton branch, taking into account single-mode and intermode elastic polariton-polariton interactions. In this case, pumping is carried out by two laser pulses with close frequencies. It is shown that at the initial phase difference equal to $\theta_0 = \pi/2$, the regime of quantum self-trapping of exciton-polaritons is observed. Periodic and aperiodic modes of evolution of a system of quasiparticles are obtained.

Keywords: exciton-polariton states, quantum self-trapping, periodic and apeiodic regimes.

DOI: 10.21883/EOS.2022.12.55243.3410-22

Introduction

Exciton-polaritons have recently become a promising platform for the study of such effects of fundamental physics as Bose-Einstein condensation [1–4], superfluidity [5] and Josephson oscillations [6–8]. The properties of exciton polaritons are supposed to be used in various optical and quantum devices, such as optical transistors [9–11], diodes [12], interferometers [13], routers [14], couplers [15–17], lasers [18]. An energy-efficient neural network logic output using exciton polaritons in a microresonator was obtained in [19].

Recent experiments in exciton-polariton systems have made it possible to make high-precision measurements of the polariton-polariton interaction constants, which are key parameters determining the nonlinear dynamics of condensed exciton-polaritons [20–22]. In [23], the self-trapping of exciton-polariton condensates is theoretically predicted and experimentally obtained in [24], which is explained by the formation of a new polaron-like state. The trapped state of exciton-polaritons is stabilized due to the scattering of excitons in a polariton condensate. The interaction of exciton-polariton condensates in a semiconductor microresonator with acoustic phonons is analyzed in [25]. It was shown that parametric instability in the system leads to the generation of a coherent acoustic wave and additional polariton harmonics.

In [26,27], two identical pumping photons on the lower branch of the polariton dispersion law were used to study the properties of an optical parametric oscillator. The quantum self-trapping regime in the dynamics of exciton-polaritons in microresonators was theoretically obtained. However, in [28,29] it was shown that two different pump beams can be converted into two signal and idle modes degenerate at the photon frequency. The presence of two different pumping beams provides great opportunities for generating signal and idle beams with predetermined properties. In [30], the dynamics of polaritons is theoretically studied when pumping is carried out by two lasers with close frequencies without taking into account single-mode and intermode elastic polariton-polariton interactions. Aperiodic and periodic regimes of transformation of a pair of pump polaritons into signal and idle mode polaritons have been found. Analytical solutions of a system of nonlinear differential equations with equal attenuation constants [31] are obtained. It was shown that the introduction of two independent pumps leads to an increase in the degrees of freedom of the system.

Problem formulation. Main results

The purpose of this work is to study the dynamics of exciton-polaritons in the parametric oscillator regime, when pumping is carried out by two pulses close in frequency, taking into account elastic polariton-polariton interactions. We consider the situation when polaritons are excited on the lower branch of the law of dispersion at the „magic angle“ (Fig. 1). In this case, the process of parametric scattering of two different pump polaritons into the signal and idle modes is described by a Hamiltonian of the form

