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Capillary-wave warp-drive

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The generation of surface water flow in channels with sources and resonators of capillary oscillations is detected and investigated. Moving devices with a capillary-wave accelerator of the surface fluid flow are demonstrated. The surface flow of liquid in the channel is generated due to the local deformation of the liquid surface by capillary vibrations and the formation of an excess liquid surface on average near the source and the transport of this surface by waves.

Keywords: capillary oscillations and waves, surface liquid flow, channel.

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Capillary oscillations and waves excited by vibrating rods and plates on a free liquid surface produce surface flows in liquid [1]. The excitation of two-dimensional capillary flows (specifically, Faraday ripple) and the formation of surface flows are threshold-type processes, which is typical of parametric excitation of non-linear waves of a finite amplitude. If the amplitude of source oscillations is sufficiently large, capillary oscillations and waves may exhibit nonlinearity related to a local deformation of a free liquid surface near the source. The effective vibrational liquid tension and Marangoni vibrational convection may be manifested in this case [2].

The aim of the present study is to analyze the generation of a surface liquid flow by capillary oscillations excited by a sources and a resonator in a planar channel.

Observations of flows produced on a water surface by a horizontal vibrating plate revealed that flows may be generated by sources distributed along the free liquid surface. The study of capillary waves excited on a water surface in a Patri dish by the tips of two rods oscillating in phase demonstrated that these sources generate multidirectional flows in the same way as a part of a vibrating plate does. In order to produce a unidirectional flow, the liquid surface was bounded by a channel with a quarter-wave resonator of capillary waves, which had a *U*-shaped cavity, at the center. The channel and the resonator were fabricated from 0.1-mm-thick steel plates. When capillary oscillations of a certain frequency are excited by the source exposed to liquid in the channel opposite to the open resonator cavity, standing capillary waves are excited within the liquid surface region bounded by the cavity. Getting superimposed onto capillary oscillations near the source, these waves enhance the amplitude of oscillations. The source together with the resonator excite semicylindrical traveling waves on the liquid surface in the channel, which interfere with waves reflected off the channel walls. Two-dimensional travelling capillary waves are thus excited on the liquid surface in the channel. This surface acts as a planar waveguide, which is bounded by the channel width and its length, for them. If gaps are left between the resonator and the channel walls, capillary oscillations in the channel produce a directional surface liquid flow that enters the channel through these gaps.

It was found that an effective surface water flow is produced in the channel when the width of gaps between the resonator and the channel walls is equal to the resonator cavity width. Figure 1, *a* shows the pattern of flows and waves on the water surface in a Petri dish under excitation of capillary oscillations within the water surface region bounded by a channel 28 × 12 mm in size with a rod source of capillary oscillations located opposite to an open cavity of a *U*-shaped resonator 8 × 4 mm in size that is mounted at the channel center. The flow consists of two plane flows the particles in which are accelerated within the water surface regions being directly adjacent to the edges of the capillary oscillation resonator (opposite to the source). Currents in these flows close outside the channel structure. In addition, secondary eddy flows on both sides of the flow coming from the channel are observed on the water surface. A surface flow with a velocity of 18 mm/s produced by the same source was also observed in a circular channel 15 mm in width with a resonator (with a 5-mm-wide cavity) mounted within it (Fig. 1, *b*).

The moving flow of liquid surface particles in the channel carries momentum. Therefore, the conservation laws make it possible to propel a free-floating device with a channel, a resonator, and a source of capillary oscillations along the liquid surface in the direction opposite to the flow in the channel.

The device with a capillary-wave drive is shown in Fig. 2, *a*. Its body is assembled from two rectangular polymer trays 145 × 45 × 8 mm in size. A 15 mm-wide gap is left between them. *L*-shaped plates are glued to these trays on the opposite sides of the gap. The long sides of plates and the tray sides facing them form *U*-shaped cavities 10 × 5 mm in size. A 5-mm-wide gap is left between the

