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Rotational and translational galloping of prisms in the air stream

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In experiments in a wind tunnel, oscillations of three prisms with a rectangular cross section are studied. The prisms are located perpendicular to the velocity vector of the incoming flow and are limited from the ends by end plates that prevent air flow. The elastic suspension allows body oscillation with six degrees of freedom. It turned out that under the action of the air flow, two modes of oscillation of prism realized: translational oscillations in the direction perpendicular to the generatrix of prismatic bodies and the flow velocity, and rotational oscillations around an axis that is parallel to the generatrix, passes through the center of the prism and is perpendicular to the velocity of the incoming air flow. The tension of the two springs included in the elastic suspension is measured by the strain gauge method during the oscillation. The calibration experiment makes it possible to link the amplitudes of spring tension oscillations and phase shift with the amplitudes of rotational and translational oscillations of prisms. It turned out that a prism with a height-to-width ratio of 0.22 in the flow is subject to rotational oscillations. An increase in the ratio of height to width to 0.36 leads to a decrease in the amplitude of rotational oscillations and the appearance of translational ones. The ranges of existence of rotational and translational oscillations overlap. A further increase in the ratio of height to width to 0.43 is accompanied by intensive translational galloping.

Keywords: galloping, bluff body, wind tunnel, strain gauge, translational oscillations, rotational oscillations.

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Introduction

In this paper, we study the aeroelastic galloping of extended bodies. Bluff elastic or elastically fixed bodies under the action of wind can perform translational or rotational vibrations. In addition to galloping, another mechanism for oscillations excitation of extended bodies in an air stream is possible, associated with the formation of a periodic vortex chain in the wake of the body. In the present paper, it is assumed that the frequency of vortex shedding forming the Karman vortex chain is much higher than the natural oscillation frequency of the elastically fixed body; therefore, the aerodynamic forces arising due to vortex shedding are averaged and do not affect the slower oscillation process during galloping. The purpose of this paper is to determine the modes of oscillation in the air stream of elastically fixed long prisms, depending on the proportions of their rectangular cross section.

Prediction of wind oscillation modes of buildings, structures, bridges, cargo carried by cranes and aircrafts, having a shape close to the shape of prisms, is important to prevent the dangerous consequences of such oscillations.

To describe the translational galloping of a bluff body a quasi-stationary model is proposed in [1]. This model is based on the hypothesis that the aerodynamic forces acting on the body depend only on the relative stream velocity and on the angles describing the body orientation relative to the air stream velocity vector. For a transverse air stream around the extended body oscillating in a direction perpendicular to the stream, the normal aerodynamic force acting in the direction of motion depends only on the instantaneous angle of attack α. The normal aerodynamic force coefficient c_y depending on the angle of attack α can be determined in a wind tunnel in experiments with a fixed body. In the paper [1] the dependence $c_y(\alpha)$ of the square prism was approximated by a fifth-order polynomial. Later [2] it was found that the model performs better if $c_y(\alpha)$ is approximated by a seventh order polynomial. The quasi-stationary model was widely used later to describe the translational galloping of rectangular cylinders of various proportions [3], cylinders with a triangular cross-section [4], diamond-shaped cross-section [5], oblique cylinders [6]. The influence of the aspect ratio during translational galloping was studied in the paper [3]. The results of this study are presented in the monograph [7].

The oscillation model of the elastically fixed body with two degrees of freedom was developed in the papers [8,9]. The authors of these papers made attempts to extend the quasi-stationary galloping model to the rotational oscillations of two bluff bodies: a square prism and bluff angle section. The difficulty in applying these models lies in the fact that different points of the rotating body have different speeds and, consequently, the instantaneous angle of attack of these points is different. We have to choose the characteristic point to determine the instantaneous angle of attack, this is some arbitrary rule.

1. Experimental method and results processing

The experiments were carried out in the wind tunnel AT-12 located on-premises Saint Petersburg State University [10]. The wind tunnel has an open test section, the outlet part of the circular section nozzle has a diameter of 1500 mm. The speed of the air stream can be smoothly adjusted in the range from 0 to 40 m/s.

