03

Spectral-angular characteristics of the radiation of a charged particle in the Redmond field

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Received June 05, 2022
Revised July 26, 2022
Accepted August 03, 2022

Based on the solution of the equation of motion of a charge in an electromagnetic field, the classical theory of radiation of a relativistic charged particle linearly accelerated by a high-intensity laser pulse in the presence of a static component of the magnetic field is constructed. Solutions obtained by Kopytov G.F. and Pogorelov A.V., were used to study the spectral-angular characteristics of the radiation of a charged particle in a combination of the field of a plane monochromatic electromagnetic wave and a constant magnetic field, the so-called Redmond field. According to the calculated formulas for the radiation intensity of particles in the Redmond field, graphs of the dependence on the magnitude of the magnetic field, phase and phase-angular distributions are plotted. The Fourier transform of the intensity of the electric field of the radiation and the spectral density of the radiation of the particle in the case of linear polarization of the wave is obtained.

Keywords: Redmond field, spectral-angular characteristics, charged particle, Lawson-Woodward theorem, superpower laser radiation.

DOI: 10.21883/EOS.2022.11.55099.3774-22

Introduction

In [1] spectral-angular radiation characteristics of a charge accelerated by high-intensity electromagnetic radiation were obtained. The theory of charged particles acceleration in plasma using laser radiation was suggested as early as 1979 [2] and is refining up to now [3]. Currently the highest intensities achievable in the focus of laser beam are $\sim 10^{23}$ W/cm^2 [4]. These strong electric fields become achievable with the emergence and development of laser plants that allow generating pulses in optical range with a duration of a few femtoseconds (10^{-15} s). However, as it is known from the Lawson−Woodward theorem, a particle in the unlimited vacuum space without static component of electric or magnetic field can not absorb the energy from laser pulse, no matter how strong is the pulse [5]. The issue of particle oscillation in a combination of a plane electromagnetic wave (EMW) field and a constant magnetic field was investigated for the first time in [6] and remains in the focus of researchers up to now in the point of view of classical and quantum theories [7–16]. Results of these studies are of special interests, because the problem formulated in them corresponds to real technical systems.

In [17] a technique was developed to determine spectral-angular and polarization characteristics for a beam of relativistic charged particles in undulator.

In this study the authors made an attempt to investigate the issue of spectral-angular radiation characteristics of a charged particle in the Redmond field (a combination of a plane EMW field and a constant magnetic field) based on the results obtained in [14], where on the basis of the classical equation of charge motion in electromagnetic field the energy characteristics of a charged particle accelerated by laser radiation in a constant magnetic field were calculated without taking into account the radiative friction. It is known from [18], that energy loss of electron for the hard radiation are achieved at an energy of 1 GeV, which corresponds a laser field intensity of $\sim 10^{22}$ W/cm^2. In this study all the characteristics were calculated at an intensity of 10^{19} W/cm^2. However, in the case of prolongated interaction between the wave and the particle, even a small parameter of radiative friction can significantly contribute to the particle?s dynamics, therefore it is assumed that the monochrome electromagnetic wave in the study represents an ultrashort laser pulse. The characteristics of interest are the intensity of charge radiation and its angular and phase-angular distribution, as well as the Fourier-image of electric field strength of the particle radiation and evaluation of its spectral density modulus for linear and circular polarization of the electromagnetic wave.

Formulation of the problem

The classical equation of particle motion with a charge q and a mass m is as follows:

$$\frac{d\mathbf{p}}{dt} = q\mathbf{E} + \frac{q}{c}\left[\mathbf{v} \times \mathbf{H}_\Sigma\right], \qquad (1)$$

where $\mathbf{H}_\Sigma = \mathbf{H} + \mathbf{H}_0$; $\mathbf{H}_0 = \mathbf{k}H_0$; $\mathbf{H}$ — vector of magnetic field strength of the electromagnetic wave, $\mathbf{H}_0$ — vector of the external magnetic field strength.

Let us supplement equation (1) with initial conditions for velocity and coordinates of the charged particle:

$$\mathbf{v}(0) = \mathbf{v}_0, \quad \mathbf{r}(0) = \mathbf{r}_0.$$

The relativistic factor γ is related to the intensity of electromagnetic field I as follows: $\gamma = \sqrt{1 + I/I_{\text{rel}}}$ as well as equal to $\gamma = mc(1 - v_{z0})/\sqrt{1 - v_0^2/c^2}$, where the relativistic intensity I_{rel} (W/cm^2) is defined as follows:

$$I_{\text{rel}} = \frac{m^2c^3\omega^2}{8\pi q^2} = \frac{1.37 \cdot 10^{18}}{\lambda^2},$$

where λ — wavelength, c — speed of light in vacuum.

Let us select the direction of electromagnetic wave along axis z, in this case components of vectors of electric and magnetic fields are defined by the following system:

$$\begin{cases} E_x = H_y = b_x \cos\Phi, \\ E_y = -H_x = f b_y \sin\Phi, \\ E_z = H_z = 0, \end{cases} \tag{2}$$

where axes x and y coincide with directions of semi-axes of the wave polarization ellipse b_x and b_y, and $b_x \geq b_y \geq 0$; $\Phi = \omega\xi + \varphi_0$; $\xi = t - z/c$; ω — carrier frequency; $f = \pm 1$ — polarization parameter, with superscript for right-hand polarization E_y and subscript for left-hand polarization.

Effect of constant magnetic field on the radiation intensity of a charged particle in a field of plane monochrome electromagnetic wave

By applying vector multiplication to equation (1) and vector H we get the Umov–Poynting vector in the following form:

$$\mathbf{S} = \frac{c}{4\pi}[\mathbf{E} \times \mathbf{H}] = \frac{c}{4\pi q}[\mathbf{F} \times \mathbf{H}] - \frac{1}{4\pi}\left[[\mathbf{v} \times \mathbf{H}_\Sigma] \times \mathbf{H}\right], \tag{3}$$

where $\mathbf{F} = d\mathbf{p}/dt$.