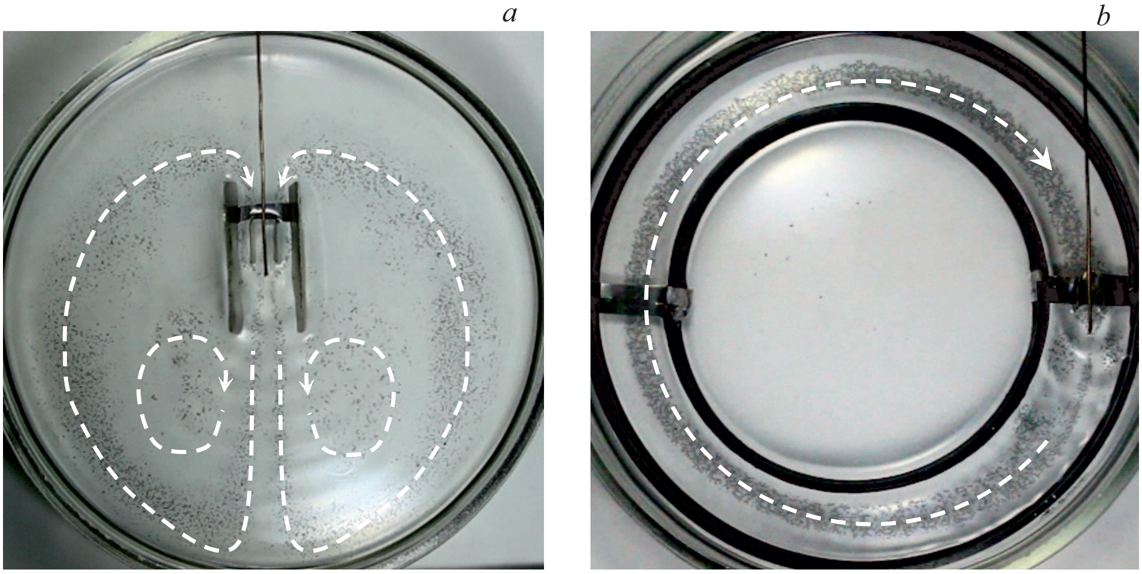

Figure 1. Waves and flows produced by capillary oscillations with a frequency of 56 Hz in rectangular (*a*) and circular (*b*) channels.

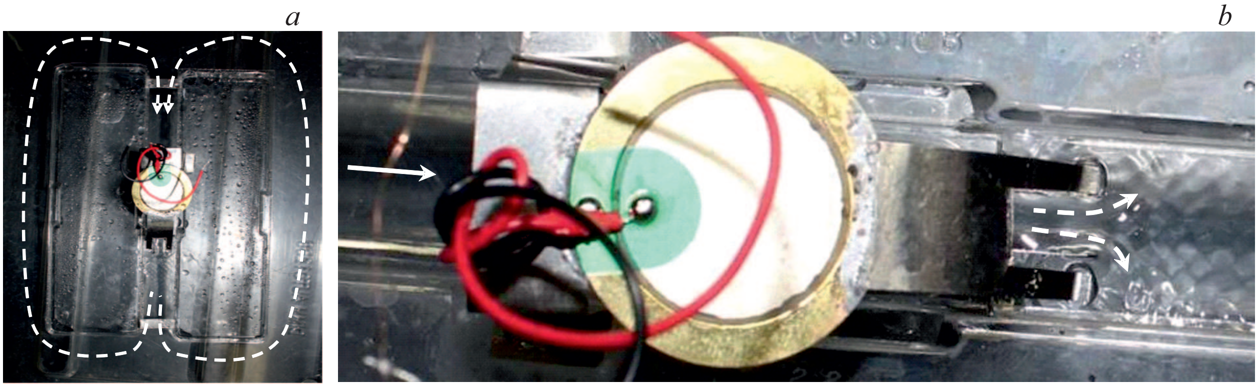

Figure 2. Device with a capillary-wave drive: *a* — overall view, *b* — waves and flow in the channel under oscillations with a frequency of 214 Hz.

plates. An FML-27T-3.9A1-100 piezoelectric transducer with a slab waveguide (12 mm in width and 0.1 mm in thickness) is mounted on the device body. Two short 2-mm-wide oscillator leads extend away from the free edge face of the plate. The mass of the device is $16 \cdot 10^{-3}$ kg. When the device is positioned on the water surface, its body is wetted by water, and a channel forms between the trays. When the piezoelectric transducer is connected to an AC supply, oscillators excite travelling capillary waves with a distributed amplitude in the channel. A directional flow is produced in the channel if capillary oscillations are excited in a specific frequency interval. The device then moves in the direction opposite to the flow in the channel. The flow velocity is maximized at oscillation frequencies of 76 and 214 Hz. Having seeded the water surface with tracer particles, we found that the flow exiting the channel gets divided into two closed flows that go round the device body and enter the channel on the opposite side. The device did also move at other wave excitation frequencies (616 and 704 Hz), but their small amplitude makes these waves virtually invisible. When voltage with a frequency of 3.5 kHz is applied to the piezoelectric transducer electrodes, the device acquires a fairly high speed of 60 mm/s, since resonance oscillations of the transducer itself are excited in this case, and outcoming water jets form under the oscillator edges.