$$H = \hbar\omega_{p1}\hat{a}^+_{p1}\hat{a}_{p1} + \hbar\omega_{p2}\hat{a}^+_{p2}\hat{a}_{p2} + \hbar\omega_s\hat{a}^+_s\hat{a}_s + \hbar\omega_i\hat{a}^+_i a_i + \hbar\nu_{p1}\hat{a}^+_{p1}\hat{a}^+_{p1}\hat{a}_{p1}\hat{a}_{p1}/2 + \hbar\nu_{p2}\hat{a}^+_{p2}\hat{a}^+_{p2}\hat{a}_{p2}\hat{a}_{p2}/2 + \hbar\nu_s\hat{a}^+_s\hat{a}^+_s\hat{a}_s\hat{a}_s/2 + \hbar\nu_i\hat{a}^+_i\hat{a}^+_i\hat{a}_i\hat{a}_i/2 + \hbar\nu_{p1s}\hat{a}^+_{p1}\hat{a}_{p1}\hat{a}^+_s\hat{a}_s + \hbar\nu_{p2s}\hat{a}^+_{p2}\hat{a}_{p2}\hat{a}^+_s\hat{a}_s + \hbar\nu_{p1i}\hat{a}^+_{p1}\hat{a}_{p1}\hat{a}^+_i\hat{a}_i + \hbar\nu_{p2i}\hat{a}^+_{p2}\hat{a}_{p2}\hat{a}^+_i\hat{a}_i + \hbar\nu_{p1p2}\hat{a}^+_{p1}\hat{a}_{p1}\hat{a}^+_{p2}\hat{a}_{p2} + \hbar\nu_{si}\hat{a}^+_s\hat{a}_s\hat{a}^+_i\hat{a}_i + \hbar\mu(\hat{a}^+_s\hat{a}^+_i\hat{a}_{p1}\hat{a}_{p2} + \hat{a}^+_{p1}\hat{a}^+_{p2}\hat{a}_s\hat{a}_i), \quad (1)$$

where ω_{p1}, ω_{p2}, ω_s, ω_i — natural frequencies of two different pump polaritons, signal and idle modes, respectively; $\hat{a}_{p1}$, $\hat{a}_{p2}$, $\hat{a}_s$, $\hat{a}_i$ — polariton destruction operators; ν_{p1}, ν_{p2}, ν_s, ν_i and ν_{p1p2}, ν_{p1s}, ν_{p2s}, ν_{p1i}, ν_{p2i}, ν_{si} — constants of single-mode and intermode elastic polariton-polariton interactions; μ — constant of parametric polariton-polariton conversion. Using (1), we obtain a system of nonlinear Heisenberg equations for the operators $\hat{a}_{p1}$, $\hat{a}_{p2}$, $\hat{a}_s$, $\hat{a}_i$. Further, averaging this system of equations and using the mean field approximation, we obtain a system of nonlinear evolutionary equations for complex amplitudes of polaritons $a_{p_1} = \langle\hat{a}_{p_1}\rangle$; $a_{p_2} = \langle\hat{a}_{p_2}\rangle$, $a_s = \langle\hat{a}_s\rangle$ and $a_i = \langle\hat{a}_i\rangle$:

$$i\dot{a}_{p1} = \omega_{p1}a_{p1} + (\nu_{p1}a^*_{p1}a_{p1} + \nu_{p1s}a^*_s a_s + \nu_{p1i}a^*_i a_i + \nu_{p1p2}a^*_{p2}a_{p2})a_{p1} + \mu a^*_{p2}a_s a_i,$$

$$i\dot{a}_{p2} = \omega_{p2}a_{p2} + (\nu_{p2}a^*_{p2}a_{p2} + \nu_{p2s}a^*_s a_s + \nu_{p2i}a^*_i a_i + \nu_{p1p2}a^*_{p1}a_{p1})a_{p2} + \mu a^*_{p1}a_s a_i,$$

$$i\dot{a}_s = \omega_s a_s + (\nu_s a^*_s a_s + \nu_{p1s}a^*_{p1}a_{p1} + \nu_{p2s}a^*_{p2}a_{p2} + \nu_{si}a^*_i a_i)a_s + \mu a^*_i a_{p1}a_{p2},$$

$$i\dot{a}_i = \omega_i a_i + (\nu_i a^*_i a_i + \nu_{p1i}a^*_{p1}a_{p1} + \nu_{p2i}a^*_{p2}a_{p2} + \nu_{si}a^*_s a_s)a_i + \mu a^*_s a_{p1}a_{p2}. \quad (2)$$

The system of equations (2) must be supplemented with initial conditions, which can be written as follows:

$$a_{p1|t=0} = a_{p10}\exp(i\varphi_{p10}), \qquad a_{p2|t=0} = a_{p20}\exp(i\varphi_{p20}),$$

$$a_{s|t=0} = a_{s0}\exp(i\varphi_{s0}), \qquad a_{i|t=0} = a_{i0}\exp(i\varphi_{i0}), \quad (3)$$

where a_{p10}, a_{p20}, a_{s0}, a_{i0} and φ_{p10}, φ_{p20}, φ_{s0}, φ_{i0} — the actual amplitudes and initial phases of the corresponding exciton-polariton states.