By writing vector products in equation (3), we can obtain components of the Umov–Poynting vector:

$$S_x(t) = \frac{1}{4\pi} H_y \left[(v_x H_y - v_y H_x) - \frac{c}{q} F_z\right], \tag{4}$$

$$S_y(t) = -\frac{1}{4\pi} H_x \left[(v_x H_y - v_y H_x) - \frac{c}{q} F_z\right], \tag{5}$$

$$S_z(t) = \frac{1}{4\pi}\left[v_x(H_x^2 + H_y^2) + \frac{c}{q}(E_x F_x + E_y F_y)\right]. \tag{6}$$

The Lorentz force acting on the particle in the Redmond field in a component form can be written as follows:

$$F_x = \frac{1}{1+g}\left\{q\left[b_x + \frac{\eta(b_x\eta \mp b_y)}{1-\eta^2}\right]\cos\Phi + \frac{R\omega_c^2}{c}\gamma\cos\Phi_c\right\}, \tag{7}$$

$$F_y = \frac{1}{1+g}\left\{q\left[\pm b_y - \frac{\eta(b_x \mp \eta b_y)}{1-\eta^2}\right]\sin\Phi + \frac{R\omega_c^2}{c}\gamma\sin\Phi_c\right\}, \tag{8}$$

$$F_z = \frac{\gamma\omega}{1+g}\left[\frac{q^2(b_x^2 - b_y^2)}{2\gamma^2\omega^2(1-\eta^2)^2}\sin 2\Phi + \frac{q(b_x \mp b_y)}{2\gamma\omega}\frac{R\omega_c}{c}\frac{1+\eta}{1-\eta}\sin(\Phi + \Phi_c) - \frac{q(b_x \pm b_y)}{2\gamma\omega}\frac{R\omega_c}{c}\frac{1+\eta}{1-\eta}\sin(\Phi - \Phi_c)\right], \tag{9}$$

where $\Phi_c = \omega_c\xi + \psi_0$.

As can be seen from (9), in the Redmond field in the initial moment of time $t = 0$ the longitudinal component of pulse is $dp_\parallel/dt \neq 0$, which means that in this case the Lawson–Woodward theorem is not fulfilled.

Let us use the formula for the longitudinal component of the particle momentum with correction for the relativistic factor γ from [14]

$$g = h - \frac{q^2}{4\gamma^2\omega^2}\frac{b_x^2 - b_y^2}{1-\eta^2}\cos 2\Phi - \frac{R\omega_c}{c}\frac{q}{2\gamma\omega} \times \frac{(b_x \pm b_y)(1-\eta)\cos(\Phi - \Phi_c) - (b_x \mp b_y)(1+\eta)\cos(\Phi + \Phi_c)}{1-\eta^2},$$

where h — constant part of the longitudinal component of particle velocity defined by the initial conditions and parameters of the accelerating field, respectively, equal to:

$$h = \frac{1}{2}\left[\frac{m^2c^2}{\gamma^2} - 1 + \frac{R^2\omega_c^2}{c^2} + \frac{1}{2}\frac{q^2}{\gamma^2\omega^2} \times \frac{(b_x - f\eta b_y)^2 + (\eta b_x - f b_y)^2}{(1-\eta^2)^2}\right],$$

where $\omega_c = qH_0/\gamma$ — cyclotron frequency, $\eta = \omega_c/\omega$, R — constant defined by initial conditions [14].

By substituting (7)–(9) and velocity values from [14] to formulae (4)–(6), we get components of the Umov–Poynting vector in the following form:

$$S_x(t) = \frac{1}{8\pi}\frac{cb_x}{1+g}\frac{R\omega_c}{c}\left[(b_x \pm b_y)\left(1 - \frac{1-\eta}{1+\eta}\right) \times \sin(\Phi - \Phi_c) - (b_x \mp b_y)\left(1 - \frac{1+\eta}{1-\eta}\right)\sin(\Phi + \Phi_c)\right]\cos\Phi,$$

$$S_y(t) = \pm\frac{1}{8\pi}\frac{cb_y}{1+g}\frac{R\omega_c}{c}\left[(b_x \pm b_y)\left(1-\frac{1-\eta}{1+\eta}\right)\right.$$

$$\left.\times\sin(\Phi-\Phi_c)-(b_x \mp b_y)\left(1-\frac{1+\eta}{1-\eta}\right)\sin(\Phi+\Phi_c)\right]\sin\Phi,$$

$$S_z(t) = \frac{1}{8\pi}\frac{c}{1+g}\left\langle\left\{(b_x^2-b_y^2)\left[(1+g)+\frac{\eta^2}{1-\eta^2}\right]\right.\right.$$

$$\left.\times\cos 2\Phi+(b_x^2+b_y^2)\left[(1+g)+\frac{1}{1-\eta^2}\left(\eta^2 \mp \frac{2\eta b_x b_y}{b_x^2 b_y^2}\right)\right]\right\}$$

$$+\frac{R\omega_c}{c}\left(\frac{q}{\gamma\omega}\right)^{-1}\eta[(b_x \pm b_y)\cos(\Phi-\Phi_c)$$

$$\left.+(b_x \mp b_y)\cos(\Phi+\Phi_c)]\right\rangle.$$

For the case of circularly polarized electromagnetic wave $b_x = b_y = b/\sqrt{2}$, we get modulus of the vector in the following form:

$$|\mathbf{S}(t)| = \sqrt{S_x^2(t)+S_y^2(t)+S_z^2(t)}, \tag{10}$$

where $I_{\mathrm{cir}} = cb^2/8\pi$, $\mu = q^2b^2/\gamma^2\omega^2 = I_{\mathrm{lin}}/2I_{\mathrm{rel}} = I_{\mathrm{cir}}/I_{\mathrm{rel}}$.

With presence of a constant magnetic field, oscillation of the particle takes place with two periods $\tilde{T} = 2\pi(1+h)/\omega$ and $\tilde{T}_c = 2\pi/\omega_c$. Since the particle motion is a superposition of two types of periodic motions with frequencies ω and ω_c, the intensity will be averaged using the following formula [14]:

$$\overline{f}(t) = \frac{1}{2\pi}\int\limits_{\Phi(t)}^{\Phi(t)+2\pi}\frac{1}{\tilde{T}}\int\limits_{\Phi_c(t)}^{\Phi_c(t)+2\pi} f(t')\,\frac{1+g}{\omega}d\Phi' d\Phi_c'.$$

Now let us evaluate the radiation intensity averaged over the period of particles oscillation in the field of electromagnetic wave with circular polarization in the presence of a constant magnetic field

$$I_{\mathrm{rad}}^{\mathrm{cir}} = \frac{I_{\mathrm{cir}}}{1+\frac{\mu}{4}\left[\left(1-\frac{f\eta}{1+f\eta}\right)^2+\left(\frac{1}{1+f\eta}\right)^2\right]}$$

$$\times\sqrt{\begin{array}{c}\left\{1+\frac{\mu}{4}\left[\left(1-\frac{f\eta}{1+f\eta}\right)^2+\left(\frac{1}{1+f\eta}\right)^2\right]-\frac{f\eta}{1+f\eta}\right\}^2+\\ +\frac{\mu}{2}\left(1-f\,\frac{\eta}{1+f\eta}\right)^2\left[\left(1-\frac{1-f\eta}{1+f\eta}\right)^2+\frac{\eta^2}{\mu}\right]\end{array}}. \tag{11}$$

With magnetic field switched off formulae (10), (11) becomes equivalent to formulae derived in [1].