The oscillation of three prismatic bodies was studied. All bodies made of wood had the same length $L = 700$ mm, the same width $W = 95$ mm and different heights H — 21, 34 and 41 mm. Thus, the aspect ratio H/W was 0.22, 0.36, and 0.43, respectively. End plates were installed on the bodies, they were discs with a diameter of 200 mm, which limit the air stream through the ends. The bodies were suspended in the test section of the wind tunnel on a wire suspension containing eight steel springs. The layout of the bodies in the test section is shown in Fig. 1. In the equilibrium position the small side face of the prismatic bodies is oriented perpendicular to the speed vector of the oncoming stream. Bodies suspended in this way could move with six degrees of freedom near the equilibrium position. However, only two types of noticeable oscillations were observed during the experiment: translational oscillations of the center of gravity in the vertical direction and rotational oscillations around a horizontal axis perpendicular to the speed vector of the oncoming stream.

Two S-50 semiconductor strain gauges register the tension of the two lower springs. The Velleman-PCS500 PC oscilloscope converts the analog output signals of strain gauges into digital ones and transmits them to the control computer. Readout frequency is 100 Hz or 1250 Hz. The duration of readings recording is 17 s or 3.3 s, respectively. The amplitude of the steady oscillations was determined, as a rule, with a readout frequency of 1250 Hz. Damped oscillations were studied with a readout frequency of 100 Hz, since this mode made it possible to cover a longer time interval.

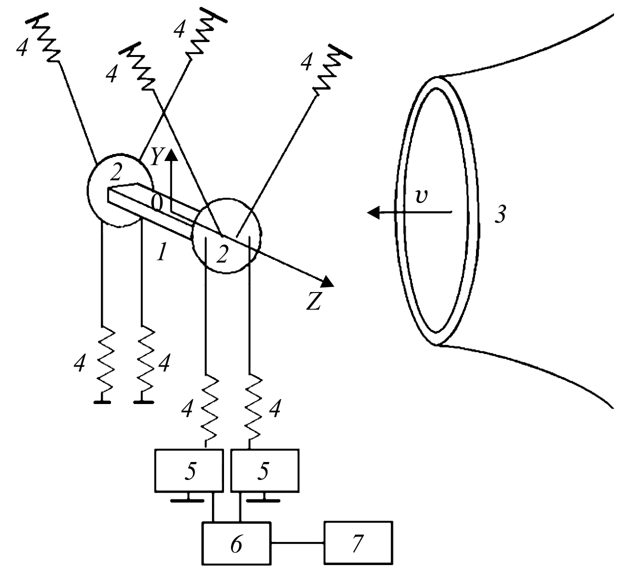

Figure 1. Scheme of experiment: *1* — model, *2* — end plates, *3* — wind tunnel nozzle, *4* — springs, *5* — semiconductor strain gauges, *6* — PC oscilloscope, *7* — computer.

The calibration procedure made it possible to relate the amplitudes of the tension oscillations of the two springs and the phase difference with the amplitudes of the vertical periodic displacement of the two points of the model, to which the wire rods connected with the springs were attached. A load of known weight was suspended to the model during the experiment. At the same time, a change in the readings of the instruments and a vertical displacement of the rod attachment points were recorded.

Fig. 2 shows examples of the vertical displacements of two model points y_1 and y_2 vs. time.

The graph in Fig. 2, *a* shows a fragment of a record of stable translational oscillations of a prismatic body with aspect ratio of $H/W = 0.43$. Air stream speed is 6.5 m/s. The horizontal distance between points is $l = 180$ mm. The signal reading frequency is 100 Hz. Oscillations of two points occur in phase. It is noticeable that the amplitude of oscillations of the rear point slightly exceeds the amplitude of oscillations of the front point. This means that simultaneously with translational oscillations the rotational oscillations occur with the same frequency, the amplitude of which is very small.

The graph in Fig. 2, *b* shows a fragment of a record of steady rotational oscillations of a prism with an aspect ratio of $H/W = 0.22$. Air stream speed is 15.6 m/s. The horizontal distance between points is $l = 180$ mm. The signal reading frequency is 1250 Hz. Oscillations of two points occur in antiphase.

The oscillation frequencies of rotational and translational oscillations do not depend on the ongoing stream speed and are determined by the elasticity of the springs, the reduced weight of the model, and the moment of inertia of the model.