We will further introduce the density of quasiparticles into consideration

$$n_{p1} = a^*_{p1}a_{p1}, \quad n_{p2} = a^*_{p2}a_{p2}, \quad n_s = a^*_s a_s, \quad n_i = a^*_i a_i$$

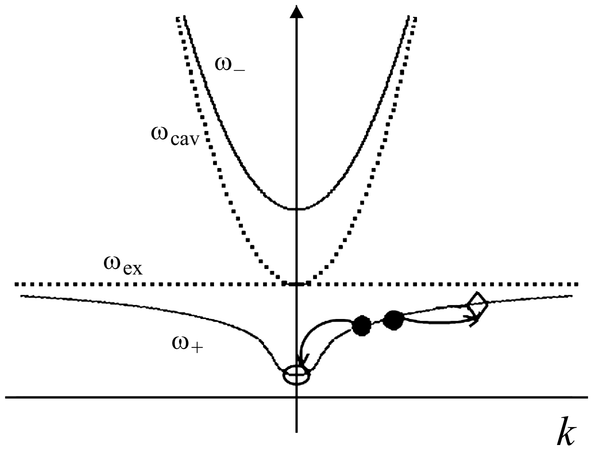

Figure 1. The energies of the polaritons of the upper and lower branches ($\omega_\pm$) of the law of dispersion. Dispersion of the natural frequencies of the microresonator ω_{cav} and exciton ω_{ex}. Two different pump polaritons dissipate into signal and idle modes.

and two „components of" polarization

$$Q = i(a_{p1}a_{p2}a^*_s a^*_i - a_s a_i a^*_{p1}a^*_{p2}),$$

$$R = a_{p1}a_{p2}a^*_s a^*_i + a_s a_i a^*_{p1}a^*_{p2}$$

and we obtain the following system of nonlinear differential equations describing the dynamics of exciton-polaritons taking into account elastic interparticle interactions:

$$\dot{n}_{p1} = \dot{n}_{p2} = \mu Q, \qquad \dot{n}_s = \dot{n}_i = -\mu Q,$$

$$\dot{Q} = \big[\Delta + (\nu_{p1} + \nu_{p1p2} - \nu_{p1s} - \nu_{p1i})n_{p1} + (\nu_{p2} + \nu_{p1p2} - \nu_{p2s} - \nu_{p2i})n_{p2} + (\nu_{p1i} + \nu_{p2i} - \nu_i - \nu_{si})n_i + (\nu_{p1s} + \nu_{p2s} - \nu_s - \nu_{si})n_s\big]R + 2\mu[n_s n_i(n_{p1} + n_{p2}) - n_{p1}n_{p2}(n_s + n_i)],$$

$$\dot{R} = -[\Delta + (\nu_{p1} + \nu_{p1p2} - \nu_{p1s} - \nu_{p1i})n_{p1} + (\nu_{p2} + \nu_{p1p2} - \nu_{p2s} - \nu_{p2i})n_{p2} + (\nu_{p1i} + \nu_{p2i} - \nu_i - \nu_{si})n_i + (\nu_{p1s} + \nu_{p2s} - \nu_s - \nu_{si})n_s]Q, \quad (4)$$

where $\Delta = \omega_{p1} + \omega_{p2} - \omega_s - \omega_i$ — resonance disorder.