For the case of linearly polarized electromagnetic wave $b_x = b$, $b_y = 0$, the intensity of wave is:

$$|\mathbf{S}(t)| = \frac{1}{2}\frac{I_{\mathrm{lin}}}{1+g}\sqrt{\begin{array}{l}\frac{1}{2}\frac{R^2\omega_c^2}{c^2}\left[\left(1-\frac{1-\eta}{1+\eta}\right)\sin(\Phi-\Phi_c)\right.\\ \left.-\left(1-\frac{1-\eta}{1+\eta}\right)\sin(\Phi-\Phi_c)\right]^2\\ \times(1+\cos 2\Phi)\\ +\left\{\left[(1+g)+\frac{\eta^2}{1-\eta^2}\right](1+\cos 2\Phi)\right.\\ \left.+2\frac{R\omega_c}{c}\left(\frac{qb}{\gamma\omega}\right)^{-1}\eta\cos\Phi\cos\Phi_c\right\}^2\end{array}}, \tag{12}$$

where $I_{\mathrm{lin}} = cb^2/4\pi$.

Now let us evaluate the radiation intensity averaged over the period of particles oscillation in the field of electromagnetic wave with linear polarization in the presence of a constant magnetic field provided that at the initial moment of time the particle was in rest:

$$I_{\mathrm{rad}}^{\mathrm{linear}} = \frac{1}{2}\frac{I_{\mathrm{lin}}}{1+h}\sqrt{\begin{array}{l}\frac{1}{4}\frac{R^2\omega_c^2}{c^2}\left[\left(1-\frac{1-\eta}{1+\eta}\right)^2+\left(1-\frac{1-\eta}{1+\eta}\right)^2\right.\\ \left.+\frac{3\mu(1+\eta^2)}{(1-\eta^2)^2}+\frac{4\eta^2}{\mu}\right]+\frac{7\mu^2}{128}\frac{1}{(1-\eta^2)^2}\\ +\frac{3}{2}\left[(1+h)^2+\frac{\eta^4}{(1-\eta^2)^2}\right]+\frac{1}{2}\frac{\eta^2}{1-\eta^2}\\ \times\left[6(1+h)-\frac{\mu}{1-\eta^2}\right]\end{array}}, \tag{13}$$

where values of $R^2\omega_c^2/c^2$ and h are equal to, respectively:

$$\frac{R^2\omega_c^2}{c^2} = \mu\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]$$

and

$$h = \frac{\mu}{4}\left\{\frac{1+\eta^2}{(1-\eta^2)^2}+2\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\},$$

where $\Phi_0 = \omega\xi_0+\varphi_0$, $\xi_0 = -z/c$.

Thus, the minimum radiation intensity corresponds to the phase of $\Phi_0 = 0$ and $\Phi_0 = \pi$ and is defined by the following formula:

$$I_{\min}^{\text{linear}} = \frac{I_{\text{lin}}}{2 + \frac{\mu}{2}\frac{1+3\eta^2}{(1-\eta^2)^2}} \sqrt{\begin{aligned} &\frac{\mu}{4}\frac{\eta^2}{(1-\eta^2)^2}\left[\left(1-\frac{1-\eta}{1+\eta}\right)^2 \right.\\ &\left. + \left(1-\frac{1+\eta}{1-\eta}\right)^2 + \frac{4\eta^2}{\mu}\right] + \frac{7\mu^2}{128} \\ &\times \frac{1}{(1-\eta^2)^2} + \frac{3\mu^2}{4}\frac{\eta^2(1+\eta^2)}{(1-\eta^2)^4} \\ &+\frac{3}{2}\left\{\left[1+\frac{\mu}{4}\frac{1+3\eta^2}{(1-\eta^2)^2}\right]^2 \right.\\ &\left. + \frac{\eta^4}{(1-\eta^2)^2}\right\} + \frac{1}{2}\frac{\eta^2}{1-\eta^2} \\ &\times \left\{6\left[1+\frac{\mu}{4}\frac{1+3\eta^2}{(1-\eta^2)^2}\right]\right. \\ &\left. - \frac{\mu}{1-\eta^2}\right\} \end{aligned}}.$$

The maximum radiation intensity corresponds to the phase of $\Phi_0 = \pi/2$ and $\Phi_0 = 3\pi/2$ and is defined by the following formula:

$$I_{\min}^{\text{linear}} = \frac{I_{\text{lin}}}{2 + \frac{\mu}{2}\frac{3+\eta^2}{(1-\eta^2)^2}} \times \sqrt{\begin{aligned} &\frac{\mu}{4}\frac{1}{(1-\eta^2)^2}\left[\left(1-\frac{1-\eta}{1+\eta}\right)^2 + \left(1-\frac{1+\eta}{1-\eta}\right)^2 + \frac{4\eta^2}{\mu}\right] \\ &+ \frac{7\mu^2}{128}\frac{1}{(1-\eta^2)^2} + \frac{3\mu^2}{4}\frac{(1+\eta^2)}{(1-\eta^2)^4} + \frac{3}{2} \\ &\times \left\{\left[1+\frac{\mu}{4}\frac{3+\eta^2}{(1-\eta^2)^2}\right]^2 + \frac{\eta^4}{(1-\eta^2)^2}\right\} + \frac{1}{2}\frac{\eta^2}{1-\eta^2} \\ &\times \left\{6\left[1+\frac{\mu}{4}\frac{3+\eta^2}{(1-\eta^2)^2}\right] - \frac{\mu}{1-\eta^2}\right\} \end{aligned}}.$$