We assume that the time dependences of y_j $(j = 1, 2)$ are sums of harmonic functions, a constant value D_j and errors ξ, which are a random variable:

$$y_{ji} = B_j \cos \omega t_i + C_j \sin \omega t_i + D_j + \xi_{ji}, \qquad (1)$$

where $i = 1, 2, \ldots, n$ — count number. We also assume that the dispersion of the random variable ξ_{ji} does not depend on i. Then the coefficients B_j, C_j and D_j in equation (1) can be determined by the least squares method, minimizing the sum of squared errors $\sum_{i=1}^{n} \xi_{ji}^2$. The oscillation frequency was determined either using the Fourier transform or by counting the number of oscillation periods in a known time interval. For oscillations with a steady amplitude, the sample size n covered several oscillation periods. When damped oscillations were analyzed, the number of elements in the sample n corresponded to the number of readings in one oscillation period. The

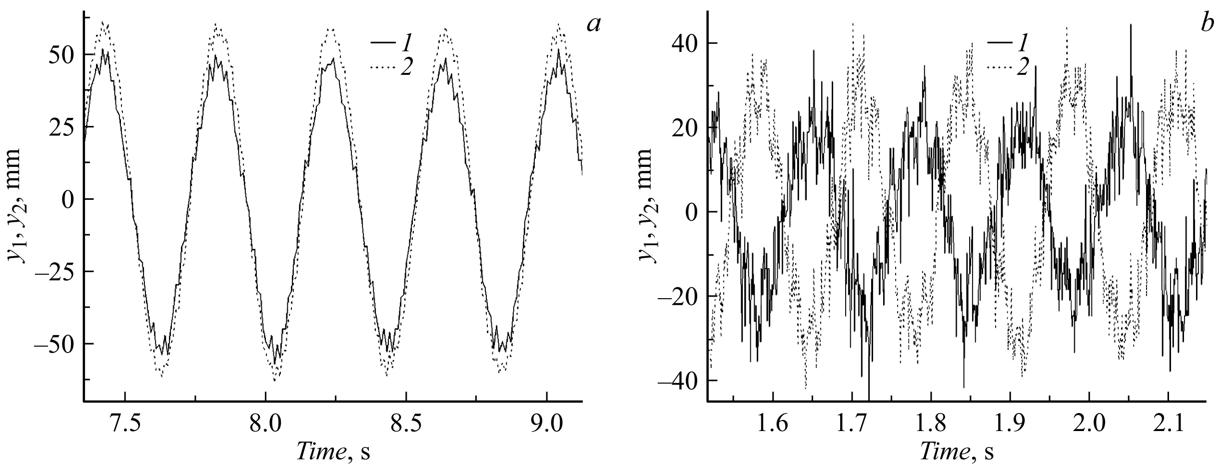

Figure 2. Movement of two points on the model vs. time: *1* — front point, *2* — back point.

oscillation amplitudes of the rod attachment points a_j and the oscillation phase shift φ are expressed by the formulas

$$a_j = \sqrt{B_j^2 + C_j^2}, \quad \mathrm{tg}\,\varphi = \frac{C_1B_2 - C_2B_1}{B_1B_2 + C_1C_2}. \qquad (2)$$

All measurements were repeated at least five times, which made it possible to estimate the random component of the experimental error. The amplitude of the translational oscillations of the center of gravity of the prisms along the vertical axis ρ_y and the rotational oscillations ρ_θ were determined by the formulas

$$\rho_y = \frac{\sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\varphi}}{2},$$

$$\rho_\theta = \frac{\sqrt{a_1^2 + a_2^2 - 2a_1a_2\cos\varphi}}{l}. \qquad (3)$$

The dimensionless amplitudes of translational oscillations ρ_Y were obtained from the dimensional ones by multiplying by the angular frequency and dividing by the air stream speed: $\rho_Y = \rho_y\omega/v$. The amplitude of translational oscillations, nondimensionalized in this way, is equal to the amplitude of oscillations of the instantaneous angle of attack.

2. Experimental results

Three prismatic bodies oscillated in the stream in different modes. The experiments were carried out in the range of Reynolds numbers determined from the width of prisms W, $2.5 \cdot 10^4 < \mathrm{Re} < 1.2 \cdot 10^5$. Scruton's number $\mathrm{Sc} = 2m/(\rho W^2)\delta$, where m — mass of unit prism length, ρ — air density, δ — logarithmic oscillation decrement, was in the range of $10 < \mathrm{Sc} < 11$. For all elastically fixed prisms, the natural frequency of rotational oscillations was higher than the natural frequency of translational oscillations. Natural frequencies f of translational and rotational oscillations are given in the Table.