Using the initial conditions for the amplitudes of polaritons (3), we obtain the initial conditions for the densities of quasiparticles and the components of „polarization":

$$n_{p1|t=0} = |a_{p10}|^2 = n_{p10}, \qquad n_{p2|t=0} = |a_{p20}|^2 = n_{p20},$$

$$n_{s|t=0} = |a_{s0}|^2 = n_{s0}, \qquad n_{i|t=0} = |a_{i0}|^2 = n_{i0},$$

$$Q_{|t=0} = Q_0 = 2\sqrt{n_{p10}n_{p20}n_{s0}n_{i0}}\sin\theta_0,$$

$$R_{|t=0} = R_0 = 2\sqrt{n_{p10}n_{p20}n_{s0}n_{i0}}\cos\theta_0, \quad (5)$$

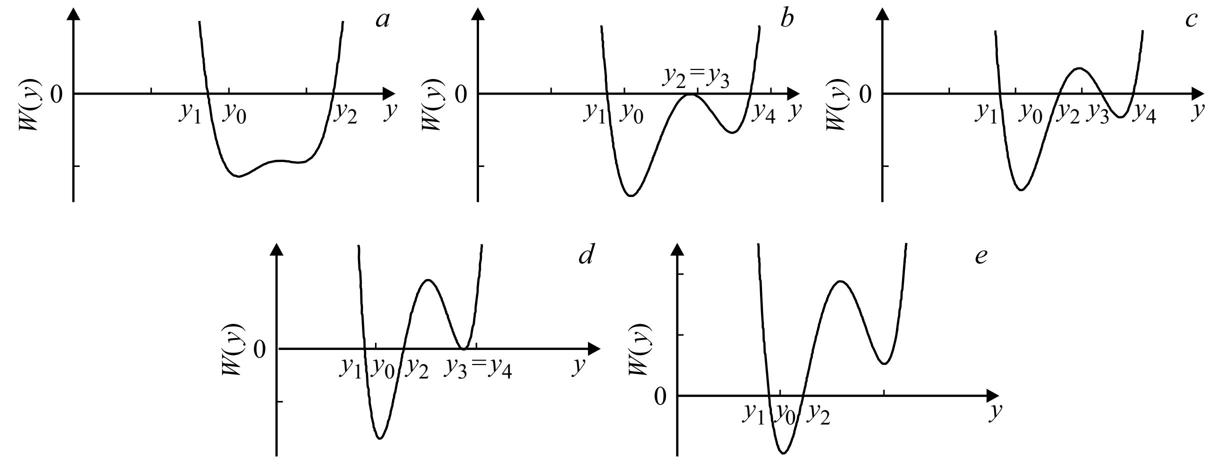

Figure 2. Graph of the potential energy of a nonlinear oscillator (8) under conditions of exact resonance, depending on the normalized density of the first pulse pumping polaritons at fixed values of the initial phase difference $\theta_0 = \pi/2$, the normalized initial densities of quasiparticles $\bar{n}_{p20} = 0.3$, $\bar{n}_{s0} = 0.9$, constant polariton-polariton elastic interactions $\bar{\nu}_{p1} = 4$, $\bar{\nu}_{p2} = 7$, $\bar{\nu}_s = -4$, $\bar{\nu}_i = -9$ and various $\bar{n}_{i0} = 1$ (*a*), 1.337 (*b*), 1.4 (*with*), 1.512 (*d*), 1.7 (*e*).

where $\theta_0 = \varphi_{s0} + \varphi_{i0} - \varphi_{p10} - \varphi_{p20}$ — initial phase difference.

From a system of nonlinear differential equations (4) it is not difficult to obtain integrals of motion

$$n_{p1} - n_{p2} = n_{p10} - n_{p20}, \qquad n_{p1} + n_s = n_{p10} - n_{s0},$$

$$n_s - n_i = n_{s0} - n_{i0}, \qquad n_{p2} + n_i = n_{p20} + n_{i0},$$

$$Q^2 + R^2 = 4 n_{p1} n_{p2} n_s n_i,$$

$$R = R_0 + \frac{\Delta}{\mu}(n_{p10} - n_{p1}) + \frac{(\nu_{p1} + \nu_{p1p2} - \nu_{p1s} - \nu_{p1i})}{2\mu}(n_{p10}^2 - n_{p1}^2) + \frac{(\nu_{p2} + \nu_{p1p2} - \nu_{p2s} - \nu_{p2i})}{2\mu}(n_{p20}^2 - n_{p2}^2) + \frac{(\nu_{p1s} + \nu_{p2s} - \nu_s - \nu_{si})}{2\mu}(n_s^2 - n_{s0}^2) + \frac{(\nu_{p1i} + \nu_{p2i} - \nu_i - \nu_{si})}{2\mu}(n_i^2 - n_{i0}^2). \qquad (6)$$