The average radiation intensity of a charged particle in the Redmond field can be determined by the following formula:

$$\overline{f}(t) = \sup \lim_{t\to\infty} \frac{1}{t}\int\limits_0^t f(t)dt.$$

Thus, the radiation intensity of charged particles in the Redmond field averaged by initial phase Φ_0 is as follows:

$$I_{\text{rad}}^{\text{linear}} = \frac{I_{\text{lin}}}{2} \times \sqrt{\begin{aligned} &\frac{2\mu(1-\eta^2)^2[\mu(1-10\eta^2+\eta^4)+4(1-\eta^4)(1-\eta^2)]}{\left\{\begin{array}{c}\sqrt{\mu(3+\eta^2)+4(1-\eta^2)^2}\times \\ \times\sqrt{\mu[4-3(1-\eta^2)]+4(1-\eta^2)^2}\end{array}\right\}^3} \\ &\times\left[\left(1-\frac{1-\eta}{1+\eta}\right)^2+\left(1-\frac{1+\eta}{1-\eta}\right)^2+\frac{4\eta^2}{\mu}+\frac{3\mu(1+\eta^2)}{(1-\eta^2)^2}\right] \\ &+\frac{7\mu^2(1-\eta^2)[\mu(1+\eta^2)+2(1-\eta^2)^2]}{4\left\{\begin{array}{c}\sqrt{\mu(3+\eta^2)+4(1-\eta^2)^2}\times \\ \times\sqrt{\mu[4-3(1-\eta^2)]+4(1-\eta^2)^2}\end{array}\right\}^3} \\ &+\frac{3}{2}\left\langle 1+\frac{32\eta^4[\mu(1-\eta^4)(1-\eta^2)+2(1-\eta^2)^4]}{\left\{\begin{array}{c}\sqrt{\mu(3+\eta^2)+4(1-\eta^2)^2}\times \\ \times\sqrt{\mu[4-3(1-\eta^2)]+4(1-\eta^2)^2}\end{array}\right\}^3}\right\rangle \\ &+12\eta^2(1-\eta^2) \\ &\times\left\langle\frac{\begin{array}{c}[\mu(3+\eta^2)+4(1-\eta^2)^2]\times \\ \times[4\mu-3\mu(1-\eta^2)+4(1-\eta^2)^2]\end{array}}{\left\{\begin{array}{c}\sqrt{\mu(3+\eta^2)+4(1-\eta^2)^2}\times \\ \times\sqrt{\mu[4-3(1-\eta^2)]+4(1-\eta^2)^2}\end{array}\right\}^3}\right\rangle \\ &-16\eta^2(1-\eta^2) \\ &\times\left\langle\frac{\mu[\mu(1+\eta^4)+2(1-\eta^2)^3]}{\left\{\begin{array}{c}\sqrt{\mu(3+\eta^2)+4(1-\eta^2)^2}\times \\ \times\sqrt{\mu[4-3(1-\eta^2)]+4(1-\eta^2)^2}\end{array}\right\}^3}\right\rangle \end{aligned}}. \quad (14)$$

Formulae (11) and (14) can be used to plot the change in radiation intensity of a particle in the Redmond field as a function of magnetic field strength (Fig. 1).

It can be seen from the figure that up to the level of $\eta = 2.5$ the constant magnetic field results in a little increase in the particle radiation intensity if the electromagnetic wave is circularly polarized, while the intensity for the linearly polarized wave decreases.

By differentiating relationship (11) with respect to Φ_0, we get the phase distribution of radiation intensity:

$$\frac{dI_{\text{rad}}^{\text{linear}}}{d\Phi_0} = \frac{I_{\text{lin}}}{2} \times \frac{\left\langle 1+\frac{\mu}{4}\left\{\frac{1+\eta^2}{(1-\eta^2)^2}+2\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}\right\rangle\kappa - \frac{\mu}{2}\frac{\sin 2\Phi_0}{1-\eta^2}\theta}{\left\langle 1+\frac{\mu}{4}\left\{\frac{1+\eta^2}{(1-\eta^2)^2}+2\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}\right\rangle^2}, \quad (15)$$

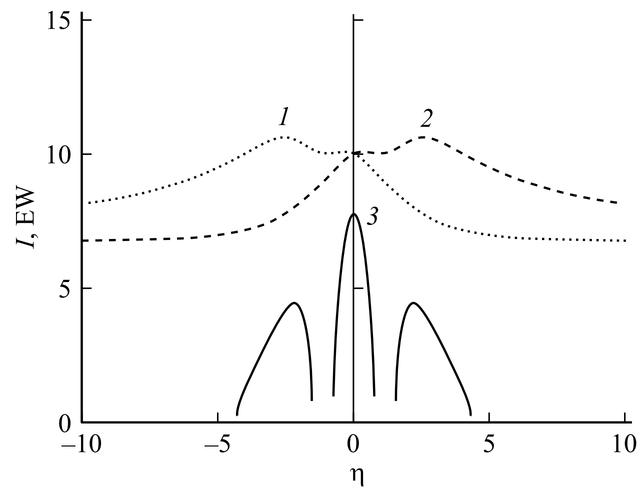

Figure 1. Radiation intensity of a particle as a function of magnetic field strength (1 - circular right-hand, *2* — circular left-hand, *3* — linear).

where θ and κ, respectively, are:

$$\theta = \sqrt{\begin{aligned}&\frac{\mu}{4}\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\left[\left(1-\frac{1-\eta}{1+\eta}\right)^2\right.\\&\left.+\left(1-\frac{1+\eta}{1-\eta}\right)^2+\frac{3\mu(1+\eta^2)}{(1-\eta^2)^2}+\frac{4\eta^2}{\mu}\right]\\&+\frac{7\mu^2}{128}\frac{1}{(1-\eta^2)^2}+\frac{3}{2}\left\langle 1+\frac{\mu}{4}\left\{\frac{1+\eta^2}{(1-\eta^2)^2}\right.\right.\\&\left.\left.+2\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}\right\rangle^2\\&+\frac{3\eta^4}{2(1-\eta^2)^2}+\frac{3\eta^2}{1-\eta^2}\left\langle 1+\frac{\mu}{4}\left\{\frac{1+\eta^2}{(1-\eta^2)^2}\right.\right.\\&\left.\left.+2\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}\right\rangle-\frac{\mu\eta^2}{2(1-\eta^2)^2}\end{aligned}}\ .$$

$$\begin{aligned}\kappa=\frac{d\theta}{d\Phi_0}&=\frac{1}{2\theta}\left\langle\frac{\mu}{4}\frac{\sin 2\Phi_0}{1-\eta^2}\left[\left(1-\frac{1-\eta}{1+\eta}\right)^2\right.\right.\\&+\left.\left(1-\frac{1+\eta}{1-\eta}\right)^2+\frac{3\mu(1+\eta^2)}{(1-\eta^2)^2}+\frac{4\eta^2}{\mu}\right]+\frac{3\mu}{2}\left\langle 1+\frac{\mu}{4}\right.\\&\times\left.\left\{\frac{1+\eta^2}{(1-\eta^2)^2}+2\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}\right\rangle\frac{\sin 2\Phi_0}{1-\eta^2}\\&\left.+\frac{3\mu}{2}\frac{\eta^2}{(1-\eta^2)^2}\sin 2\Phi_0\right\rangle.\end{aligned}$$

Formula (15) can be used to plot the phase distribution of radiation intensity of a charged particle in the Redmond field with an intensity of $I_{\text{lin}} = 10^{19}\,\text{W/cm}^2$ on the phase plane (along the axis of abscissa $\sin\Phi_0$ and along the axis of ordinate $dI^{\text{linear}}_{\text{rad}}/d\Phi_0$) for different magnetic field strengths (Fig. 2).