Figure 2, *b* presents the two-dimensional wave pattern in the device channel excited by two oscillators operating in phase at a frequency 214 Hz. The solid arrow points in the direction of surface flow in the free water surface region that is left unperturbed by waves in the channel; dashed arrows denote sections of the motion trajectories of tracer particles in the flow near the edges of quarter-wave resonators of capillary oscillations, where the acceleration of particles moving in the surface flow occurs. At the specified wave excitation frequency, the average particle speed in the region of an unperturbed free water surface was 20 mm/s, while the device speed was 3 mm/s.

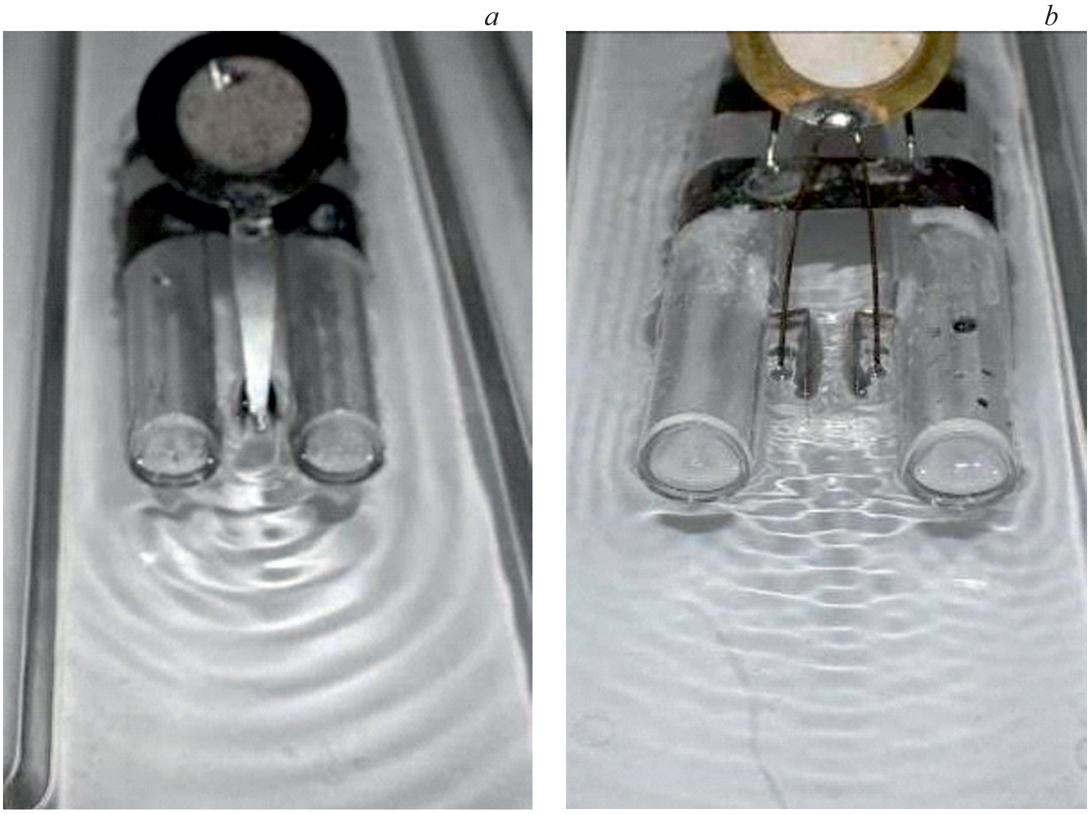

Figure 3. Moving devices and patterns of two-dimensional capillary waves corresponding to a frequency of 46 (*a*) and 137 Hz (*b*) of oscillations of sources in these devices.

Moving devices with a body made of hollow polymer cylinders 12 mm in diameter and 75 mm in length were also constructed. A resonator, a piezoelectric transducer, and a source of capillary oscillations mounted on these cylinders generated a directional flow in the channel. Figure 3, *a* shows the device with one resonator in the channel with a width of 10 mm and one slab source of capillary oscillations connected to a 7NB-31R2DM-1 piezoelectric transducer. When AC voltage with an amplitude of 20 V and a frequency of 46 Hz is applied to the piezoelectric transducer electrodes, the device with a mass of $8 \cdot 10^{-3}$ kg moves with a speed of 8 mm/s. The second device with a capillary-wave drive (Fig. 3, *b*) has a body made of hollow polymer cylinders with a 15-mm-wide gap between them. Bent *L*-shaped plates 8 mm in width are glued symmetrically to the cylinders in the gap. The long sides of plates and the cylinder sides facing them form *U*-shaped cavities 12×5 mm in size. A 5-mm-wide gap is left between the plates. Cylinders are fastened together with metal staples with an FML-27T-3.9A1-100 piezoelectric transducer, which features two oscillators made of metal rods 0.6 mm in diameter and 30 mm in length, mounted on one of them. When the device is set afloat and an AC voltage with a frequency of 137 Hz is applied to the piezoelectric transducer electrodes, the rods vibrate at one of the resonance frequencies of capillary oscillations and excite two-dimensional traveling waves in the channel, which generate a directional surface flow of water. If voltage with an amplitude of 20 V is applied to the piezoelectric transducer electrodes, the device moves with a speed of 20 mm/s. The oscillation amplitude of the rod tips in contact with water does not exceed 0.1 mm.