Further consideration will be carried out for normalized values $y = \frac{n_{p1}}{n_{p10}}$, $\alpha = \frac{\Delta}{\mu n_{p10}}$, $\tau = t\mu n_{p10}$, $\bar{n}_{p20} = \frac{n_{p20}}{n_{p10}}$, $\bar{n}_{s0} = \frac{n_{s0}}{n_{p10}}$, $\bar{n}_{i0} = \frac{n_{i0}}{n_{p10}}$, $\bar{\nu}_{p1} = \frac{\nu_{p1} - \nu_{p1p2} - \nu_{p1s} - \nu_{p1i}}{\mu}$, $\bar{\nu}_{p2} = \frac{\nu_{p2} - \nu_{p1p2} - \nu_{p2s} - \nu_{p2i}}{\mu}$, $\bar{\nu}_s = \frac{\nu_{p1s} + \nu_{p2s} - \nu_s - \nu_{si}}{\mu}$, $\bar{\nu}_i = \frac{\nu_{p1i} + \nu_{p2i} - \nu_i - \nu_{si}}{\mu}$.

Using the integrals of motion and the introduced normalizations, it is possible to bring a system of nonlinear differential equations (4) to one nonlinear differential equation for the normalized density of the pump polaritons y of the first pulse:

$$\frac{1}{2}\left(\frac{dy}{d\tau}\right)^2 + W(y) = 0, \qquad (7)$$

where

$$W(y) = 2\Bigg(-2y(y - 1 + \bar{n}_{p20})(1 + \bar{n}_{s0} - y)(1 + \bar{n}_{i0} - y) + \Bigg[\sqrt{\bar{n}_{p20}\bar{n}_{s0}\bar{n}_{i0}}\cos\theta_0 + \frac{1}{2}\alpha(1 - y) + \frac{\bar{\nu}_{p1}}{4}(1 - y^2) + \frac{\bar{\nu}_{p2}}{4}(1 - y)(y - 1 + 2\bar{n}_{p20}) + \frac{\bar{\nu}_s}{4}(1 - y)(1 - y + 2\bar{n}_{s0}) + \frac{\bar{\nu}_i}{4}(1 - y)(1 - y + 2\bar{n}_{i0})\Bigg]^2\Bigg). \qquad (8)$$

Differential equation (7) is a special case of the equation of a nonlinear oscillator with a total energy equal to zero, where the term $\frac{1}{2}\left(\frac{dy}{d\tau}\right)^2$ plays the role of kinetic, and the term $W(y)$ — potential energy, respectively. The type of solution $y(\tau)$ is determined by the roots of the algebraic equation $W(y) = 0$, which, as can be seen from (8), depend on the initial densities of quasiparticles $\bar{n}_{p20}$, $\bar{n}_{s0}$, $\bar{n}_{i0}$, the initial phase difference θ_0, the normalized resonance detuning α and from the normalized constants of elastic polariton-polariton interactions $\bar{\nu}_{p1}$, $\bar{\nu}_{p2}$, $\bar{\nu}_s$, $\bar{\nu}_i$.