In can be seen from Fig. 2, that with increase in the magnetic field strength the maximum of radiation intensity of charged particle shifts towards $\Phi_0 = \pi/2, -\pi/2, \ldots, \pm\pi/2 + 2\pi n$.

Instantaneous angular distribution of radiation intensity of charged particles will be as follows:

$$\begin{aligned}\frac{dI^{\text{linear}}_{\text{rad}}}{d\Omega}&=\frac{I_{\text{lin}}}{4\pi}\\&\times\frac{\left\langle 1+\frac{\mu}{4}\left\{\frac{1+\eta^2}{(1-\eta^2)^2}+2\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}\right\rangle\kappa-\frac{\mu}{2}\frac{\sin 2\Phi_0}{1-\eta^2}\theta}{\left\langle 1+\frac{\mu}{4}\left\{\frac{1+\eta^2}{(1-\eta^2)^2}+2\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}\right\rangle^2\cos\Phi_0}.\end{aligned}\tag{16}$$

By differentiating distribution (16) with respect to Φ_0, we get the phase-angular distribution of radiation intensity as a function of initial phase of the electromagnetic wave:

$$\begin{aligned}\frac{d^2I^{\text{linear}}_{\text{rad}}}{d\Phi_0 d\Omega}&=\frac{I_{\text{lin}}}{4\pi}\\&\times\left\langle\frac{\Gamma}{1+\frac{\mu}{4}\left\{\frac{1+\eta^2}{(1-\eta^2)^2}+2\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}}\right.\\&\left.-\frac{\mu\left\langle 2\kappa\sin\Phi_0+\theta\cos\theta\Phi_0-\frac{\frac{\mu\theta}{1-\eta^2}\sin 2\Phi_0\sin\Phi_0}{1+\frac{\mu}{4}\left\{\frac{1+\eta^2}{(1-\eta^2)^2}+2\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}}\right\rangle}{(1-\eta^2)\left\langle 1+\frac{\mu}{4}\left\{\frac{1+\eta^2}{(1-\eta^2)^2}+2\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}\right\rangle^2}\right\rangle,\end{aligned}\tag{17}$$

where

$$\begin{aligned}\Gamma=\frac{\cos\Phi_0}{2\theta}&\left\langle\frac{\mu}{2(1-\eta^2)}\left[1-\left(1-\frac{1-\eta}{1+\eta}\right)^2\right.\right.\\&+\left.\left(1-\frac{1+\eta}{1-\eta}\right)^2+\frac{3\mu(1+\eta^2)}{(1-\eta^2)^2}+\frac{4\eta^2}{\mu}\right]+\frac{3\mu}{1-\eta^2}\\&\times\left\langle 1+\frac{\mu}{4}\left\{\frac{1+\eta^2}{(1-\eta^2)^2}+2\left[3\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}\right.\\&\left.\left.+\frac{\eta^2}{1-\eta^2}\right\rangle-2\left(\frac{\gamma}{\cos\Phi_0}\right)^2\right\rangle.\end{aligned}$$

Formula (17) can be used to plot the phase-angular distribution of radiation intensity of a charged particle for the radiation intensity of $I_{\text{lin}} = 10^{19}\,\text{W/cm}^2$ on the phase plane (along the axis of abscissa $\sin\Phi_0$ and along the axis of ordinate $d^2I^{\text{linear}}_{\text{rad}}/d\Phi_0 d\Omega$) for different magnetic field strengths (Fig. 3).

Also, it can be seen in Fig. 3, that the maximum of intensity shifts towards $\Phi_0 = \pi/2, -\pi/2, \ldots, \pm\pi/2 + 2\pi n$,