Natural oscillation frequencies

H/W	Frequency f, Hz	
	Translational vibrations	Rotational vibrations
0.22	3.7	7.6
0.36	2.5	4.5
0.43	2.4	4.2

The prism with the smallest aspect ratio $H/W = 0.22$ performed rotational oscillations. Figure 3, *a* shows the amplitude of rotational oscillations (in radians) vs. $1/\mathrm{Sh}$, where $\mathrm{Sh} = fW/v$ — Strouhal number. The variable $1/\mathrm{Sh}$ is the dimensionless speed of the oncoming stream. The same Figure shows the dependence of dimensionless amplitude of translational oscillations having the same frequency 7.6 Hz.

To estimate the frequency of vortex shedding in the Karman chain, we assume that the Strouhal number fH/v of the vortex chain is 0.13. Such a Strouhal number is typical for a square prism [11]. In this case, for the minimum speed in the experiment 9 m/s, the frequency of the Karman chain vortex shedding is 56 Hz, which is much higher than the frequency of rotational oscillations 7.6 Hz.

The mathematical model proposed earlier for the rotational vibrations of some bluff bodies, such as a small aspect ratio cylinder [12] and a bridge segment [13], predicts that the dependence of the squared amplitude on the Strouhal number is linear, if the stream speed is high enough. To test this model applicability, such a dependence is plotted in Fig. 3, *b*. The points in this Figure lie near a straight line.

The graphs in Fig. 3 also show 95% confidence intervals obtained from an estimate of the repeated measurements scattering.

The prism can oscillate translationally in the vertical direction with a frequency close to the natural oscillation frequency 3.7 Hz. We excited such oscillations by shifting the model in the vertical direction by a distance of

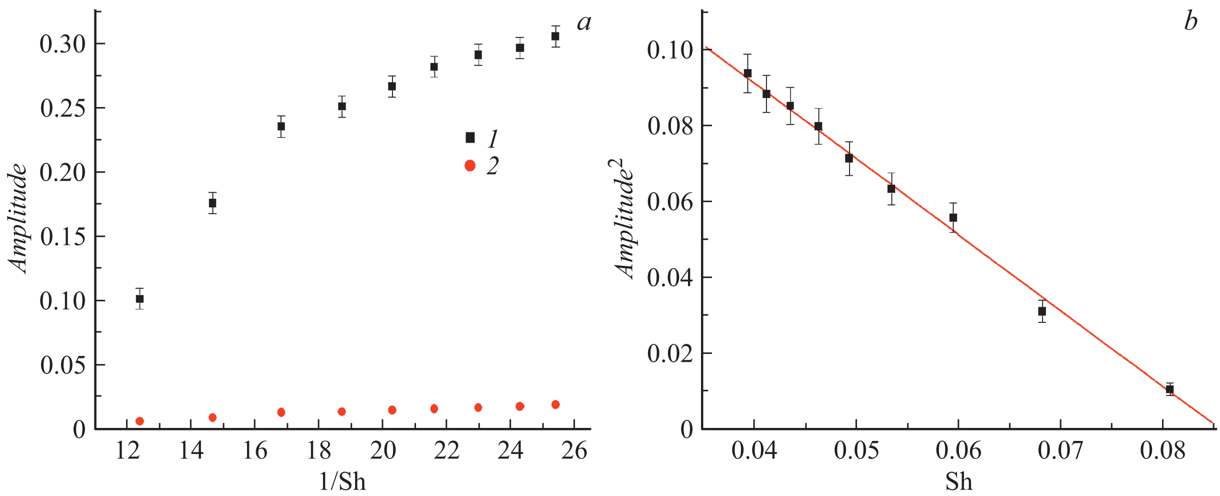

Figure 3. Dependences: *a* — oscillation amplitudes of prism with aspect ratio 0.22 vs. 1/Sh, *b* — squared amplitude vs. Sh: *1*— rotational oscillations with a frequency of 7.6 Hz, *2* — translational oscillations with the same frequency.