We will look for a solution to the differential equation (7) in the case when the initial phase difference is $\pi/2$. Then the equation $W(y) = 0$ can have either two or four real roots (Fig. 2, 3). Fig. 3 shows the dependence of the real roots on the normalized density of the polaritons of the idle mode at fixed values of the normalized initial densities of the polaritons of the pumps of both pulses and the initial density of the polaritons of the signal mode and the normalized resonance detuning $\alpha = 0$. It can be seen that for small values of $\bar{n}_{i0}$ there are two real roots —$y_4 > y_1$ and two complex conjugate — $y_{2,3} = a \pm ib$. The amplitude of the oscillations of the normalized density of the polaritons

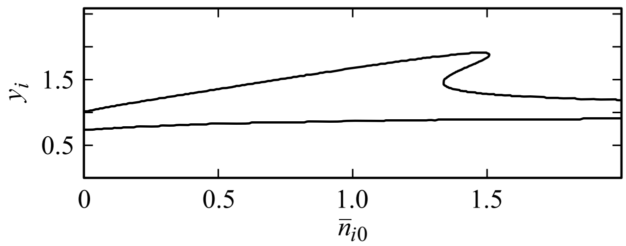

Figure 3. Dependence of the real roots of the potential energy equation of a nonlinear oscillator under conditions of precise resonance on the normalized initial density of polaritons of the idle mode at fixed values of the system parameters: $\bar{n}_{p20} = 0.3$, $\bar{n}_{s0} = 0.9$, $\bar{\nu}_{p1} = 4$, $\bar{\nu}_{p2} = 7$, $\bar{\nu}_s = -4$, $\bar{\nu}_i = -9$.

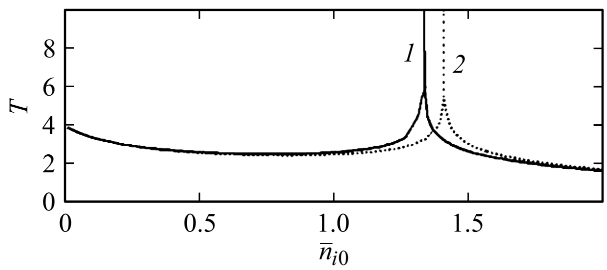

Figure 4. The period of fluctuations in the density of the polaritons of the first pulse pumping, depending on the normalized initial density of the polaritons of the idle mode and various values of $\alpha = 0$ (*1*), 0.5 (*2*) and for fixed values of system parameters: $\bar{n}_{p20} = 0.3$, $\bar{n}_{s0} = 0.9$, $\bar{\nu}_{p1} = 4$, $\bar{\nu}_{p2} = 7$, $\bar{\nu}_s = -4$, $\bar{\nu}_i = -9$.

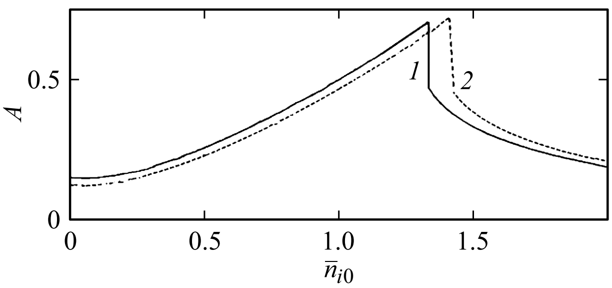

Figure 5. The amplitude of fluctuations in the density of polaritons pumping the first pulse, depending on the normalized initial density of polaritons of the idle mode and various values $\alpha = 0$ (*1*), 0.5 (*2*) and for fixed values of system parameters: $\bar{n}_{p20} = 0.3$, $\bar{n}_{s0} = 0.9$, $\bar{\nu}_{p1} = 4$, $\bar{\nu}_{p2} = 7$, $\bar{\nu}_s = -4$, $\bar{\nu}_i = -9$.