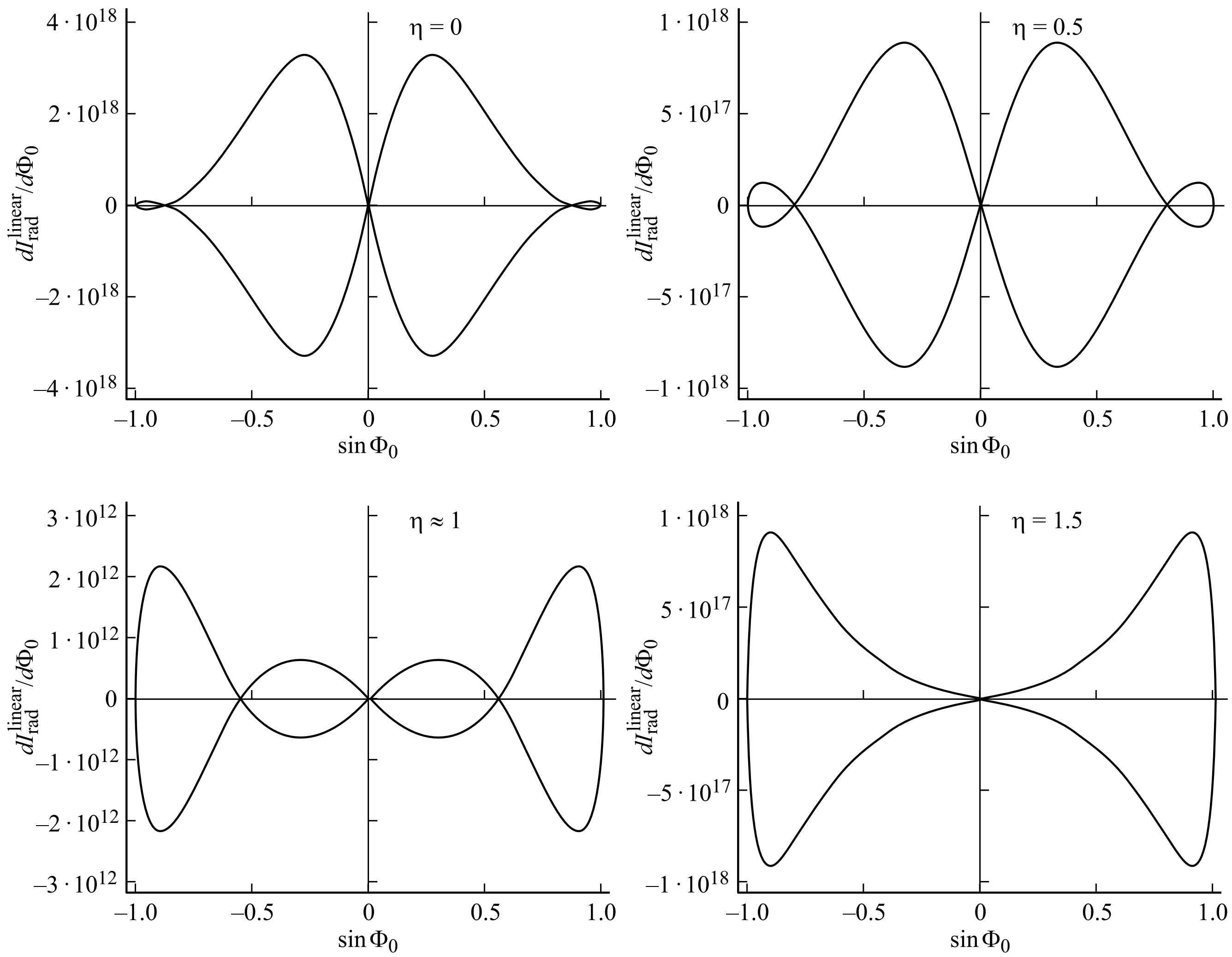

Figure 2. Phase distribution of radiation intensity of particles at various magnetic field strengths.

and in conditions close to the resonance the phase-angular distribution has the form of ellipse.

Fourier-image of electric field strength in the Redmond field

$$\mathbf{E}(\mathbf{r},t)=\sum_{\omega=-\infty}^{\omega=\infty}\mathbf{E}_\omega(\mathbf{r})\exp(-i\Phi). \tag{18}$$

The Fourier-image can be represented in the form of a periodic function:

$$\mathbf{E}_\omega(\mathbf{r})=\frac{1}{\tilde{T}}\int\limits_t^{\tilde{t}}\mathbf{E}(\mathbf{r},t)\exp(-i\Phi)dt. \tag{19}$$

Let us express $\mathbf{E}(\mathbf{r},t)$ from equation (1) and substitute it to function (19), then proceed from time integration to integration over phases Φ and Φ_c, which yields the following expressions for real and imaginary parts of $\mathbf{E}_\omega(\mathbf{r})$:

$$\mathrm{Re}\big(\mathbf{E}_\omega(\mathbf{r})\big)=\frac{1}{2\pi}\int\limits_{\Phi_c(t)}^{\Phi_c(\tilde{t})}\frac{1}{\tilde{T}}\int\limits_{\Phi(t)}^{\Phi(\tilde{t})}\left(\frac{1}{q}\frac{d\mathbf{p}}{dt}-\frac{1}{c}\left[\mathbf{v}\times\mathbf{H}_\Sigma\right]\right)\times\cos\Phi\,\frac{1+g}{\omega}\,d\Phi d\Phi_c.$$

$$\mathrm{Im}\big(\mathbf{E}_\omega(\mathbf{r})\big)=\frac{1}{2\pi}\int\limits_{\Phi_c(t)}^{\Phi_c(\tilde{t})}\frac{1}{\tilde{T}}\int\limits_{\Phi(t)}^{\Phi(\tilde{t})}\left(\frac{1}{q}\frac{d\mathbf{p}}{dt}-\frac{1}{c}\left[\mathbf{v}\times\mathbf{H}_\Sigma\right]\right)\times\sin\Phi\,\frac{1+g}{\omega}\,d\Phi d\Phi_c. \tag{20}$$

Components of transformation of Fourier-function (20) will be as follows:

$$\mathrm{Re}(\mathbf{E}_{\omega,x})=\frac{b_x}{2(1+h)}\left(1+h-\frac{q^2(b_x^2-b_y^2)}{8\gamma^2\omega^2(1-\eta^2)}\right);$$