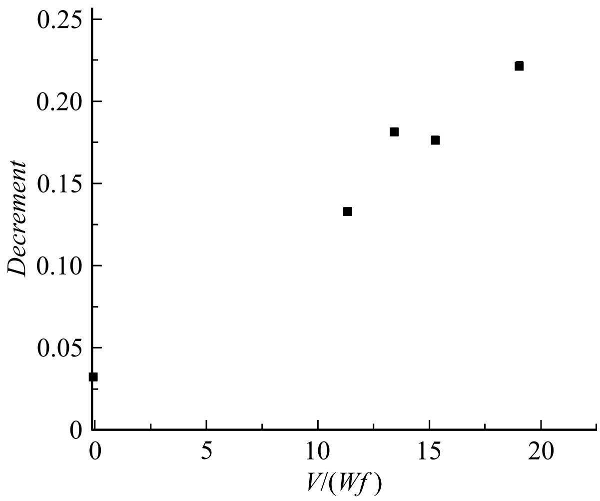

Figure 4. Logarithmic decrement of the translational vibrations of prism with aspect ratio of $H/W = 0.22$ vs. the dimensionless stream speed.

$0.3-0.5W$ and releasing it. In this case, the oscillations are damped. Within a few seconds, the dependence of the logarithm of the oscillation amplitude on time is close to linear. At this time interval, the slope of the logarithm of the amplitude vs. time was determined. During translational oscillations, rotational oscillations are not observed.

Fig. 4 shows the logarithmic oscillation decrement vs. the dimensionless speed of the oncoming stream. It turned out that the air stream speeds up the attenuation process. The point corresponding to the stream speed $v = 0$ was obtained with the wind tunnel fan turned off.

Unlike the prism with small aspect ratio $H/W = 0.22$, the prism with aspect ratio $H/W = 0.43$ performs translational oscillations in the air stream. Translational oscillations of bluff bodies in stream are often described using the quasi-stationary approximation. Within the framework of the approximation, it is assumed that the aerodynamic forces depend only on the instantaneous angle of attack. The tangent of the angle of attack is equal to the ratio of the speed of the body vertical movement, taken with the opposite sign, and the speed of the oncoming stream. The aerodynamic normal force can be determined in the wind tunnel on fixed models. The dependence of the normal force coefficient on the angle of attack is in many cases expressed by a high-order polynomial. In particular, for a square prism a seventh-order polynomial gives good results. If the dependence of the normal force on the angle of attack is limited to a term of the third order, the mathematical model gives that the dependence of the square of the dimensionless amplitude on the inverse speed becomes linear [11]. Fig. 5 shows two experimental graphs. On the graph in Fig. 5, *a* there is the dependence of the dimensionless amplitude on 1/Sh, the graph in Fig. 5, *b* shows the dependence of the squared amplitude on the Strouhal number Sh. The last graph shows noticeable deviations from the linear relationship.

Just as rotational oscillations of the prism with small aspect ratio are accompanied by translational oscillations with small amplitude of the same frequency, the translational oscillations of prism with aspect ratio of 0.43 are accompanied by small rotational oscillations with translational oscillations frequency. The amplitudes of these small oscillations are not shown in Fig. 5.

The third prismatic body with an average aspect ratio $H/W = 0.36$ is subject to both translational and rotational oscillations. Moreover, the speed ranges in which oscillations are realized overlap. The joint existence of steady rotational and translational oscillations is also observed. There is a competition between two modes of oscillations. Previously, such competition between rotational and translational oscillations was predicted by the mathematical model