The generation of a directional surface liquid flow in channels with sources and resonators of capillary oscillations and waves may be explained in the following way.

A source and a resonator periodically excite capillary oscillations of a finite amplitude, thus distorting and deforming the free liquid surface in a channel. Free liquid surface element ΔS_0 near the source acquires, on average, excess surface ΔS_v. Average relative deformation $\overline{\delta}$ (referred to below simply as „deformation“

According to experimental data, capillary oscillations excited by the source on the liquid surface in the channel propagate in the form of traveling waves only in one direction (due to reflection off the resonator). Waves carry energy, and the surface energy flux density carried by a capillary wave is equal to the Umov vector value: $\mathbf{S}_{\mathrm{U}} = w_s c_c \mathbf{n}$, where w_s is the surface energy density, c_c is the capillary wave velocity, and $\mathbf{n}$ is a unit vector. The average Umov vector value is equal to wave intensity $I_s = \overline{w}_s c_c$. Just as in a cylindrical wave, the energy density in a semicylindrical wave decreases with distance from the source. Therefore, the expression for the excess surface energy density in a wave takes the following form:

$\overline{w}_s = \sigma\overline{\delta}_0(r_0/r)$, where $\overline{\delta}_0$ is the maximum liquid surface deformation per a surface element at distance r_0 from the center of source–liquid contact; $r \geqslant r_0$ is the distance to the source.

Thus, $I_s = \sigma\overline{\delta}_0(r_0/r)c_c$, and the average momentum density in a wave is

$$\overline{p}_s = (\overline{w}/c^2)c = I/c^2.$$

Average surface energy flux dW_s/dt at distance r from the source of a semicylindrical capillary wave is equal to the product of the wave intensity and arc $L = \pi r$: $dW_s/dt = I_s K = \pi r_0 \sigma \overline{\delta}_0 c_c$. The energy carried by a wave within period T of source oscillations may be derived from integral $dW_s = \int\limits_0^T \pi r_0 \sigma \overline{\delta}_0 c_c dt$. This energy is equal to $\Delta W_s = \pi r_0 \sigma \overline{\delta}_0 c_c T$ and is proportional to excess liquid surface $\Delta S = \pi r_0 \overline{\delta}_0 \lambda_c$ created by the source within the oscillation period. Since $\lambda_c = c_c T$, the energy carried by a wave within the oscillation period is $\Delta W_s = \sigma \Delta S$. A wave then also carried excess liquid surface S_δ, the flux of which is written as $dS_\delta/dt = (1/\sigma)dW_s/dt = \pi r_0 \overline{\delta}_0 c_c$ and expressed in m^2/s. The excess liquid surface flux density is equal to liquid surface deformation flux $s_\delta = \overline{\delta} c_c$ with a dimension of [m/s].

The acceleration of particles in liquid in a surface flow in the channel may be induced by the gradient of surface deformation by capillary oscillations. The derivative of the wave momentum density is equal to the product of the surface tension and the deformation gradient: $d\overline{p}_s/dt = \partial \overline{w}_s/\partial r = \sigma \operatorname{grad} \overline{\delta}$. Assuming that the wave momentum is transmitted to liquid surface particles, we obtain the following equation of motion for liquid surface particles: $\rho_s du_s/dt = -\sigma \operatorname{grad} \overline{\delta}$. Thus, the acceleration of liquid surface particles is $du_s/dt = (\sigma k/\rho_l) \operatorname{grad} \overline{\delta} = c_c^2 \operatorname{grad} \overline{\delta}$.

The discussed mechanism of generation of a directional surface flow in a channel with a source and a resonator of capillary oscillations may probably find application in the design of generators of directional electromagnetic fluxes in waveguides with built-in resonators and sources of electromagnetic oscillations.

Moving devices with a directional surface flow in a channel are somewhat similar to the hypothetical Alcubierre warp-drive [3], which should move in space due to a local deformation of spacetime in front of it and a local expansion of space behind. A source and a resonator of capillary oscillations in devices with a capillary-wave generator of a surface liquid flow deform locally the free liquid surface in a channel, thus forming, on average, an excess curved surface that is carried by waves in the channel in the form of a surface flow expanding within the channel and beyond it.

Conflict of interest

The author declares that he has no conflict of interest.

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