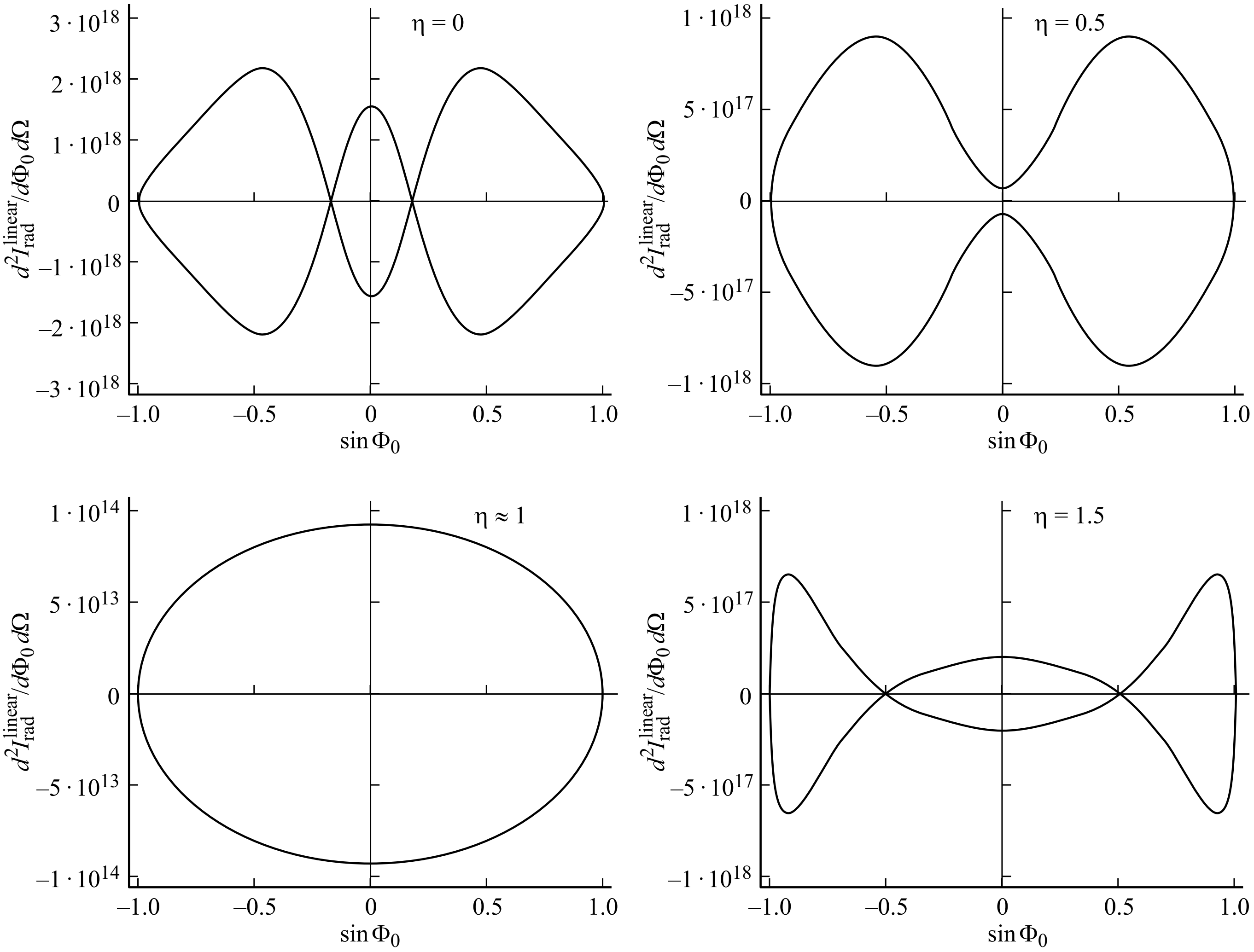

Figure 3. Phase-angular distribution of radiation intensity of particles at various magnetic field strengths.

$$\mathrm{Im}(\mathbf{E}_{\omega,x}) = 0;$$

$$\mathrm{Re}(\mathbf{E}_{\omega,y}) = 0;$$

$$\mathrm{Im}(\mathbf{E}_{\omega,y}) = \pm\frac{b_y}{2(1+h)}\left(1+h+\frac{q^2(b_x^2-b_y^2)}{8\gamma^2\omega^2(1-\eta^2)}\right);$$

$$\mathrm{Re}(\mathbf{E}_{\omega,z}) = \mathrm{Im}(\mathbf{E}_{\omega,z}) = 0. \quad (21)$$

Let us consider cases of circular and linear polarization.

Circular polarization. In [1] it was found out that in the case of circular polarization only amplitude information is kept, while phase information is lost completely, therefore in this case of wave polarization the external magnetic field will not make any contribution.

Linear polarization. In this case from (21) we get the following for the Fourier-image:

$$\mathrm{Re}(\mathbf{E}_\omega) = \frac{b\left\langle 1+\frac{\mu}{8}\left\{\frac{1+3\eta^2}{(1-\eta^2)^2}+4\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}\right\rangle}{2+\frac{\mu}{2}\left\{\frac{1+\eta^2}{(1-\eta^2)^2}+2\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}}. \quad (22)$$

By substituting (22) into formula (18), we get the radiation spectrum of the particle at the initial moment of time:

$$\mathrm{Re}\big((\mathbf{E}_\omega(\mathbf{r},t_0)\big) = b$$

$$\times\sum_{\omega=-\infty}^{\omega=\infty}\frac{1+\frac{\mu}{8}\left\{\frac{1+3\eta^2}{(1-\eta^2)^2}+4\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}}{2+\frac{\mu}{2}\left\{\frac{1+\eta^2}{(1-\eta^2)^2}+2\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\}}\cos\Phi_0. \quad (23)$$

The radiation spectrum has the following phase distribution:

$$\mathrm{Re}\left(\left|\frac{d\mathbf{E}(\mathbf{r},t_0)}{d\Phi_0}\right|\right) = b$$

$$\times\left|\sum_{\omega=-\infty}^{\omega=\infty}\left(1-\frac{2A}{B}\right)\frac{\mu\sin\Phi_0\cos^2\Phi_0}{B(1-\eta^2)}-\frac{A}{B}\sin\Phi_0\right|, \quad (24)$$

where

$$A = 1+\frac{\mu}{8}\left\{\frac{1+3\eta^2}{(1-\eta^2)^2}+4\left[\frac{\sin^2\Phi_0}{1-\eta^2}+\frac{\eta^2}{(1-\eta^2)^2}\right]\right\};$$