of the pumping of the first pulse will be determined by the expression $A = y_4 - y_1$. The dynamics of the system will be periodic transformations of quasiparticles, while the oscillation period will be equal to

$$T = \frac{2K(k)}{m}, \tag{9}$$

where $K(k)$ — is a complete elliptic integral of the first kind with the module k [32,33]. The parameter m is defined by the expression

$$m = (m_1 m_2)^{\frac{1}{4}}, \tag{10}$$

where

$$m_1 = (y_2 - y_4)(y_3 - y_1), \quad m_2 = (y_2 - y_1)(y_3 - y_4).$$

Solution of the equation (7) will be expressed in terms of Jacobi elliptic functions:

$$y = \frac{y_1 + y_4}{2} + \frac{y_1 - y_4}{2} \frac{c - d + (c + d)cn(m\tau + F(\varphi_0, k))}{c + d + (c - d)cn(m\tau + F(\varphi_0, k))}, \tag{11}$$

where

$$c = \sqrt{(y_2 - y_4)(y_3 - y_4)}, \quad d = \sqrt{(y_2 - y_1)(y_3 - y_1)}.$$

In (11) $F(\varphi_0, k)$ — an incomplete elliptic integral of the first kind with module k and parameter φ_0 [32,33], which are defined by the following expressions:

$$k^2 = \frac{1 - (m_1 + m_2)/(2\sqrt{m_1 m_2})}{2},$$

$$\varphi_0 = \arccos\left(\frac{y_4 c + y_1 d - c - d}{y_1 d - y_4 c + c - d}\right). \tag{12}$$

In addition, as can be seen from Fig. 2, *b* and Fig. 3, with an increase in the initial polariton density of the signal mode, the two middle roots of the equation $W(y) = 0$ turn out to be degenerate, $y_2 = y_3$, which corresponds to the aperiodic regime of evolution of quasiparticles. The solution in this case will be written as

$$y = \frac{y_1(y_4 - y_2) + y_4(y_2 - y_1)\mathrm{th}^2\left(\frac{\sqrt{(y_4 - y_2)(y_2 - y_1)}\tau}{2}\right)}{y_4 - y_2 + (y_2 - y_1)\mathrm{th}^2\left(\frac{\sqrt{(y_4 - y_2)(y_2 - y_1)}\tau}{2}\right)}. \tag{13}$$

As the normalized initial density of polaritons of the idle mode $\bar{n}_{i0}$ increases, a region of existence of four real roots of the equation $W(y) = 0$ arises (Fig. 2, *c*, Fig. 3). We will arrange the roots in descending order: $y_4 > y_3 > y_2 > y_1$. The temporal evolution of exciton-polaritons in this case will be periodic transformations of quasiparticles. The amplitude and period of the oscillations will be determined by the following expressions:

$$A = y_2 - y_1, \quad T = \frac{4K(k)}{\sqrt{(y_4 - y_2)(y_3 - y_1)}}. \tag{14}$$

Then the solution of equation (7) can be obtained in the form

$$y = \frac{y_1(y_4 - y_2) + y_4(y_2 - y_1) \times \mathrm{sn}^2\left(\frac{\sqrt{(y_4 - y_2)(y_3 - y_1)}\tau}{2} \pm F(\varphi_0, k)\right)}{y_4 - y_2 + (y_2 - y_1) \times \mathrm{sn}^2\left(\frac{\sqrt{(y_4 - y_2)(y_3 - y_1)}\tau}{2} \pm F(\varphi_0, k)\right)}. \tag{15}$$

The module k and the parameter φ_0 in (14) and (15) are defined as

$$k^2 = \frac{(y_2 - y_1)(y_3 - y_4)}{(y_2 - y_4)(y_3 - y_1)},$$

$$\varphi_0 = arcsin\sqrt{\frac{(y_4 - y_2)(1 - y_1)}{(y_2 - y_1)(y_4 - 1)}}. \quad (16)$$

Thus, we observe a transition from a periodic regime of evolution to an aperiodic regime and again to a periodic one when only the initial normalized density of polaritons of the idle mode changes. With an increase in the normalized resonance detuning, the point of the bifurcation transition from the periodic to the aperiodic regime of evolution shifts to the region of higher normalized initial densities of polaritons of the idle mode (Fig. 4).