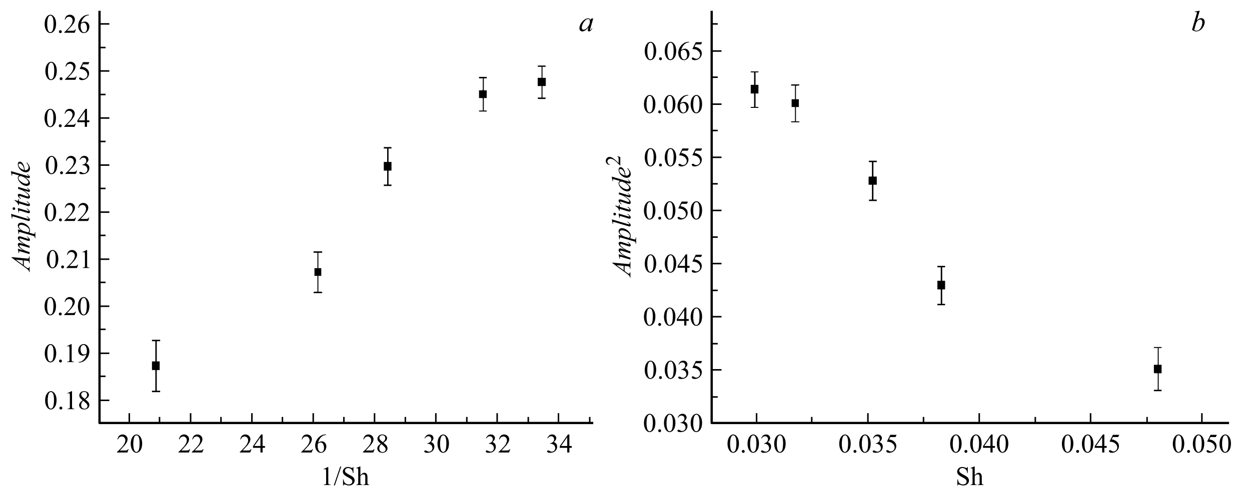

Figure 5. Dependences: *a* — dimensionless amplitude of translational vibrations of prism with aspect ratio of 0.43 with frequency of 2.4 Hz vs. 1/Sh, *b* — amplitude squared vs. Strouhal number Sh.

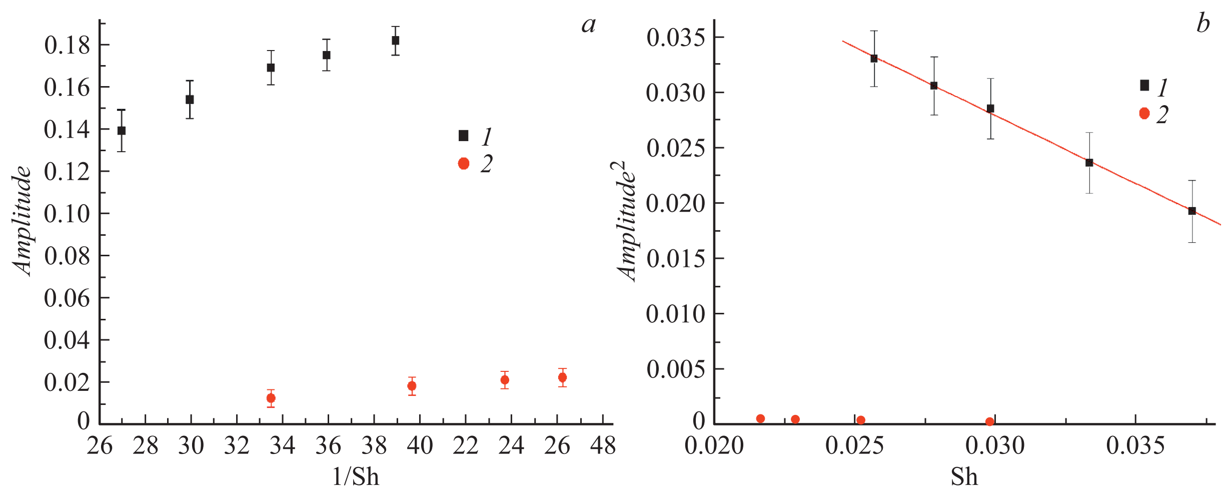

Figure 6. Dependences: *a* — prism oscillation amplitudes with aspect ratio 0.36 on 1/Sh, *b* — amplitude squared on Sh: *1* — translational oscillations with a frequency of 2.5 Hz, *2* — rotational oscillations with a frequency of 4.5 Hz, the sampling frequency of the instrument is 1250 Hz.

proposed in [12]. In the experiment with testing the bridge segment model, its predictions were verified.

The amplitude of rotational and translational oscillations vs. 1/Sh for prismatic body with aspect ratio of 0.36 is shown in Fig. 6, *a*. The Strouhal number is based on the frequency of translational vibrations.

The dependence of the squared amplitude of rotational and translational oscillations on the Strouhal number Sh is shown in Fig. 6, *b*. The dependence is close to linear one.

The amplitude of rotational oscillations is much less than the amplitude of dimensionless translational oscillations. In Fig. 7, *a* the amplitude of rotational oscillations vs. 1/Sh is shown in different scale, while Fig. 7, *b* shows – square of amplitude of oscillations vs. Sh. It turned out that rotational oscillations with a frequency of 4.5 Hz occur against the background of translational oscillations with a small amplitude. The frequency of translational oscillations is close to 2.5 Hz. The Strouhal number is based on the frequency of rotational vibrations.