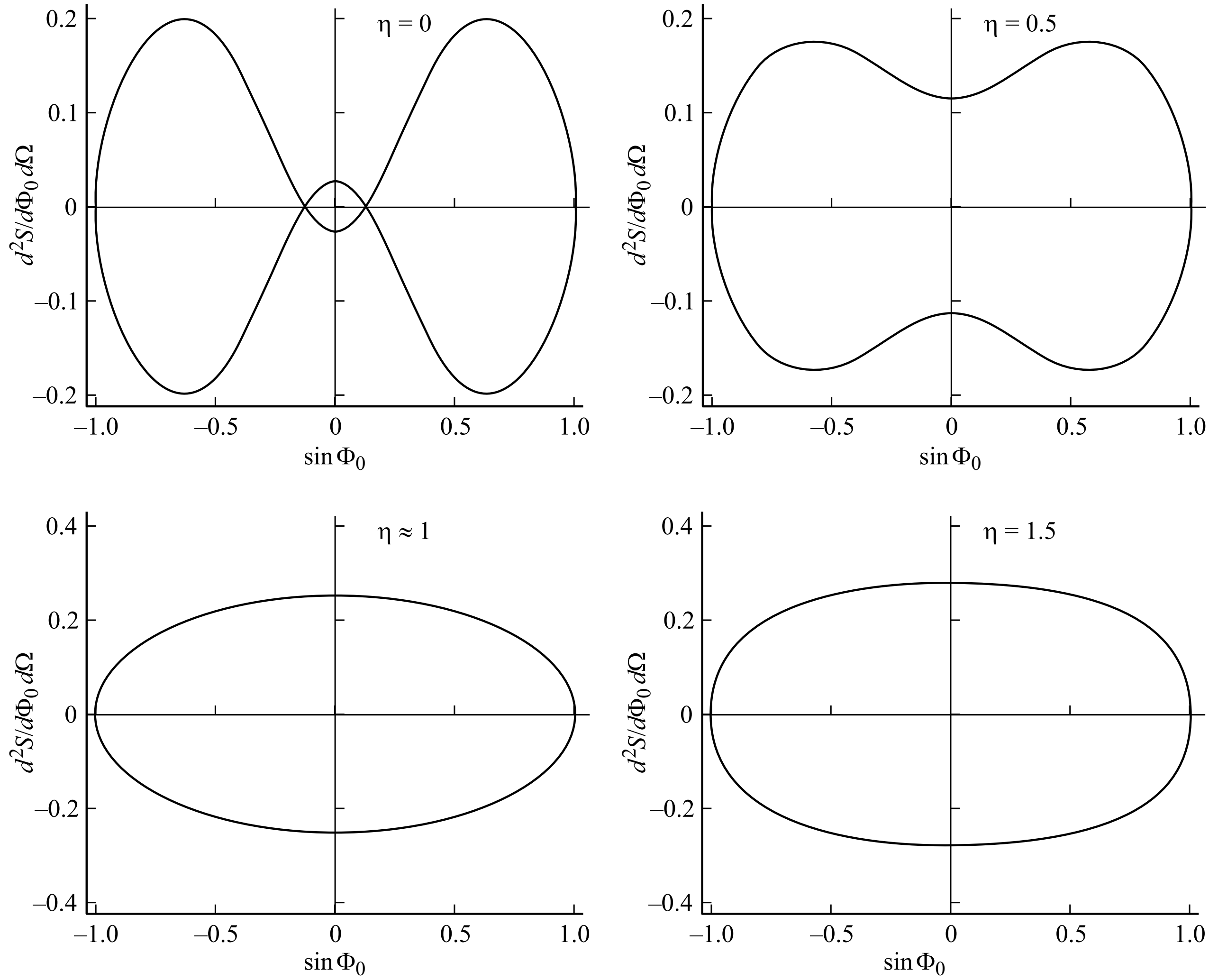

Figure 4. Phase-angular distribution of spectral density at various magnetic field strengths.

$$B = 2 + \frac{\mu}{2}\left\{\frac{1+\eta^2}{(1-\eta^2)^2} + 2\left[\frac{\sin^2\Phi_0}{1-\eta^2} + \frac{\eta^2}{(1-\eta^2)^2}\right]\right\}.$$

The radiation spectrum of a charge in a unit solid angle is defined by the following formula:

$$\mathrm{Re}\left(\left|\frac{d\mathbf{E}(\mathbf{r},t_0)}{d\Omega}\right|\right) = \frac{b}{2\pi}\left|\sum_{\omega=-\infty}^{\omega=\infty}\left(1-\frac{2A}{B}\right)\right.$$

$$\left.\times\frac{\mu\sin 2\Phi_0}{B(1-\eta^2)} - \frac{A}{B}\,\mathrm{tg}\,\Phi_0\right|, \qquad (25)$$

The phase-angular distribution of this radiation spectrum is as follows:

$$\mathrm{Re}\left(\left|\frac{d^2\mathbf{E}(\mathbf{r},t_0)}{d\Phi_0 d\Omega}\right|\right) = \frac{b}{2\pi}\left|\sum_{\omega=-\infty}^{\omega=\infty}\frac{1-2\frac{A}{B}}{B}\,\frac{\mu}{1-\eta^2}\right.$$

$$\left.(1-3\sin^2\Phi_0) - \frac{A}{B}\sec^2\Phi_0\right|. \qquad (26)$$

Let us represent the function that characterize spectral density of the radiation:

$$\mathrm{Re}\big(S(\omega)\big) = \mathrm{Re}\left(|\mathbf{E}(\mathbf{r},t_0)|^2\right)$$

$$= b^2\left|\sum_{\omega=-\infty}^{\omega=\infty}\frac{A^2}{B^2}\cos^2(\omega\xi_0)\right|. \qquad (27)$$

The phase distribution of pre-defined density of spectral radiation is expressed by the following formula:

$$\frac{dS(\omega)}{d\Phi_0} = b^2\left|\sum_{\omega=-\infty}^{\omega=\infty}\left\{\left[\frac{1}{A} - 2\frac{1}{B}\right]\frac{\mu}{1-\eta^2}\cos^2\Phi_0 - 1\right\}\right.$$

$$\left.\times\frac{A^2}{B^2}\sin 2\Phi_0\right|. \qquad (28)$$

The spectral density of radiation per unit solid angle is defined by the following expression:

$$\frac{dS(\omega)}{d\Omega} = \frac{b^2}{\pi}\left|\sum_{\omega=-\infty}^{\omega=\infty}\left\{\left[\frac{1}{A} - 2\,\frac{1}{B}\right]\frac{\mu}{1-\eta^2}\cos^2\Phi_0 - 1\right\}\right.$$
$$\left.\times\,\frac{A^2}{B^2}\sin\Phi_0\right|. \tag{29}$$

By differentiating relationship (29) with respect to Φ_0, we get the phase-angular distribution:

$$\frac{d^2S(\omega)}{d\Phi_0 d\Omega} = \frac{b^2}{\pi}$$
$$\left[\left(\frac{1}{A} - 2\,\frac{1}{B}\right)\frac{\mu}{1-\eta^2}\cos^2\Phi_0 - 1\right]\times$$
$$\times\left|\sum_{\omega=-\infty}^{\omega=\infty}\left\{\begin{array}{l}\times\left[1 + \left(\frac{1}{A} - 2\,\frac{1}{B}\right)\frac{\mu}{1-\eta^2}\sin^2\Phi_0\right] - \\ -\,\frac{2\mu}{1-\eta^2}\left[1 + \left(\frac{1}{2A} - \frac{1}{B}\right)\frac{\mu}{1-\eta^2}\times\right.\end{array}\right\}\right.$$
$$\left.\times\cos^2\Phi_0\right]\left(\frac{1}{A} - 2\frac{1}{B}\right)\sin^2\Phi_0$$
$$\left.\times\,\frac{A^2}{B^2}\cos\Phi_0\right|. \tag{30}$$

Formula (30) can be used to plot the phase-angular distribution of radiation intensity of a charged particle for the radiation intensity of $I_{\text{lin}} = 10^{19}\,\text{W/cm}^2$ on the phase plane (along the axis of abscissa $\sin\Phi_0$ and along the axis of ordinate $d^2S(\omega)/d\Phi_0 d\Omega$) for different magnetic field strengths (Fig. 4).

It can be seen from Fig. 4, that with increase in magnetic field the spectral density maximum shifts towards $\Phi_0 = \pi, -\pi, \ldots, \pm\pi n$.

Conclusion

The issue of spectral-angular radiation characteristics of a charged particle in the Redmond field is investigated. Expressions are obtained for the radiation intensity of a relativistic charge in case of circularly and linearly polarized electromagnetic wave in this field configuration. Phase and phase-angular distributions of radiation intensity are obtained for a particle moving in an electromagnetic wave with an intensity of $10^{19}\,\text{W/cm}^2$ in the presence of magnetic field of different strengths. The Fourier-image **E** of particle radiation in the Redmond field with linearly polarized electromagnetic wave is calculated. Results of this study can be used for the mathematical interpretation of experiments on the interaction of laser radiation with magnetoplasma.

Conflict of interest

The authors declare that they have no conflict of interest.

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