With a further increase in the normalized initial density of the polaritons of the idle mode, the degeneracy of the two largest roots of the equation occurs again $W(y) = 0$: $y_3 = y_4$. Solution of the equation (7) in this case is still periodic with a period

$$T = \frac{2\pi}{\sqrt{(y_3 - y_2)(y_3 - y_1)}}$$

and will be defined by the expression

$$y = \frac{y_1(y_3 - y_2) + y_3(y_2 - y_1)\sin^2\left(\frac{\sqrt{(y_3-y_2)(y_3-y_1)}\tau}{2}\right)}{y_3 - y_2 + (y_2 - y_1)\sin^2\left(\frac{\sqrt{(y_3-y_2)(y_3-y_1)}\tau}{2}\right)}. \quad (17)$$

Then there is a bifurcation transition from the four real roots of the equation $W(y) = 0$ to two real roots, $y_2 > y_1$, and two complex conjugates (Fig. 2, e). However, there is no significant effect on the amplitude of oscillations of the normalized density of polaritons of the first pulse pumping. The amplitude of the oscillations in this case monotonically decreases with the growth of the normalized density of polaritons of the idle mode and is determined by the expression $A = y_2 - y_1$. The dynamics of the system will be periodic transformations of quasiparticles, while the oscillation period will be equal to $T = \frac{4K(k)}{m}$, where $K(k)$ — a complete elliptic integral of the first kind with the module k [32,33]. Solution of the equation (7) will be expressed in terms of elliptic cosine:

$$y = \frac{y_1+y_2}{2} + \frac{y_1-y_2}{2}\,\frac{c-d+(c+d)\mathrm{cn}(m\tau+F(\varphi_0,k))}{c+d+(c-d)\mathrm{cn}(m\tau+F(\varphi_0,k))}, \quad (18)$$

where

$$c = \sqrt{(y_4 - y_2)(y_3 - y_2)},$$
$$d = \sqrt{(y_4 - y_1)(y_3 - y_1)},$$
$$m = (m_1 m_2)^{\frac{1}{4}}, \quad (19)$$
$$m_1 = (y_4 - y_2)(y_3 - y_1),$$
$$m_2 = (y_4 - y_1)(y_3 - y_2).$$

The module k and the parameter φ_0 in this case are defined by the following expressions:

$$k^2 = \frac{1 - (m_1 + m_2)/(2\sqrt{m_1 m_2})}{2},$$

$$\varphi_0 = \arccos\left(\frac{y_1 c + y_2 d - c - d}{y_2 d - y_1 c + c - d}\right). \quad (20)$$

Fig. 5 shows a graph of the dependence of the amplitude of the oscillations of the normalized density of the polaritons of the first pulse pump depending on the normalized initial density of the polaritons of the idle mode. It can be seen that the oscillation amplitude monotonically increases with an increase in $\bar{n}_{i0}$, reaches its maximum value, and at the bifurcation point of the roots of the equation $W(y) = 0$ (Fig. 2, b, 3, 5) a region of a sharp decrease in amplitude appears oscillations, which indicates the occurrence of the phenomenon of self-trapping in the system of quasiparticles. With an increase in the normalized resonance detuning α, the region of occurrence of the phenomenon of self-trapping in the system of quasiparticles shifts to the region of higher normalized densities of polaritons of the idle mode. Here, the manifestation of the phenomenon of self-trapping is not as bright as in the case of taking into account the elastic interatomic interaction under the conditions of Bose-Einstein condensation of atoms and molecules [34,35].

Conclusion

Thus, when pumping the lower polariton branch with two pulses with close frequencies, taking into account the processes of interelastic interaction of exciton-polaritons, periodic and aperiodic regimes of evolution are obtained in the system. There is a sharp decrease in the amplitude of oscillations of the normalized density of the pump polaritons, i.e., self-trapping in the system is observed. In this case, there is a slight localization of the polaritons of the pumping of the first pulse on the lower branch of the law of dispersion. It should be noted, that when pumping was carried out at one point of the law of dispersion by two identical pulses, we observed a sharp increase in the amplitude of oscillations [26] with an increase in the normalized density of polaritons of the idle mode.

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