The dimensionless amplitude of these translational oscillations increases with increase in the air stream speed, and the dependence of the squared amplitude on the reverse speed is non-linear (Fig. 7, *b*). Joint rotational and translational oscillations at stream speed of 11 m/s are steady, however, a further increase in the stream speed leads to abrupt increase in the amplitude of translational oscillations. The amplitude of translational oscillations increased to such large values that the experiment was stopped.

The change of two modes of steady oscillations at stream speed less than 11 m/s can be controlled by artificially damping oscillations of any type. For example, we were able to dampen translational vibrations by holding the model with a pointer inserted into the air stream. After a complete stop of the translational oscillations, the rotational

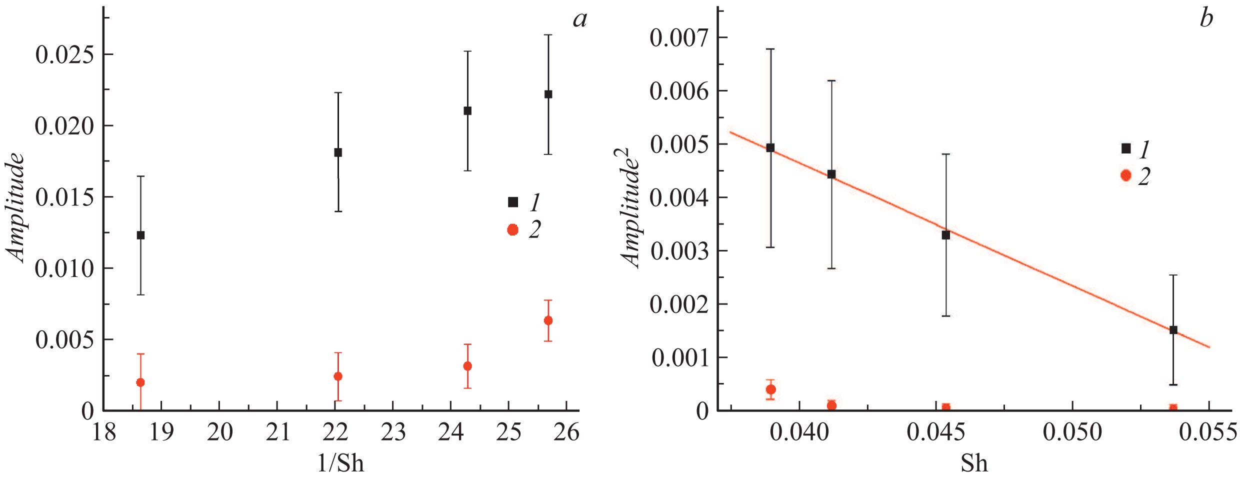

Figure 7. Dependences: *a* — prism oscillation amplitudes with aspect ratio 0.36 on 1/Sh, *b* — amplitude squared on Sh: *1* — rotational oscillations with a frequency of 4.5 Hz, *2* — translational oscillations with a frequency of 2.5 Hz, the sampling frequency of the instrument is 1250 Hz.

oscillations occurred, which prevented the appearance of translational oscillations.

Conclusion

For three prisms with end plates, differing in the ratio of the sides of a rectangular cross-section, elastically fixed in the air stream, various oscillation modes with a steady amplitude are observed. The large faces of the prisms in the equilibrium position are oriented along the speed vector of the oncoming stream. For the prism with a small height-to-width ratio, rotational oscillations are observed. The amplitude of the rotational oscillations increases with increase in the speed of the oncoming stream. The dependence of the squared amplitude of rotational oscillations on the Strouhal number is close to a linear dependence. When you try to cause translational oscillations, they are damped, and the logarithmic decrement of oscillations increases with an increase in the speed of the oncoming stream. A prism with a large height-to-width ratio oscillates only translationally. The dependence of the squared amplitude of translational oscillations on the Strouhal number differs markedly from the linear dependence. The prism with average aspect ratio of a rectangular cross section can oscillate in different modes, and the intervals of speed of different oscillation modes intersect. Translational oscillations are realized, which are accompanied by rotational oscillations of the same frequency with a small amplitude, as well as rotational oscillations against the background of translational oscillations with a small amplitude. In the latter case, the frequencies of translational and rotational oscillations are different.

Conflict of interest

The authors declare that they have no conflict of interest.

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